

McGraw-Hill Ryerson

Functions 11

Exercise and Homework Book

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Overview

Functions play an important role in many activities, from business and economics to the social, medical, and physical sciences. *McGraw-Hill Ryerson Functions 11 Exercise and Homework Book* is designed for students planning to qualify for college or university. It is designed to complement the *McGraw-Hill Ryerson Functions 11* student book.

Exercise and Homework Book Organization

- Chapter 1 distinguishes between functions and relations, including the use of the vertical line test. You will find the domain and range of functions and relations. You will examine various representations of functions, including graphical, function notation, mapping notation, and mapping diagrams. You will identify a variety of methods for determining the minimum and maximum of quadratic functions, and then apply these functions to solve related real world problems. You will practise the skills required for work with radicals. You will use the role of the discriminant to determine the number of solutions for a quadratic equation. You will determine equations of quadratic functions based on their roots and given information. The chapter ends with solving linear-quadratic systems.
- Chapter 2 extends your knowledge of transformations related to quadratic functions (grade 10 math) to general transformations of key base functions. You will then apply the transformation concepts established in this chapter to exponential functions in chapter 3 and trigonometric functions in chapter 5. The chapter begins with methods for identifying equivalent algebraic expressions. You will practise operations with rational expressions, including finding the restrictions on variables. Then, you will explore transformations of functions, including horizontal and vertical translations, reflections, and stretches and compressions. You will also examine combinations of transformations. The chapter ends with determining the inverse of a function graphically and algebraically.
- Chapter 3 introduces a new function, the exponential function. The chapter begins by studying exponential growth in populations and other real-life situations. Then, you will examine exponential decay in relation to half-life and radioactive materials. These concepts are connected to the meaning and applications of negative exponents and rational exponents. You will investigate properties and characteristics of exponential functions, as well as the impact of transformations on the graphs of exponential functions. The final section of this chapter examines mathematical modelling with exponential functions, including solving problems using the functions.
- Chapter 4 extends your knowledge of trigonometry studied in grade 10. The chapter begins by identifying special angles and triangles in terms of the unit circle. You will determine exact values of special angles, co-terminal and related angles, in degree measure, using the primary and reciprocal trigonometric ratios. The latter part of the chapter focuses on solving a variety of two dimensional and three dimensional problems involving right triangles and oblique triangles, including the ambiguous case of the sine law. In the last section of the chapter, you will work on proofs of trigonometric identities, which require the use of the Pythagorean, quotient, and reciprocal identities.
- Chapter 5 applies the concepts from chapter 4 to analyse the graphs of trigonometric functions. In the first section of the chapter, you will examine mathematical modelling with periodic functions. Then, you will graph the sine and cosine functions and identify their properties. You will investigate transformations of the sine and cosine functions and explore mathematical modelling with transformed functions. In the latter part of the chapter, you will collect data representing periodic behaviour and determine equations representing a variety of periodic situations. You will then solve related problems, including those that do not involve angles.

- Chapter 6 introduces discrete functions. The start of this chapter explores sequences as discrete functions and then examines recursive procedures. You will study Pascal's triangle and patterns in Pascal's triangle, as well as applications of the triangle. You will then explore arithmetic and geometric sequences and series and solve a variety of problems involving their application.
- Chapter 7 relates the concepts of discrete functions from chapter 6 to a focus on financial applications. The beginning of the chapter identifies and compares simple and compound interest. Then, you will explore present value, and finally, you will investigate and define ordinary annuities and the present value of an annuity. Throughout the chapter, you will solve a variety of real-world problems involving applications of these concepts, both algebraically and using technology.

Study Guide Features

- Each section begins with Key Concepts that summarize the concepts needed to complete the exercises.
- The sections continue with a worked example that guides you through the skills needed to complete the exercises.
- Exercises are organized into three sections: A (practice), B (connect and apply), and C (extend and challenge).
- A review of all the sections is included at the end of each chapter.
- Each chapter includes Math Contest questions that cover the concepts in the chapter, as well as extend your thinking and combine concepts from previous chapters.
- Selected questions in each section are marked by a star that indicates that full worked solutions are provided at the back of the book. Answers to all other questions are also provided.
- A practice exam at the end of the study guide gives you the opportunity to determine if you are ready for the final examination.

Symbols

SYMBOLS

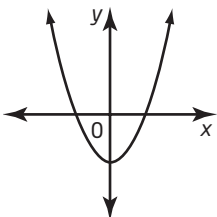
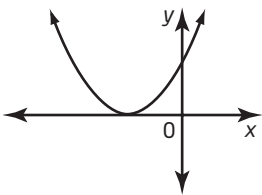
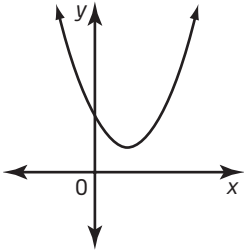
\mathbb{R}	real numbers
\mathbb{N}	natural numbers
\mathbb{Z}	integers
∞	infinity
\in	belongs to
$[a,b]$	$a \leq x \leq b$ (closed interval)
(a,b)	$a < x < b$ (open interval)

GREEK LOWER CASE LETTERS

α	alpha (a)
β	beta (b)
γ	gamma (g)
δ	delta (d) lower case
Δ	delta (d) upper case
λ	lamda (l)
ρ	rho (r)
θ	theta (th)
τ	tau (t)
π	pi (p)
ω	omega (o)

Formulas

ALGEBRA

Factoring Special Polynomials	$x^2 \pm 2xy + y^2 = (x \pm y)^2$ $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$ $x^2 - y^2 = (x - y)(x + y)$
Quadratic Formula	If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Discriminant	<p>Determine the number of solutions to a quadratic equation and the number of zeros of the related quadratic function by using the discriminant.</p> <p>If $b^2 - 4ac > 0$, there are two solutions (two distinct real roots).</p>  <p>If $b^2 - 4ac = 0$, there is one solution (two equal real roots).</p>  <p>If $b^2 - 4ac < 0$, there are no real solutions.</p> 

RULES FOR EXPONENTS

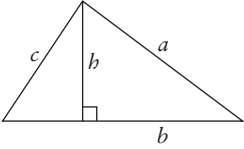

Product	$(x^a)(x^b) = x^{a+b}$
Quotient	$\frac{x^a}{x^b} = x^{a-b}$
Power	$(x^a)^b = x^{ab}$
Power of a Product	$(xy)^a = x^a y^a$
Rational Exponent	$x^{\frac{1}{a}} = \sqrt[a]{x}$
Negative Exponent	$x^{-a} = \frac{1}{x^a}$

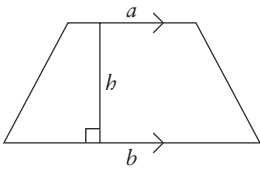
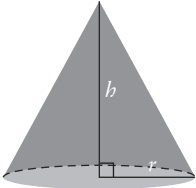
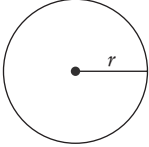
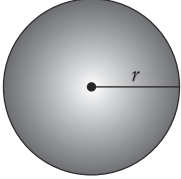
ANALYTIC GEOMETRY

Distance between Two Points Distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$	$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$
Linear Function For a line through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$	Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope y -intercept form of equation: $y = mx + b$, where b is the y -intercept Point-slope form of equation: $y - y_1 = m(x - x_1)$
Quadratic Function Equation for a parabola with vertex (p, q)	$y = a(x - p)^2 + q$ Find the vertex of a quadratic function by – graphing – completing the square: for $f(x) = a(x - h)^2 + k$, the vertex is (h, k) – partial factoring: for $f(x) = ax\left(x + \frac{b}{a}\right) + k$, the x -coordinate of the vertex is $-\frac{b}{2a}$
Circle Equation for a circle centre (h, k) and radius r	$(x - h)^2 + (y - k)^2 = r^2$

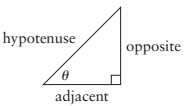
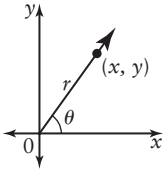
MEASUREMENT

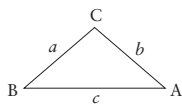
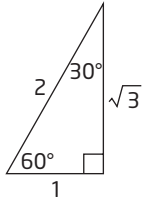
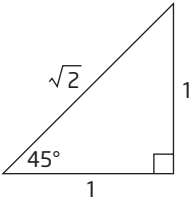
In the following, P represents perimeter, C the circumference, A the area, V the volume, and SA the surface area.

Triangle	 $P = a + b + c$ $A = \frac{1}{2}bh$
Cylinder	 $V = \pi r^2 h$ $SA = 2\pi r h + 2\pi r^2$

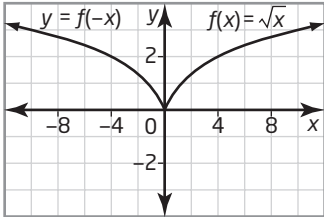
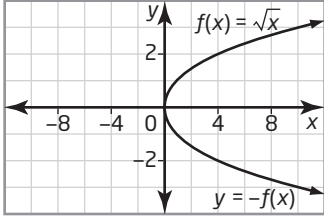
Trapezoid	 $A = \frac{1}{2}(a + b)h$
Cone	 $V = \frac{1}{3}\pi r^2 h$ $SA = \pi r^2 + \pi r s$
Circle	 $C = 2\pi r$ $A = \pi r^2$
Sphere	 $V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

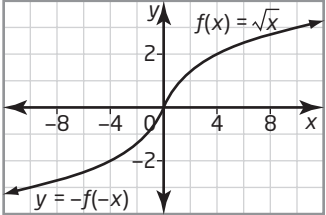
TRIGONOMETRY

Primary Trigonometric Ratios 	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
	$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$

Sine Law 	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
Cosine Law	$a^2 = b^2 + c^2 - 2bc \cos A$	$b^2 = a^2 + c^2 - 2ac \cos B$	$c^2 = a^2 + b^2 - 2ab \cos C$
Fundamental Identities	$\sin^2 \theta + \cos^2 \theta = 1$ $\tan \theta = \frac{\sin \theta}{\cos \theta}$	$1 + \cot^2 \theta = \csc^2 \theta$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$	$\tan^2 \theta + 1 = \sec^2 \theta$
Reciprocal Identities	$\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$ $\sin \theta = \frac{1}{\csc \theta}$	$\cos \theta = \frac{1}{\sec \theta}$ $\tan \theta = \frac{1}{\cot \theta}$
Unit Circle	<p>The coordinates of the point (x, y) on a unit circle are related to θ such that $x = \cos \theta$ and $y = \sin \theta$.</p> $\tan \theta = \frac{y}{x}$ <p>Exactly two angles between 0° and 360° have the same sine ratio.</p> <p>Exactly two angles between 0° and 360° have the same cosine ratio.</p> <p>Exactly two angles between 0° and 360° have the same tangent ratio.</p> <p>Find pairs of related angles using the coordinates of the endpoints of their terminal arms. Use a reference angle in the first quadrant.</p>		
Special Triangles	<p>Determine exact trigonometric ratios for special angles using special triangles:</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>The exact trigonometric ratios for 45° are $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$, and $\tan 45^\circ = 1$.</p>		

TRANSFORMATIONS

<p>Vertical and Horizontal Translations</p>	<p>The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units. If $c > 0$, the graph moves up c units. If $c < 0$, the graph moves down c units.</p> <p>The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units. If $d > 0$, the graph moves to the right d units. If $d < 0$, the graph moves to the left d units.</p> <p>Create a sketch of the graph of any transformed function by transforming the related base function.</p> <p>In general, determine the domain and range of a function of the form $g(x) = f(x - d) + c$ by adding the d-value and the c-value to restrictions on the domain and range, respectively, of the base function.</p>
<p>Reflections</p>	<p>$y = f(-x)$ A point (x, y) becomes $(-x, y)$. The graph is reflected in the y-axis.</p>  <p>Replace x with $-x$ in the expression.</p>
	<p>$y = -f(x)$ A point (x, y) becomes $(x, -y)$. The graph is reflected in the x-axis.</p>  <p>Multiply the entire expression by -1.</p>

	<p>$y = -f(-x)$ A point (x, y) becomes $(-x, -y)$. The graph is reflected in one axis and then the other.</p>  <p>First replace x with $-x$ in the expression, then multiply the entire expression by -1.</p>
Stretches	<p>The graph of $g(x) = af(x)$, $a > 0$, is a vertical stretch or a vertical compression of the graph of $f(x)$ by a factor of a. If $a > 1$, the graph is vertically stretched by a factor of a. If $0 < a < 1$, the graph is vertically compressed by a factor of a.</p> <p>The graph of $g(x) = f(kx)$, $k > 0$, is a horizontal stretch or a horizontal compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$. If $k > 1$, the graph is horizontally compressed by a factor of $\frac{1}{k}$. If $0 < k < 1$, the graph is horizontally stretched by a factor of $\frac{1}{k}$.</p>
Combinations of Transformations	<p>The parameters a, k, d, and c in the function $y = af[k(x - d)] + c$ correspond to the following transformations:</p> <ul style="list-style-type: none"> – a: vertical stretch or compression and, if $a < 0$, a reflection in the x-axis – k: a horizontal stretch or compression and, if $k < 0$, a reflection in the y-axis – d: a horizontal translation to the right or left – c: a vertical translation up or down

DISCRETE FUNCTIONS

Sequences	<p>Given the explicit formula for the nth term, t_n or $f(n)$, of a sequence, write the terms by substituting the term numbers for n. Examples: $t_n = 3n + 2$ and $f(n) = 5n + 3$.</p> <p>An explicit formula for the nth term of a sequence can sometimes be determined by finding a pattern among the terms.</p>
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Recursive	<p>A recursion formula shows the relationship between the terms of a sequence.</p> <p>Represent a sequence by a pattern, an explicit formula, or a recursion formula. Write formulas using function notation.</p> <p>For example: Pattern: 1, 3, 5, 7, ... Explicit formula: $t_n = 2n - 1$ or $f(n) = 2n - 1$ Recursion formula: $t_1 = 1, t_n = t_{n-1} + 2$, or $f(1) = 1, f(n) = f(n-1) + 2$</p>
Arithmetic Sequences	$t_n = a + (n - 1)d$ a = the first term d = the common difference n = the term number
Geometric Sequences	$t_n = a(r)^{n-1}$ a = the first term r = the common ratio n = the term number
Pascal's Triangle	$t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$ n = the horizontal row number r = the diagonal row number
Arithmetic Series	<p>Given the first term, the last term, and the number of terms of an arithmetic series, find the sum of the series using the formula $S_n = \frac{n}{2}(a + t_n)$ or the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$.</p> <p>Given the first terms of an arithmetic series, find the sum of the first n terms using the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$.</p>
Geometric Series	<p>The formula for the sum of the first n terms of a geometric series with first term a and common ratio r is $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$.</p>

FINANCIAL APPLICATIONS

Interest	$I = Prt$ P = principal, in dollars r = annual interest rate, as a decimal t = time, in years $A = P + I$ A = amount of an account earning simple interest
Compound Interest	$A = P(1 + i)^n$ P = principal i = interest rate per compounding period n = number of compounding periods

Present Value	$PV = \frac{FV}{(1 + i)^n}$ <p> <i>FV</i> = future value <i>n</i> = number of compounding periods <i>i</i> = interest rate, as a decimal per compounding period </p>
Annuities	$A = \frac{R[(1 + i)^n - 1]}{i}$ <p> <i>R</i> = regular payment <i>i</i> = interest rate per compounding period, as a decimal <i>n</i> = compounding periods </p>
Present Value of an Annuity	$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$ <p> <i>R</i> = regular withdrawal <i>i</i> = interest rate per compounding period, as a decimal <i>n</i> = number of compounding periods </p>

Tips for Success in MATH

IN CLASS

- **Listen** carefully to your teacher.
- **Focus** and **pay attention** to examples and their solutions.
- **Copy** all notes and examples. **Think** about the solutions.
- **Ask** questions when you don't understand.
- Use class time **efficiently**. Begin homework when time is given in class.
- Use **proper form** when solving questions. Don't skip key steps.
- **Ask** about homework questions you had difficulty with. Copy solutions to questions you didn't understand.
- **Concentrate**, think, pay attention, ask questions, and focus on the given examples in class.
- Create a **Math Vocabulary/Formula List** section in your notebook to record all important words, definitions, and formulas.
- Read the **instructions** to a question so that you are familiar with the wording.

AT HOME

- **Complete** your math homework **every night**. Try **every** question assigned. Refer to the text and class example solutions to help you.
- **Check** your answers with those at the back of the book. Highlight homework questions that you had **difficulty** with or **could** not do.
- **Review** examples and notes. Use them to help you with your homework.
- **Memorize** all formulas, definitions, vocabulary, and steps/procedures for solving longer questions.
- For each lesson and chapter **prepare a summary study sheet** that contains: important **formulas, definitions, vocabulary, procedures for solutions**, and **solutions** to questions from the homework that **you** found to be difficult.
- **Update** your study sheet after each lesson. This will save time when studying for tests and the exam.

PREPARING FOR A TEST

- Begin studying and reviewing at least **3 days prior** to a test. **Don't wait until the night before**.
- Review the **summary study sheets** that you have prepared.
- Review each section to be tested in the chapter. **REDO** homework questions that you found difficult.
- Do **all review** questions assigned. Try **extra homework** questions that were not assigned.
- **Memorize** formulas, definitions, vocabulary, and steps for longer solutions.
- **Study the wording of questions** so that you will understand the instructions on a test.
- Try to **categorize** the types and variety of questions done over the entire chapter.
- **DO NOT CRAM!** Studying for a math test should be easy if you have been keeping up throughout the chapter.