Name: _____ Date: _____

3.3 Rational Exponents

BLM 3–4

1. Evaluate.

a) $(27)^{\frac{2}{3}}$ **b)** $(125)^{-\frac{1}{3}}$

c) $\left(\frac{1}{16}\right)^{-\frac{3}{4}}$ **d)** $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$

2. Simplify the following expressions. Leave all answers with positive exponents.

a) $x^{\frac{3}{5}} \times x^{\frac{2}{5}}$

b) $n^{\frac{1}{2}} \times n^{\frac{1}{3}} \times n^{\frac{1}{4}}$

c) $\left(9x^2y^4\right)^{\frac{1}{2}} \times \left(125x^6y^3\right)^{\frac{2}{3}}$

d) $\left(27y^3\right)^{\frac{1}{3}} \times \left(\frac{1}{16y^4}\right)^{-\frac{3}{4}}$

3. Simplify the following expressions. Leave all answers with positive exponents.

a) $(100xy)^{\frac{1}{2}}\left(25x^3y^2\right)^{-\frac{1}{2}}$

b) $\left(27x^6\right)^{\frac{2}{3}} \div \left(9x^4\right)^{\frac{1}{2}}$

c) $\left(64x^2y^4\right)^{\frac{1}{2}} \div \left(16x^2y^4\right)^{\frac{1}{4}}$

d) $\frac{x^{-\frac{2}{3}}}{x^{-\frac{4}{5}}}$

4. The surface area, S, of a sphere is given by the equation $S = 4\pi r^2$. The volume, V, is given by the equation $V = \frac{4}{3}\pi r^3$.

a) What is the ratio of the surface area of a sphere to its volume?

b) Are there any restrictions on the variable? If so, state the restrictions.

c) Describe the type of function that represents the ratio in part a).

d) What does this function indicate about the ratio?

5. A square-based prism has a height that is twice the side length of the base.

a) Write an expression for the surface area, S, of the prism in terms of the side length, x, of the base.

b) Determine the dimensions of the prism if the area of its base is

i) 100 cm^2 **ii)** 25 m^2

c) Find the total surface area of each prism in part b).

6. Refer to question 5.

a) Write an expression for the volume, V, of the prism in terms of the side length, x, of the prism's base.

b) What is the volume of each prism in part b) of question 5?

7. Refer to questions 5 and 6.

a) Find the ratio of the surface area to volume of each prism in part b) of question 5.

b) Find an expression for the surface area to volume ratio for the prism in part a) of question 5.

c) How are your answers to parts a) and b) related? Explain.

8. Escape speed is the upward speed needed for an object to escape the gravitational pull of a celestial body. Escape speed, in metres per second, is given by the formula $v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$, where G is the universal gravitational constant of 6.67×10^{-11}; M is the mass of the body, in kilograms; and r is the radius of the body, in metres.

a) Determine the escape speed for Earth, given that the mass of Earth is 5.98×10^{24} kg and the radius of Earth is 6.38×10^6 m.

b) Determine the escape speed for the moon, given that its mass is 7.35×10^{22} kg and its diameter is 3476 km.

c) Rewrite the formula to express the radius, r, of a celestial body in terms of the other variables.

d) The escape speed at the edge of a black hole is 3×10^8 m/s (the speed of light). Use the formula from part c) to find the radius to which Earth would need to be compressed in order to form a black hole.

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