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Chapter 3 Answers

Prerequisite Skills

1. Answers may vary.

2. a) x^{10} **b)** y^{19} **c)** m **d)** h^{-2} **e)** h^5
f) x^5 **g)** y^3z^6 **h)** x^4 **i)** x^6y^6 **j)** $-x$

3. a) 243 **b)** 512 **c)** −1 **d)** 125
e) 625 **f)** 2 **g)** 256 **h)** 9

4. a)

x	y
4	$3^4 = 81$
3	27
2	9
1	3
0	1

b) Answers may vary. Sample answer: The values decrease by a factor of 3 as you proceed down the chart.

c)

x	y
4	$3^4 = 81$
3	27
2	9
1	3
0	1
−1	$\frac{1}{3}$
−2	$\frac{1}{9}$
−3	$\frac{1}{27}$

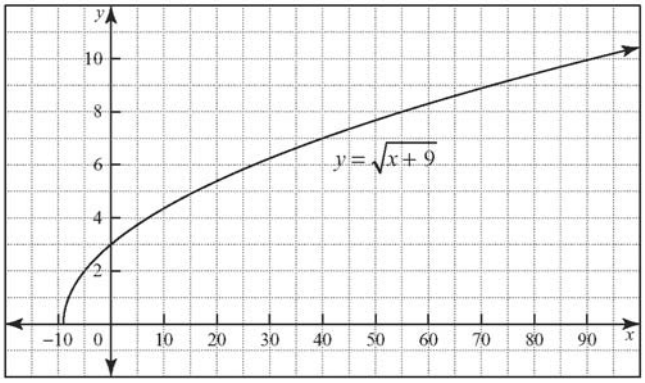
d) Answers may vary. Sample answer: Extending the pattern of dividing by 3 gives an expression of $\frac{1}{3}$, which must be equivalent to 3^{-1}.

5. a) 1 **b)** $\frac{1}{3^3}$ **c)** −1 **d)** $\frac{1}{4^3}$
e) $\frac{9}{4}$ **f)** $\frac{1}{(-2)^3}$ **g)** $\frac{1}{3^2 \times 2^3}$ **h)** $\frac{1}{6}$

6. a) $\frac{1}{-3xy}$ **b)** 1 **c)** $\frac{a^2}{b^2}$ **d)** $\frac{1}{3^3x^6y^9}$

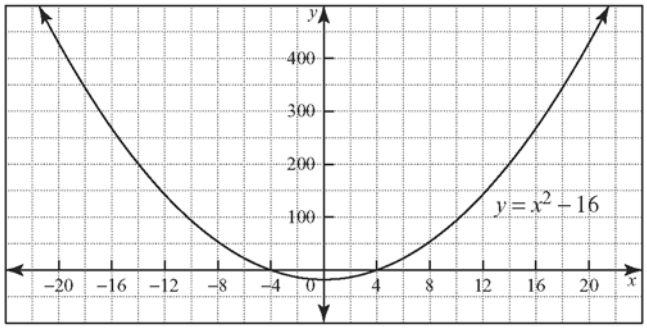
7. i) domain $\{x \in \mathbb{R}\}$ **ii)** range $\{y \in \mathbb{R}, y \le 4\}$
iii) x-intercepts at (9, 0) and (0, 4); y-intercept at (0, 0)

8. a)



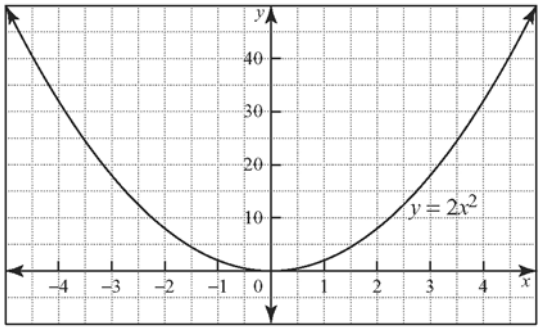
i) domain $\{x \in \mathbb{R}, x \ge -9\}$ **ii)** range $\{y \in \mathbb{R}, y \ge 0\}$
iii) (0, 0) is the only intercept

b)



i) domain $\{x \in \mathbb{R}\}$ **ii)** range $\{y \in \mathbb{R}, y \ge -16\}$
iii) x-intercepts at (−4, 0) and (4, 0); y-intercept at (0, −16)

9. a)



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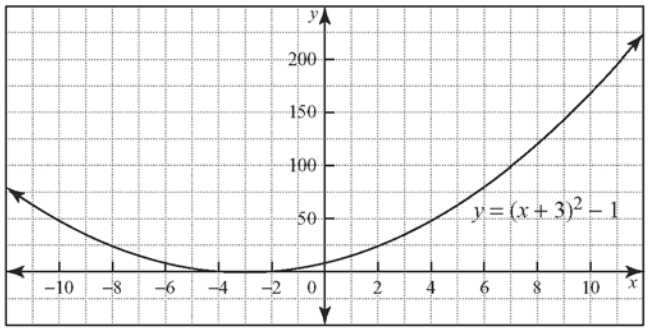
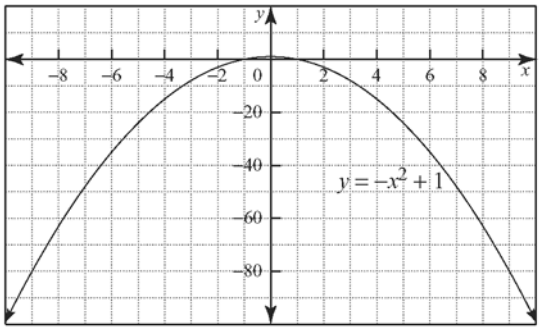
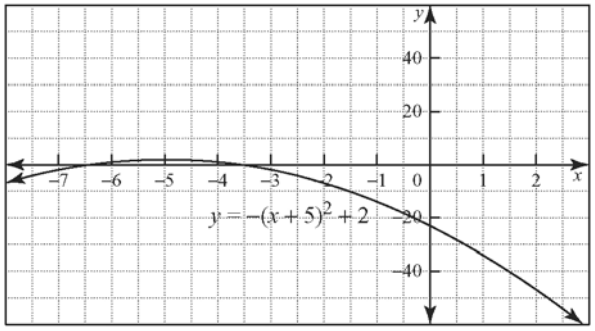
b)



c)



d)



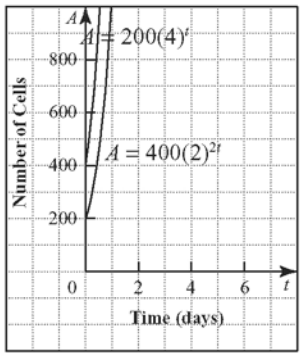
3.1 The Nature of Exponential Growth

1. a) 1, 2, 4, 8, 16, 32, 64, …
b) $N = 2^t$, where N is the number of people who are told the rumour and t is the number of 5-min intervals that have passed since the rumour started.
c) 1, 3, 7, 16, 31, 63, …
d) approximately 40 min

2. a) 125 000 **b)** 15 625 **c)** next Wednesday

3. a) $A = 400(2)^{2t}$; $A = 200(4)^t$

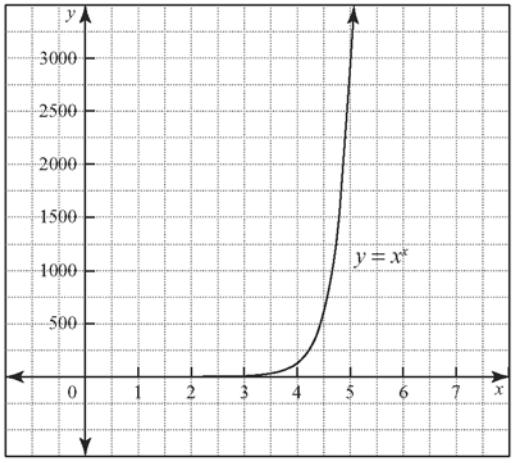
b)



c) Answers may vary. Sample answer: The functions have a similar growth rate, but the graph of $A = 400(2)^{2t}$ is always a larger value for any value of t.

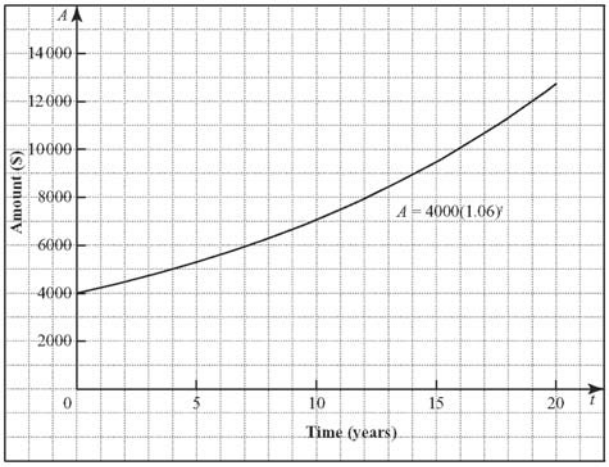
4. a) Answers may vary. Sample answer: The shape should look exponential.

b)



c) Answers may vary.

5. a) i) \$4494.40 **ii)** \$7163.39 **iii)** \$12 828.54
b) At $n = 0$, $A = \$4000$, which is the initial value of the investment.
c)



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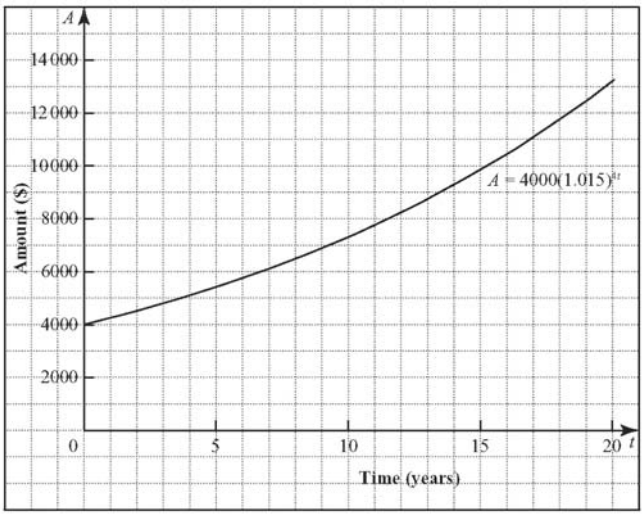
d) Answers may vary. Sample answer: The relation is exponential, because the growth rate increases as n increases.

e) approximately 27.6 years

6. a) i) \$4505.97 **ii)** \$7256.07 **iii)** \$13 162.65

b) At $t = 0$, $A = \$4000$, which is the initial value of the investment.

c)



d) Answers may vary. Sample answer: The relation is exponential, because the growth rate increases as n increases.

e) approximately 27.0 years

7. a) Answers may vary. Sample answer: The more frequently that interest is paid, the faster the investment will grow.

b) Answers may vary. Sample answer: Because interest is paid more frequently than is the case in the second investment, this investment can be expected to grow faster.

c) approximately 26 years 11 months.

3.2 Exponential Decay: Connecting to Negative Exponents

1. a) 9 **b)** $\frac{1}{4}$ **c)** $\frac{1}{16}$ **d)** 27

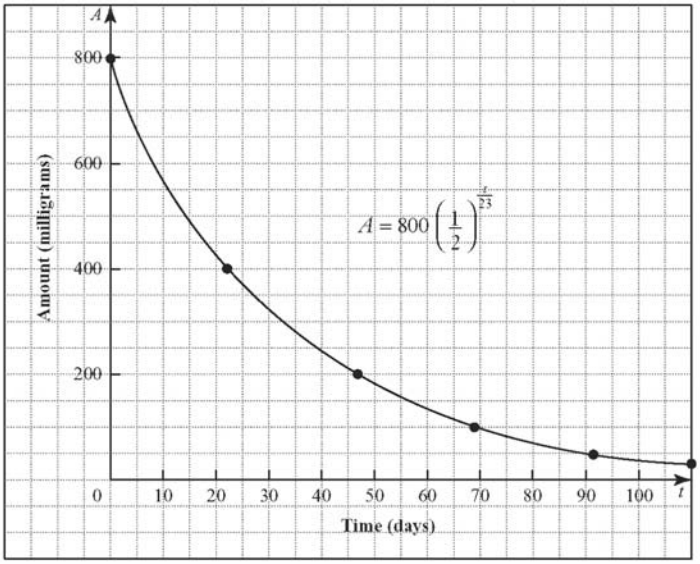
2. a) $\frac{y^4}{9x^2}$ **b)** $-\frac{8}{a^5}$ **c)** $\frac{k^2}{2}$ **d)** $\frac{3y}{x}$

3. a) a^6y^6 **b)** $16x^4$ **c)** x^9y^6 **d)** $\frac{4b^8}{9a^6}$

4. a)

Day	Amount of Material Present (mg)
0	800
23	400
46	200
69	100
92	50
115	25

b) and **d)**



c) $A = 800\left(\frac{1}{2}\right)^{\frac{t}{23}}$

Answers may vary. Sample answer: The fit is an exact fit to the data.

5. a)

Month	Amount ($)
0	10 000.00
1	9 800.00
2	9 604.00
3	9 411.92
4	9 223.68
5	9 039.21
6	8 858.42
7	8 681.26
8	8 507.63
9	8 337.48
10	8 170.73
11	8 007.31
12	7 847.17

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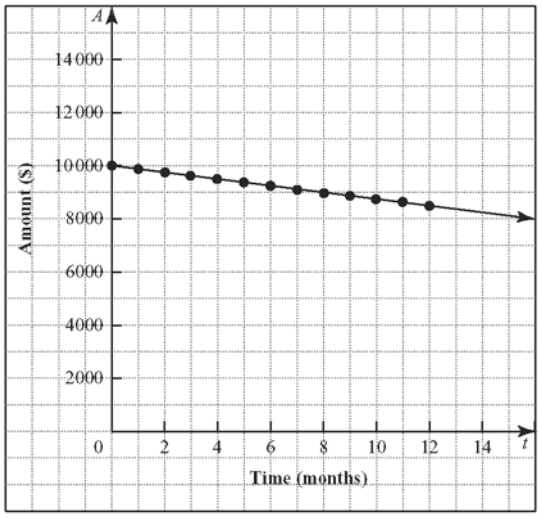
b)



c) Answers may vary. Sample answer: This is an example of a decreasing exponential function.

d) approximately 17.7 months

6. 2.7 N

7. a) 5.8×10^{-8} N

b) Answers may vary. Sample answer: The negative sign indicates that the force is an attractive force.

8. a) 2.57×10^{-47} N

b) Answers may vary. Sample answer: Because of the large difference in values, it can be seen that a gravitational force is much smaller (and thus weaker) than the electric force of attraction within an atom.

3.3 Rational Exponents

1. a) 9 **b)** $\frac{1}{5}$ **c)** 8 **d)** $\frac{9}{4}$

2. a) x **b)** $n^{\frac{13}{12}}$ **c)** $75x^5y^4$ **d)** $24y^4$

3. a) $\frac{2}{xy^{\frac{1}{2}}}$ **b)** $3x^2$ **c)** $4x^{\frac{1}{2}}y$ **d)** $x^{\frac{2}{15}}$

4. a) $\frac{S}{V} = \frac{3}{r}$ **b)** $r > 0$

c) Answers may vary. Sample answer: This function is a reciprocal function.

d) Answers may vary. Sample answer: This tells us that the volume grows faster than the surface area as the radius increases.

5. a) $S = 10x^2$

b) i) 10 cm by 10 cm by 20 cm **ii)** 5 m by 5 m by 10 m

c) i) 1000 cm^2 **ii)** 250 m^2

6. a) $V = 2x^3$

b) i) 2000 cm^3 **ii)** 250 m^3

7. a) $\frac{S}{V} = \frac{5}{x}$

b) i) $\frac{S}{V} = \frac{1}{2}$ **ii)** $\frac{S}{V} = 1$

c) Answers may vary. Sample answer: The ratios based on the calculated values are identical to the ratio that is obtained by the formula in part a) once the value of x is substituted into the expression.

8. a) 11 182 m/s **b)** approximately 2375 m/s

c) $r = \frac{2GM}{\left(v_{\text{escape}}\right)^2}$ **d)** 0.008 864 m (approximately 9 mm)

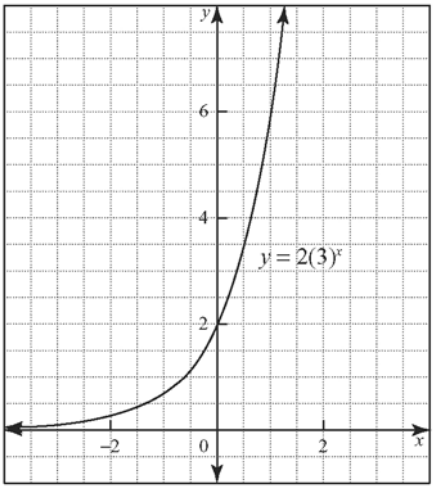
3.4 Properties of Exponential Functions

1. a) $A = 2000\left(\frac{1}{2}\right)^{\frac{t}{14}}$

b) i) 250 mg **ii)** 0.49 mg

c) approximately 71 h

2. a) i)



ii) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$

iii) no x-intercept; y-intercept at $\left(0, \frac{1}{2}\right)$

iv) always increasing

v) $y = 0$

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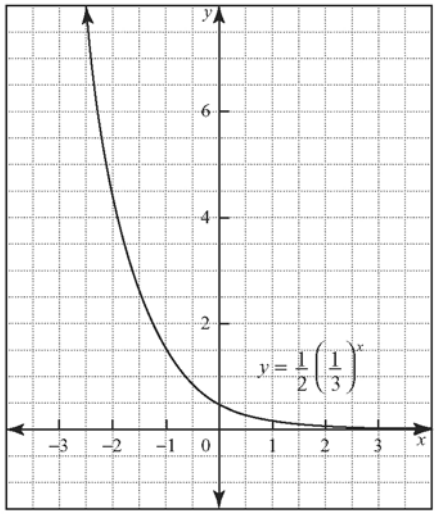
b) i)



ii) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}, y > 0\}$

iii) no x-intercept; y-intercept at $\left(0, \frac{1}{2}\right)$

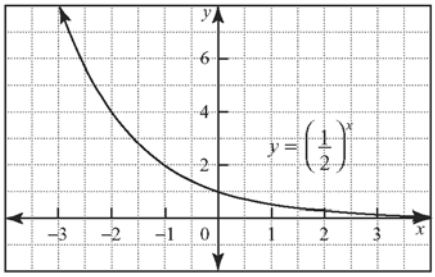
iv) always decreasing **v)** $y = 0$

3. a) i) 5.25 m **ii)** 5.8125 m

b) Answers may vary. Sample answer: The tree will never reach 6 m, because $h = 6$ is the equation of the horizontal asymptote of the function used to model the tree's growth.

4. $y = 3\left(\frac{1}{2}\right)^x$

5. a)



b) Answers may vary. Sample answer: They both are decreasing functions for all of x, and have an asymptote at $y = 0$. The function $y = \frac{1}{2x}$ has a vertical asymptote of $x = 0$ that the first does not have. As well, the function $y = \frac{1}{2x}$ drops below the x-axis but the first does not.

6. a) i) 1.37 m **ii)** 0.38 m

b) Answers may vary. Sample answer: The factor 5 represents the height from which the ball is dropped. The factor 0.65 represents the percent (as a decimal) of the height of the previous bounce that the ball will reach on the next bounce.

c) Answers may vary. Sample answer: If each bounce were more elastic, the factor 0.65 would increase to a value closer to 1.

7. a) $y = 320(0.60)^t$, where t is the number of half-hour increments. Answers may vary. Sample answer: The assumption is made that the decrease of medication in the bloodstream is at a constant rate of 60% each half hour.

b) i) 41.472 mg **ii)** 14.93 mg

c) approximately 2:15 p.m.

3.5 Transformations of Exponential Functions

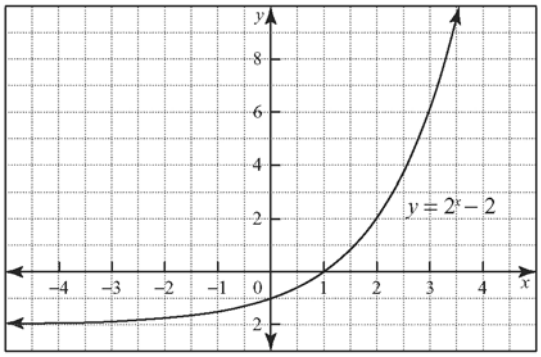
1. a) Answers may vary. Sample answer: a translation 2 units down

b) Answers may vary. Sample answer: a translation 3 units to the left

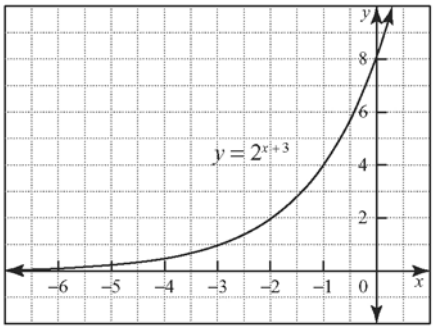
c) Answers may vary. Sample answer: a horizontal compression by a factor of $\frac{1}{2}$

d) Answers may vary. Sample answer: a vertical stretch by a factor of 3, a translation 1 unit to the right, and a translation 1 unit up

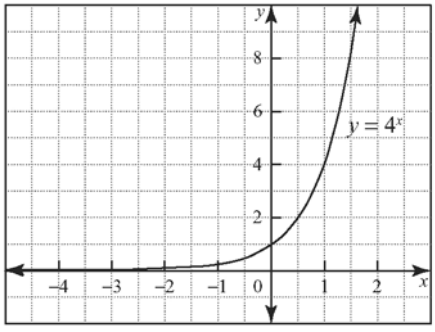
2. a)



b)



c)



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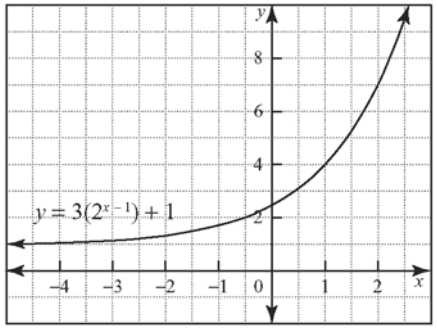
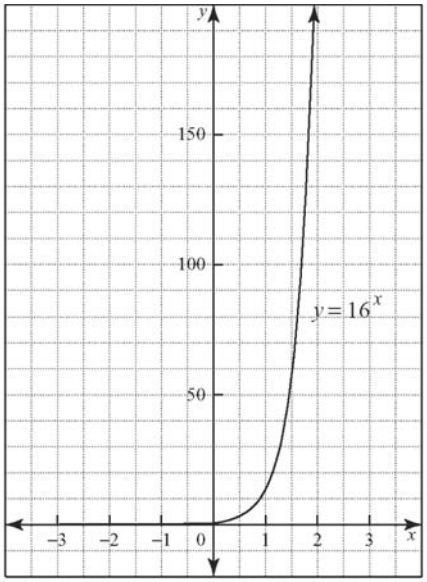
d)



3. a)



b) i) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$

ii) no x-intercept; y-intercept at (0, 1)

iii) $y = 0$

4. a) $y = 4^{2x}$ **b)** $y = 2^{4x}$

c) Answers may vary. Sample answer: The function $y = 4^{2x}$ is a horizontal compression of $y = 16^2$ by a factor of $\frac{1}{2}$. The function $y = 2^{4x}$ is a horizontal compression by a factor of $\frac{1}{4}$.

d) Answers may vary. Sample answer: The three functions can be graphed on the same set of axes to show that they all overlap.

5. a) Answers may vary. Sample answer: $y = 3(2^x) + 1$

b) Answers may vary. Sample answer: There are many exponential functions that have these properties, so this is not a unique answer.

6. a) Answers may vary. Sample answer: Her graph is not correct because she has not applied the transformations in the correct order.

b) Answers may vary. Sample answer: As long as the stretch and reflection are done before the translations are done, there is more than one order that will result in a correct graph.

7. Answers may vary.

3.6 Making Connections: Tools and Strategies for Applying Exponential Models

1. a) iii) **b)** ii) **c)** iv) **d)** i)

2. a) $A = 2^t$, where $t = 0$ represents the amount at the end of the first week, and t is measured in weeks

b) i) \$0.16 **ii)** \$2.56 **iii)** \$655.36

c) Answers may vary. Sample answer: The values would all be multiplied by 2.

3. a) $a = 7000$, $b = 1.06$

b) Answers may vary. Sample answer: The value of a represents the amount invested. The value of b relates to the interest rate being applied to the investment in the expression $1 + i$.

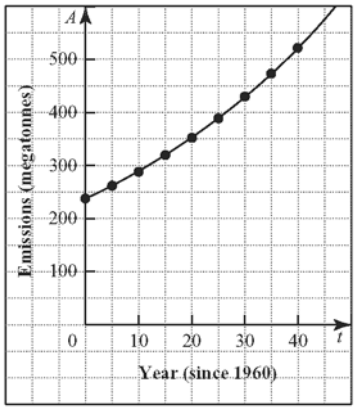
c) $A = 7000(1.06)^t$

d) Answers may vary.

4. a)

Year	CO_2 Emissions (Mt)
0	189
5	254
10	334
15	381
20	414
25	385
30	432
35	452
40	505

b) and **d)**



c) $y = 237(1.02)^t$

Answers may vary. Sample answer: The fit is not perfect, but it does seem to approximate the data.

e) approximately 704 Mt

f) Answers may vary. Sample answer: There are too many factors that could change over 15 years for the calculated value to be accurate.

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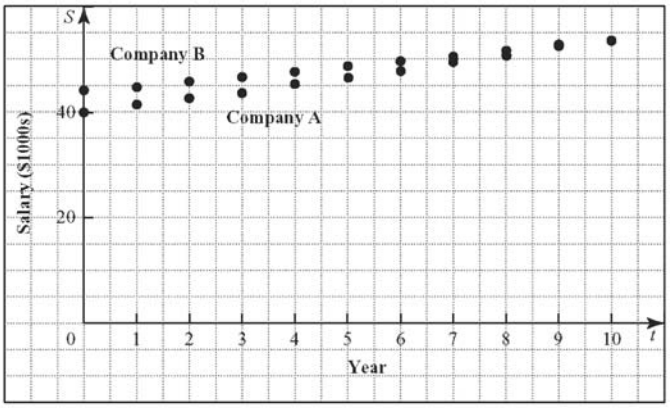
5. a)

Company A	
Year	**Salary ($)**
0	40 000.00
1	41 200.00
2	42 436.00
3	43 709.08
4	45 020.35
5	46 370.96
6	47 762.09
7	49 194.95
8	50 670.80
9	52 190.93
10	53 756.66

Company B	
Year	**Salary ($)**
0	44 000.00
1	44 880.00
2	45 777.60
3	46 693.15
4	47 627.02
5	48 579.56
6	49 551.15
7	50 542.17
8	51 553.01
9	52 584.07
10	53 635.75

b)

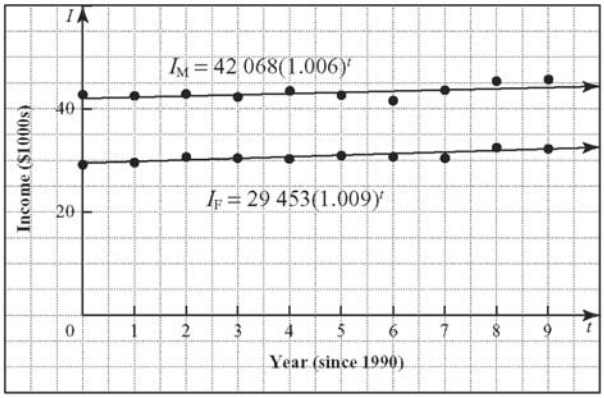


c) approximately 9.8 years **d)** approximately $53 440
e) i) Company B **ii)** Company A

6. a)

	Average Income ($)	
Year	**Female**	**Male**
0	29 050	42 913
1	29 654	42 575
2	30 903	42 984
3	30 466	42 161
4	30 274	43 362
5	30 959	42 338
6	30 606	41 897
7	30 484	43 804
8	32 553	45 070
9	32 026	45 800

b) and **e)**



c) Female: $y = 29\,453(1.009)^t$; Male: $y = 42\,068(1.006)^t$
d) Answers may vary. Sample answer: Women's salaries are growing at a slightly faster rate than men's salaries in these data.
Answer may vary. Sample answer: The fit for both equations is not perfect, but both seem to model the data to some extent.
f) Female: $36 848; Male: $48 854
g) Answers may vary. Sample answer: It is unlikely that these trends will continue exactly as seen, because it is not possible for the modelling equations to predict periods of economic slowdown or rapid economic growth.

Chapter 3 Review

1. C
2. a)

Day	Number of Items
0	25
1	50
2	100
3	200
4	400
5	800
6	1600
7	3200

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b) Answers may vary. Sample answer: Each term has a value that is a constant multiplication of the term before it.

3. a) ii) **b)** i) **c)** iii)

4. a) $\frac{1}{5}$ **b)** 9 **c)** $\frac{3}{16}$

d) $\frac{5}{3}$ **e)** 4 **f)** $\frac{25}{4}$

5. a) $A = 2000\left(\frac{1}{2}\right)^{\frac{t}{10}}$

b) 125 mg **c)** 80 days

6. a) $A = 2000\left(2\right)^{-\frac{t}{10}}$

b) Answers may vary. Sample answer: You can graph the two equations and notice that they overlap each other.

7. a) $\frac{1}{a}$ **b)** $2x^3y^7$ **c)** $\frac{32b^4}{a^6}$ **d)** $45u^2v^{10}$

8. a) $\frac{1}{3}$ **b)** $\frac{3}{2}$ **c)** 125 **d)** 8

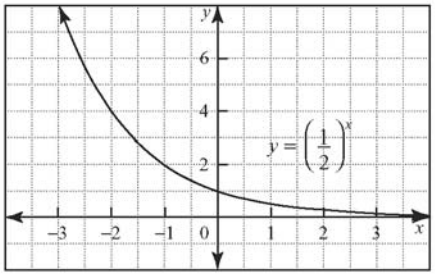
9. a) \$18 061.11 **b)** \$18 193.97

c) Answers may vary. Sample answer: As the frequency of interest payments increases, the amount that the investment is worth increases, all other factors being equal.

10. a) \$7401.22 **b)** \$10 955.62

c) Answers may vary. Sample answer: As time increases, the amount that the investment is worth increases, all other factors being equal.

11.



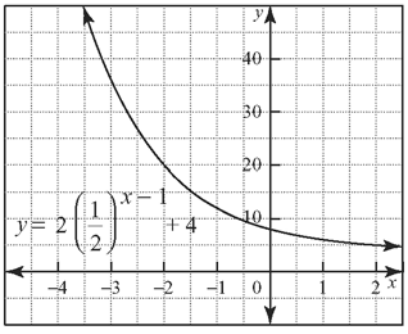
a) domain $\{x \in \mathbb{R}\}$ **b)** range $\{y \in \mathbb{R}, y > 0\}$

c) no x-intercept; y-intercept at (0, 1)

d) always decreasing **e)** $y = 0$

12. $y = 5(3)^x$

13.



a) domain $\{x \in \mathbb{R}\}$ **b)** range $\{y \in \mathbb{R}, y > 4\}$

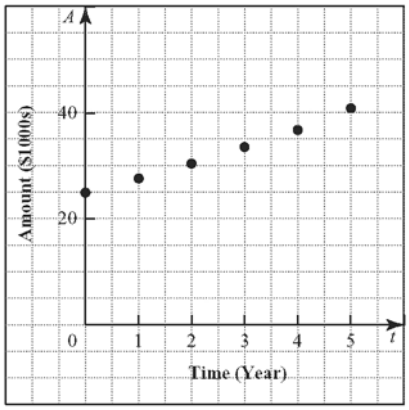
c) $y = 4$

14. a) vertical stretch by a factor of 5, and translated 1 unit to the right

b) translated 4 units up

c) a vertical compression by a factor of 2, a translation 2 units to the left and 2 units down

15.



Answers may vary. Sample answer: Yes, the data seem to be exponential because the rate of growth of the investment appears to be increasing over time.

b) Answers may vary. Sample answer: $A = 25\,000\,(1.1)^t$, where 25 000 represents the initial investment, and 1.1 represents the increasing factor related to the interest rate based on the expression $(1 + i)$.

c) \$64 843.56 **d)** approximately 25 years

Chapter 3 Practice Test

1. A
2. B
3. D
4. C
5. D

6. a) 32 **b)** 7 **c)** $-\frac{1}{9}$ **d)** $\frac{67}{8}$

7. a) $\frac{y^4}{x^3}$ **b)** $8v^{12}$ **c)** $\frac{48}{b^9}$ **d)** $\frac{3u}{v^2}$

8. Answers may vary. Sample answer: a vertical stretch by a factor of 2, a reflection in the y-axis, a translation 1 unit to the left, and a translation up 2 units.

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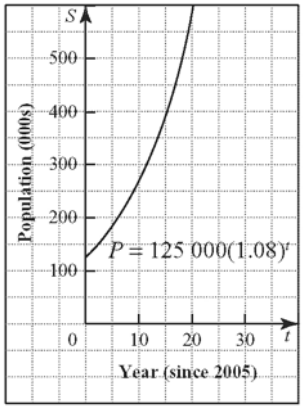
Name: _____ Date: _____

9. a) $P = 125\,000(1.08)^t$

b)



c) 396 521 **d)** 2023

e) Answers may vary. Sample answer: The assumption is that the growth continues to occur at the rate of 8% per year.

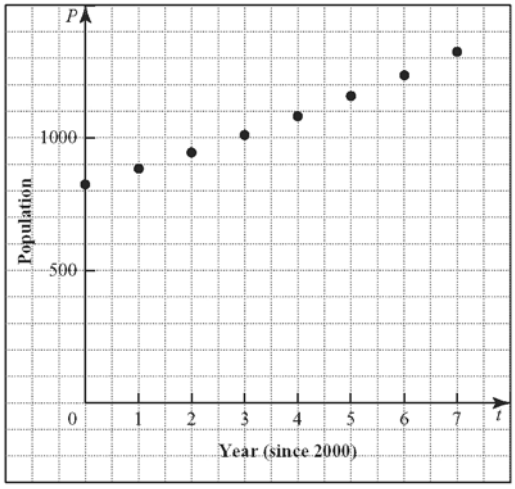
10. a) $P = A\left(1 + \frac{i}{N}\right)^{-n \times N}$ **b)** $3000

11. a) $h = \frac{S - 2\pi r^2}{2\pi r}$ **b)** $h = (S - 2\pi r^2)(2\pi r)^{-1}$ **c)** 11 cm

12. a)

Year	Population
0	825
1	883
2	945
3	1011
4	1081
5	1157
6	1238
7	1325

b)



c) Answers may vary. Sample answer: The data seem to be following an exponential relationship because the growth rate appears to be increasing over time.

d) $P = 825(1.07)^t$ **e)** 1858

f) approximately 25 years