Functions 11 Practice Exam

Part A Short Answer

1. Evaluate $\left(-\frac{3}{5}\right)^4$. Express your answer as a fraction.

2. Simplify $(-7)^{43} \times (-7)^{52} \div (-7)^{30}$. Express your answer as a power.

3. Express $\left(\frac{x^4}{y^6}\right)^7$ as a quotient of powers.

4. Express 5^{-9} using a positive exponent.

5. Express $\sqrt[5]{48}$ as a power.

6. Evaluate $\left(\sqrt[5]{-32}\right)^3$.

7. What type of function is represented by $y = 8^x$?

8. State the y-intercept of the graph of $y = \left(\frac{1}{5}\right)^x$.

9. State the equation of the horizontal asymptote of the graph of $y = 9^x$.

10. State the range of the graph of $y = \left(\frac{3}{4}\right)^x$.

11. State the equation that represents the graph of $y = 2^x$ after it is translated 3 units to the right.

12. State the equation that represents the graph of $f(x) = 6^x$ after it is compressed horizontally by a factor of $\frac{1}{2}$.

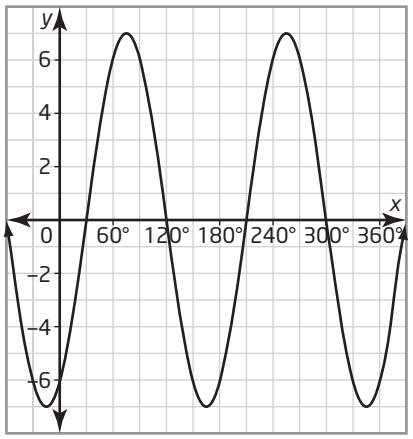
13. Is the graph of $y = 5^x$ increasing or decreasing?

14. Express $y = \left(\frac{1}{8}\right)^x$ in terms of an equivalent equation using base 2.

15. State the equation of the horizontal asymptote for $y = 3^{x+2} - 1$.

16. For each graph, write a corresponding equation in terms of sine.

a)



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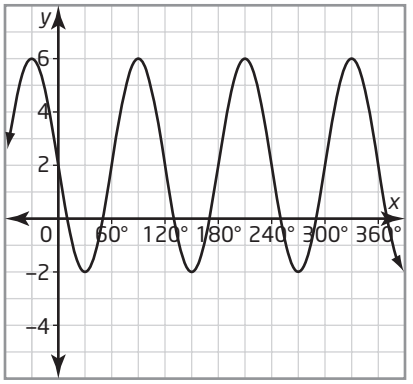


Part B Extended Answer

17. Simplify $\frac{9x}{x^2 - 5x + 6} - \frac{4x}{x^2 + x - 12}$ and state any restrictions on the variable.

18. Determine whether $f(x) = (2x - 3)^2 - (4x + 5)(x - 7)$ is equivalent to $g(x) = \frac{11x^2 - 33x - 44}{x + 1}$. Justify your answer.

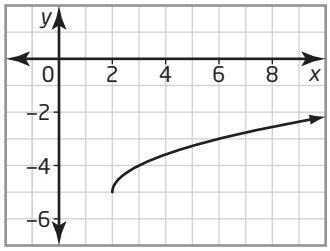
19. Identify the parameters and use them to describe the transformations that must be applied to the graph of each base function to obtain the transformed function. Write the transformed equation in simplified form.

a) $f(x) = \sqrt{x}$, to obtain the transformed function $y = -f[4(x + 3)] - 1$

b) $f(x) = \frac{1}{x}$, to obtain the transformed function $y = \frac{1}{2}f[-(x + 5)] + 2$

20. Each graph represents a transformation of one of the base functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. State the base function and the equation of the transformed function.

a)



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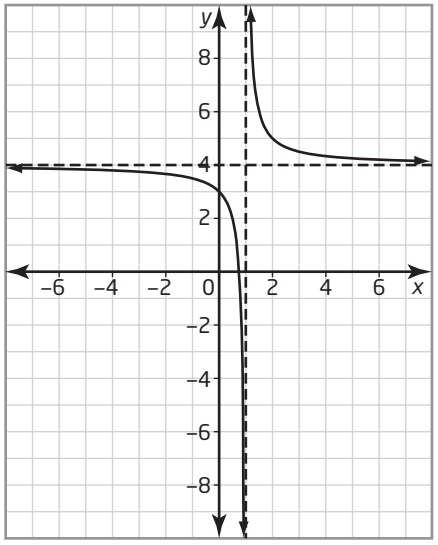

21. Consider these functions:

i) $f(x) = \frac{1}{2 - x}$

ii) $f(x) = 3x^2 + 1$

For each function,

a) state the domain and range of $f(x)$

b) evaluate $f(-5)$

c) determine the equation of the inverse function

22. Determine the vertex of each quadratic function by completing the square. Verify your answer by using partial factoring. State if the vertex is a maximum or minimum.

a) $f(x) = -3x^2 + 6x + 4$

b) $f(x) = 2x^2 - 8x + 7$

23. Find an equation in standard form for the quadratic function with zeros $x = 3 \pm \sqrt{5}$ and containing the point $(2, -8)$.

24. State the value of the discriminant and determine the number of points of intersection of each. Justify your answers.

a) the line $y = 2x + 1$ and the quadratic function $f(x) = x^2 - 3x + 3$

b) the line $y = -3x + 2$ and the quadratic function $f(x) = x^2 - 4x + 9$

25. Andrew and David are on a canoe trip between three campsites located at different points on a lake. Starting at campsite A, they canoe 11 km to campsite B. From that site, they canoe 20 km to campsite C. If the angle from campsite A to campsite C to campsite B is 30°, how far must Andrew and David canoe in order to return directly to campsite A?

26. A squash player hits a ball 2.3 m to the side wall. The ball rebounds at an angle of 100° and travels 3.1 m to the front wall. How far is the ball from the player when it hits the front wall? Assume the player does not move after her shot.

27. State the amplitude, the period, the phase shift, and the range of the graph of each sinusoidal function. Then, sketch a graph of the function.

a) $y = 5 \sin [4(x - 30°)] - 2$

b) $y = -2 \sin [3(x + 45°)] + 5$

28. Sarah is standing on a balcony that is 10 m above the ground. From her position, the angle of elevation to the top of a tower across the street is 45°. Liz is standing on a balcony that is 20 m higher than Sarah's. From Liz's position, the angle of elevation to the top of the same tower is 30°. Determine the height of the tower. Express your answer using exact values.

29. Prove each.

a) $\dfrac{\csc \theta + \cot \theta}{\csc \theta - \cot \theta} = \dfrac{1 + 2 \cos \theta + \cos^2 \theta}{\sin^2 \theta}$

b) $\dfrac{\cos \theta}{\sec \theta} - \dfrac{\sin \theta}{\cot \theta} = \dfrac{\cot \theta \cos \theta - \tan \theta}{\csc \theta}$

30. The equation $T = 18 \sin [30(t - 4)] + 11$ models the average monthly temperature for a city in southwestern Ontario. In this equation, t denotes the number of months, with January represented by 1, and T is the temperature, in degrees Celsius.

a) Determine the city's maximum average monthly temperature. When does this occur?

b) Determine the city's minimum average monthly temperature. When does this occur?

c) What is the difference between the maximum and minimum average monthly temperatures?

d) What is the relationship between your answer in part c) and the coefficient of the sine term in the equation?

e) What is the sum of the maximum and minimum average monthly temperatures?

f) What is the relationship between your answer in part e) and the value of the constant term in the equation?

g) What is the average monthly temperature in October?

h) When is the average monthly temperature 20 °C?

31. Write $t_{18, 8}$ as the difference of two terms, each in the form $t_{n, r}$.

32. Use Pascal's triangle to expand $(4x - y)^5$.

33. Determine the number of terms in each arithmetic sequence.

a) 16, 23, 30, …, 583

b) $x + 2, x - 1, x - 4, \ldots, x - 52$

34. Determine a and d and then write the formula for the nth term of each arithmetic sequence with $t_8 = 79$ and $t_{21} = 235$.

35. Determine the number of terms in the geometric sequence $-36, -18, -9, \ldots, -\dfrac{9}{128}$.

36. Determine an arithmetic series such that the sum of the first five terms of the series is 85 and the sum of the first six terms of the series is 123.

37. A jogger runs 300 m in the first minute. The distance the jogger covers decreases by 20 m in each succeeding minute. What distance does the jogger cover in the 7th minute?

38. A ball bounces to $\frac{5}{6}$ of its height when dropped on a hard surface. Suppose the ball is dropped from a height of 40 m.

a) What height does the ball bounce back up to after the 7th bounce?

b) What is the total distance travelled by the ball after 10 bounces?

39. There is a legend that the inventor of chess chose the following for his reward: 1 grain of wheat on the first square, 2 grains of wheat on the second square, 4 on the third, 8 on the fourth, and so on, for all 64 squares on the chessboard. Find an expression for the amount of wheat required to fulfill his request.

40. Identify whether each series is arithmetic or geometric. Justify your answer. Then, determine the sum of each series.

a) $100 + 90 + 80 + \cdots - 220$

b) $1 + 3 + 9 + \cdots + 2187$

41. In a geometric series, $t_2 = 10$ and $t_5 = 1250$. Determine t_4 and S_6.

42. Matteo is very excited about his new sports car! Although he paid \$24 800 for the car, its resale value will depreciate (decline) by 22% of its current value every year. The equation relating the car's depreciated value, v, in dollars, to the time, t, in years, since its purchase is $v(t) = 24\,800(0.78^t)$.

a) Explain the significance of each part of this equation.

b) How much will Matteo's car be worth in
i) 2 years?
ii) 5 years?

c) Explain why this is an example of exponential decay.

d) How long will it take for the value of Matteo's sports car to decline to 40% of its original cost?

43. Timothy invests \$2500 in an 18-month term deposit that pays simple interest at an annual rate of 3.5%. How much interest does Timothy earn?

44. Amanda buys a \$5000 guaranteed investment certificate (GIC) that will earn simple interest at the rate of 5.8% per year for 7 years.

a) Determine the amount of interest that Amanda will earn on the GIC.

b) What is the amount of the GIC at the end of 7 years?

45. Victoria invests \$5000 in a registered retirement savings plan (RRSP) that earns interest at the rate of 6.95% per annum, compounded monthly. What is the value of the RRSP at the end of 7 years?

46. Katip wants to have \$16 000 in 5 years in order to buy a new motorcycle. If Katip has \$10 000 to invest today, what rate of interest, to the nearest hundredth of a percent, compounded quarterly, does he need to achieve his goal?

47. On his 21st birthday, Carmine receives \$5000 from his grandparents, the accumulated amount of an investment they made for him when he was born. What was the original amount of the investment if it earned interest at the rate of 8.75% per year, compounded monthly?

48. How long does it take \$100 to grow to \$500 if the amount is invested at a rate of 13.5% per annum, compounded semi-annually?

49. David invests \$25 000 in an annuity that earns 7.5% interest per year, compounded semi-annually. If the annuity is to pay David twice a year for 10 years, starting 6 months from now, what is the amount of each semi-annual payment?