Chapter 5 Answers

Prerequisite Skills

1. a) 6.12 cm   **b)** 11.3 cm   **c)** 14.8 cm   **d)** 11.3 cm
e) Answers may vary. Sample answer: The answers are both 11.3 cm. This is because the co-terminal angle of 270° is –90°, which is the same distance from the initial point as 90°.

2. a) $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$, $\tan 60° = \sqrt{3}$

b) $\sin 135° = \dfrac{1}{\sqrt{2}}$, $\cos 135° = -\dfrac{1}{\sqrt{2}}$, $\tan 135° = -1$

c) $\sin 210° = -\dfrac{1}{2}$, $\cos 210° = -\dfrac{\sqrt{3}}{2}$, $\tan 210° = \dfrac{1}{\sqrt{3}}$

d) $\sin 330° = -\dfrac{1}{2}$, $\cos 330° = \dfrac{\sqrt{3}}{2}$, $\tan 330° = -\dfrac{1}{\sqrt{3}}$

3. domain $\{x \in \mathbb{R}, x \geq 1\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

4. Answers may vary. Sample answer: $y = -x^2 - 2$

5. a)
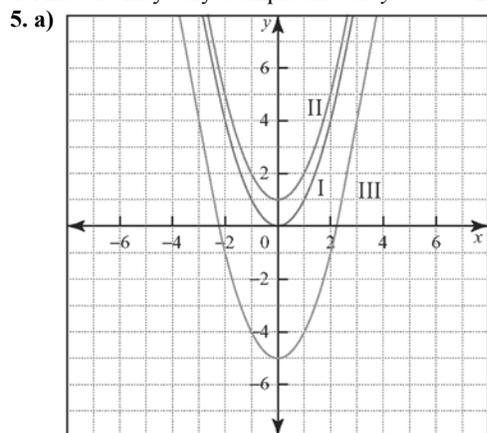
b) Answers may vary. Sample answer: The last two functions are vertical translations; up 1 unit for ii) or down 5 units for iii).

6. a)
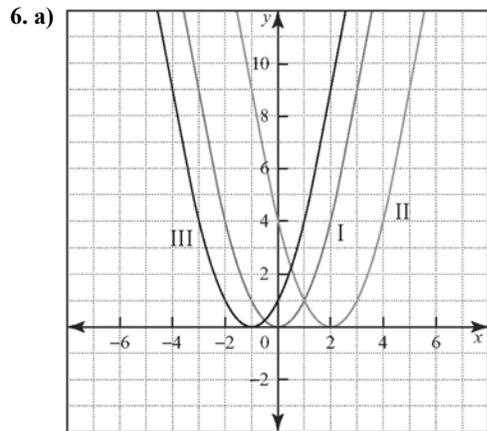
b) Answers may vary. Sample answer: The last two functions are horizontal translations; 2 units to the right for ii) and 1 unit to the left for iii).

7. $y = (x + 3)^2 + 6$

8. a) $y = (x - 3)^2 - 4$
b) Answers may vary. Sample answer: $y = x^2$ is translated 3 units to the right and 4 units down.

9. a)
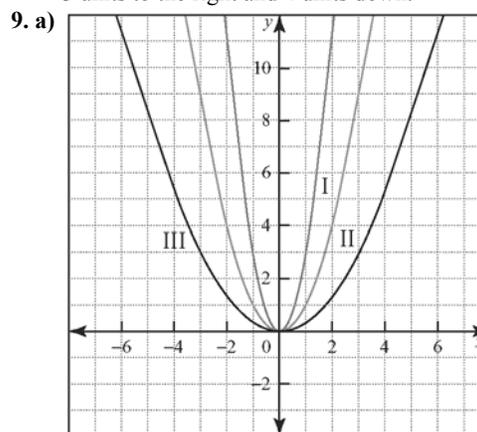
b) Answers may vary. Sample answer: The last two functions are horizontal stretches or compressions; a stretch by a factor of 3 for ii) and a compression by a factor of 3 for iii).

10. a)
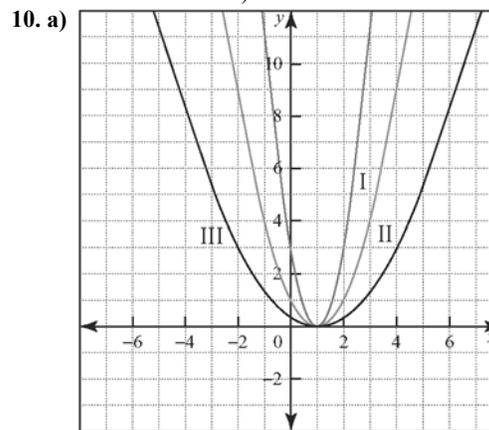
b) Answers may vary. Sample answer: The last two functions are vertical stretches or compressions; a stretch by a factor of 3 for ii) and a compression by a factor of 3 for iii).

11. $y = \dfrac{1}{2}x^2$

12. $y = 2(x - 1)^2$

Name: _____   Date: _____

13. a)
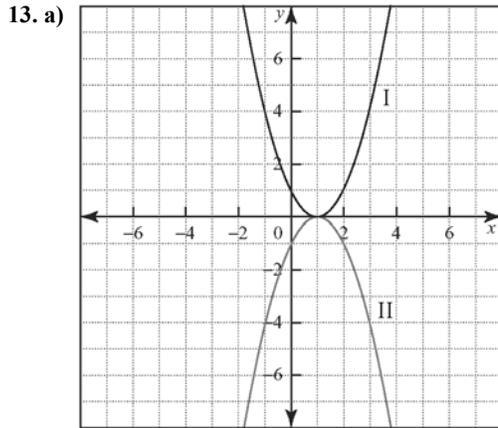
b) Answers may vary. Sample answer: The second
function is a reflection of the first over the x-axis.

14. a) I: $y = (x - 2)^2$; II: $y = -(x - 2)^2$; III: $y = -(x + 2)^2$
  b) Answers may vary. Sample answer: II is a reflection of
  I over the x-axis.
  c) Answers may vary. Sample answer: II is a reflection of
  III over the y-axis.
  d) Answers may vary. Sample answer: III is a reflection of
  I over the x-axis and the y-axis.

15. a) $y = 2(x - 2)^2 - 3$
  b)
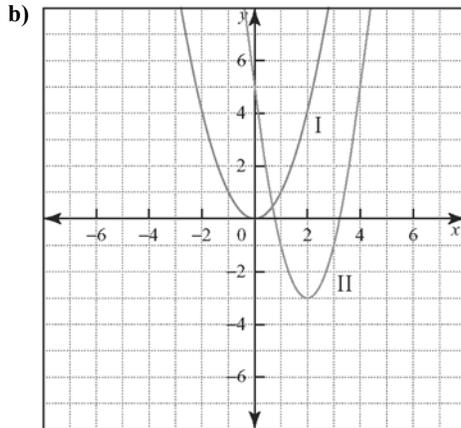
16. A reflection in the x-axis, a translation 1 unit to the left,
a compression by a factor of 3, and a translation 3 units
down.

17. a) $k = 8$ **b)** $k = 6$
18. a) 16 200   **b)** 21 600

5.1 Modelling Periodic Behaviour

1. Answers may vary. Sample answer: If he can trap the ball in
a repetitive pattern from the wall to his paddle, such that the
ball continues to bounce with the same period motion
without moving the paddle, he can leave the game on and
go to get his snack. When he returns, the pattern will not
have changed, and he can then continue to play by moving
the paddle slightly to change the path of the ball.

2. a) periodic, with a period of 5
  b) not periodic
  c) periodic, with a period of approximately 2.25

3. Answers may vary. Sample answer:
  a)
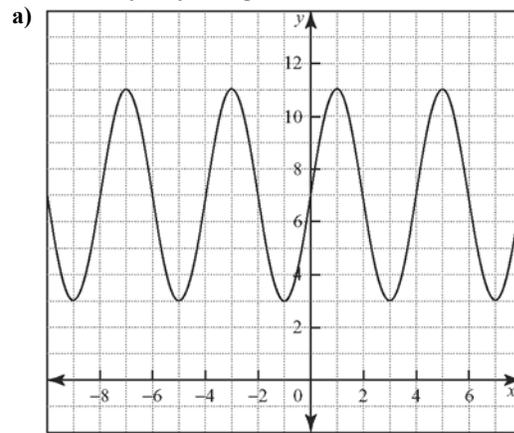
  b) 11   **c)** $x = 5, x = 9$

4. Answers may vary. Sample answer: Since three repeating
patterns exist between $x = 1$ and $x = 25$, the three patterns
repeat in a span of 24. Therefore, each pattern will repeat
every 8, which is the period.

5. a) $\dfrac{25}{4}$ **b)** -3.25 **c)** 1   **d)** $\dfrac{17}{8}$

6. a) period   **b)** amplitude   **c)** 0 m; 8 m
  d) Answers may vary. Sample answer: Yes, since the pattern
  of the position values will repeat every 45 s.

7. a)
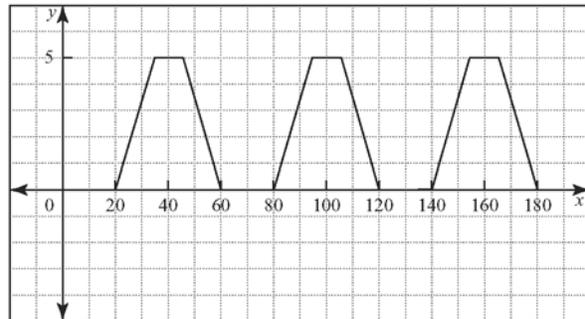
  b) 60   **c)** 480   **d)** 96 000   **e)** 3 360 000

5.2 The Sine Function and the Cosine Function

1. a)
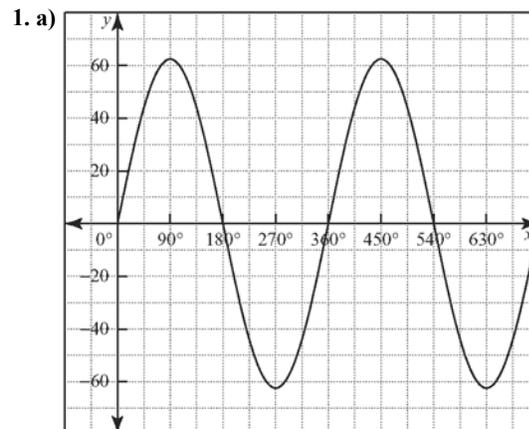
b) Answers may vary. Sample answer: A sine function describes this motion.
c) amplitude 62.5; period 360°

2. a)
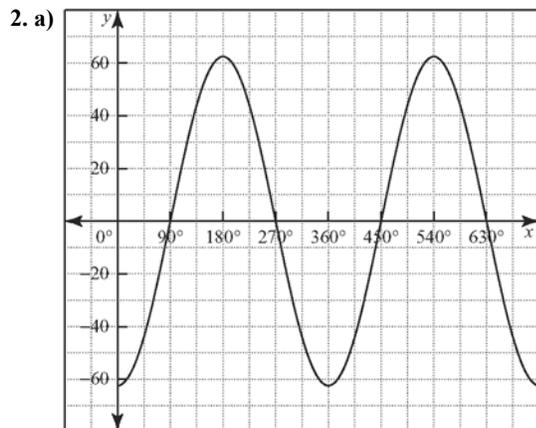
b) Answers may vary. Sample answer: A cosine function reflected in the *x*-axis describes this motion.
c) amplitude 62.5, period 360°

3. a) Answers may vary. Sample answer: Yes. An increasing interval indicates that the *y*-values of the function get larger as the *x*-values increase, and this can occur below the *x*-axis.

b)
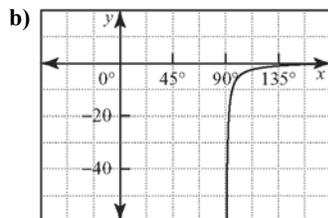
4. a) Answers may vary. Sample answer: Yes. A decreasing interval indicates that the *y*-values of the function get smaller as the *x*-values increase, and this can occur above the *x*-axis.

b)

5.

6.
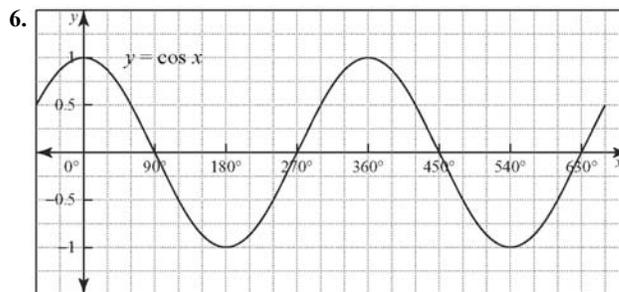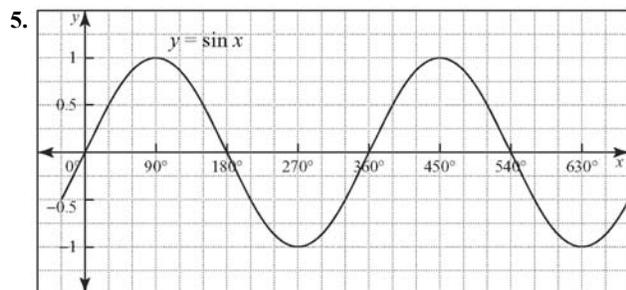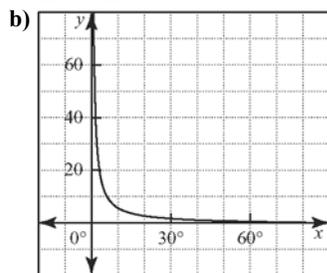
5.3 Investigate Transformations of Sine and Cosine Functions

1. a) amplitude 3, period 180°, phase shift 30° to the right, vertical shift up 1

b) amplitude $\dfrac{1}{2}$, period 120°, phase shift 45° to the right, no vertical shift

c) amplitude $\dfrac{3}{5}$, period 720°, phase shift 30° to the left, vertical shift down 2

d) amplitude 2, period 90°, phase shift 60° to the right, vertical shift up 4

2. a) $y = 2 \sin (x - 30°) - 2$

b) $y = \sin 2x + 3$

c) $y = \dfrac{1}{3} \sin \left[3 \left(x + 15° \right) \right] + 3$

3. a) $y = 3 \cos [2(x - 30°)] + 2$

b) $y = 5 \cos \left[\dfrac{4}{3} \left(x + 45° \right) \right]$

c) $y = \dfrac{3}{4} \cos \left(6x \right) - 3$

4.

5.

$y = \frac{1}{2}\cos\left[\frac{1}{2}(x - 30°)\right] - 2$
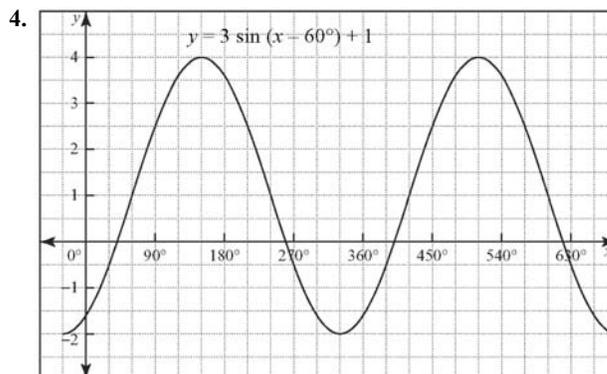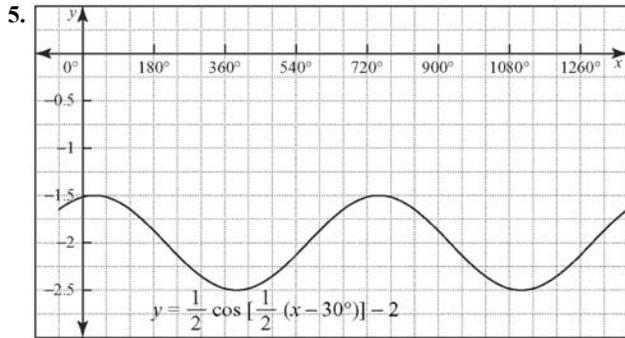
6. a) Answers may vary. Sample answer: The 2 in each function means that there will be two cycles of each function in the given domain. Since each time a cycle is drawn for the curves there are two points of intersection, there will be four points of intersection.

b)

$y = \sin 2x$ $y = \cos 2x$
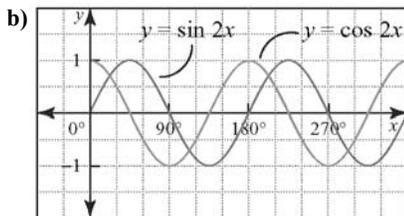
c) Answers may vary. Sample answer: The 3 in each function means that there will be three cycles of each function in the given domain. Since each time a cycle is drawn for the curves there are two points of intersection, there will be six points of intersection.

d)

$y = \sin 3x$ $y = \cos 3x$
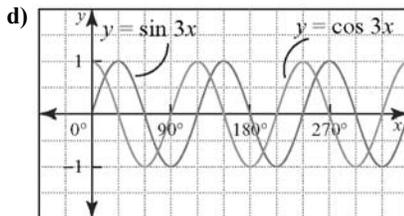
e) Answers may vary. Sample answer: There will be $2k$ points of intersection for the two functions.

7. Answers may vary. Sample answer: Changing the amplitude affects the y-values of the points of intersection, not the number of points.

8. Answers may vary. Sample answer: The phase shift affects the x-values of the points of intersection, not the number of points.

5.4 Graphing and Modelling With $y = a \sin [k(x - d)] + c$ and $y = a \cos[k(x - d)] + c$

1. a) Answers may vary. Sample answer: an increase in the amplitude by a factor of 2, a change in the period from 360° to 180°, and a shift down 2 unit

b) for $f(x)$, domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$;

for $g(x)$, domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -4 \le y \le 0\}$

c) $g(x) = 2 \cos [2(x + 45°)] - 2$

2. a) $y = 2 \sin [3(x - 30°)] + 3$ **b)** $y = 2 \cos [3(x - 60°)] + 3$

3. a) amplitude 5, period 180°, phase shift 30° to the right, vertical shift down 5

b) maximum 0, minimum −10

c) 75°, 255°, 435°

d) −105°

e) $-\dfrac{5\sqrt{3}}{2} - 5$

4. a) The amplitude increases to 10, the period changes to 120°, the function shifts 60° to the left, and then all points move down 4 units.

b) for $g(x)$, domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$;

for $f(x)$, domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -14 \le y \le 6\}$

c)

$f(x) = 10 \sin [3(x + 60°)] - 4$

$g(x) = \sin x$
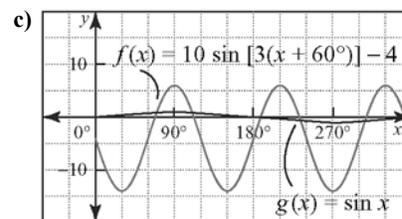
5. a) $y = 3 \sin [3(x - 30°)] + 1$

b) Answers may vary. Sample answer:
$y = 3 \cos [3(x - 60°)] + 1$

6. a)

$y = \cos x$

b)

$y = \sqrt{\cos x}$
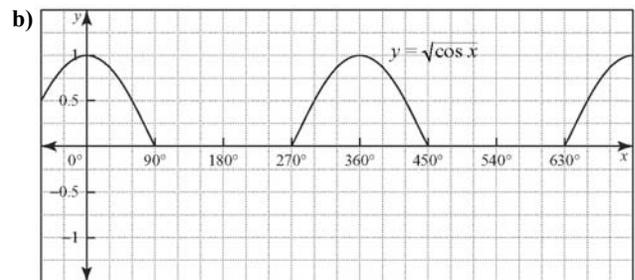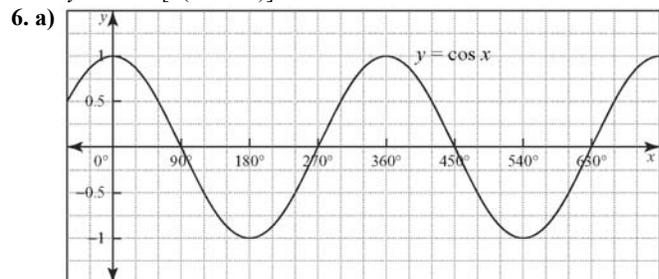
c) Answers may vary. Sample answer: Since the transformation of 1 unit down decreases all points to below the x-axis, therefore only the three points on $y = \sqrt{\cos x}$ where the function was equal to 1 will still exist. These occur at $x = 0°$, 360° and 720°.

7. For $y \cos x$, domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$.

For $y = \sqrt{\cos x}$, domain

$\{x \in \mathbb{R}, 0 \le x \le 90°, 270° \le x \le 450°, 630° \le x \le 720°\}$,

range $\{y \in \mathbb{R}, 0 \le y \le 1\}$. For $y = \sqrt{\cos x} - 1$,

domain $\{x \in 0°, 360°, 720°\}$, range $\{y \in 0\}$.

5.5 Data Collecting and Modelling

1. a) and **d)**
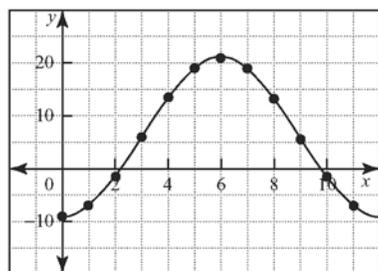
b) Answers may vary. Sample answer: Yes, since the temperature values are usually repetitive and cyclic due to the nature of the seasons.
c) Answers may vary. Sample answer:
$y = 15 \cos [30(x + 6)] + 6$
e) Answers may vary. Sample answer: The fit of the graph is very close to the data given.
2. a) Answers may vary. Sample answer: Yes, this is periodic. The y-values repeat in a cyclic pattern.
b) maximum 2.25, minimum 0.25
c) amplitude 1
d) vertical shift up 1.25
e) horizontal shift $\dfrac{5}{3}$ to the right
f) period 5
3. a) maximum 48.5, minimum 31.5
b) Answers may vary. Sample answer: This represents the height of the centre support of the turbine (measured from the ground to the centre of the blade).
c) Answers may vary. Sample answer: The amplitude of 8.5 represents the length of the blade.
d)
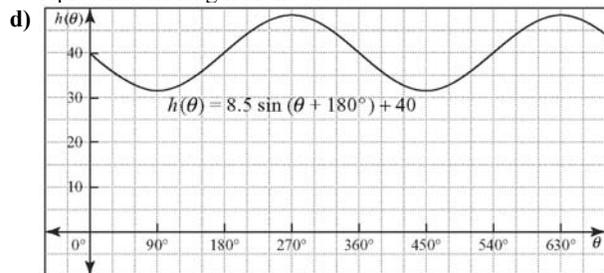
4. a) $\dfrac{6}{5}$ **b)** 50 bpm **c)** minimum 69, maximum 111

d)
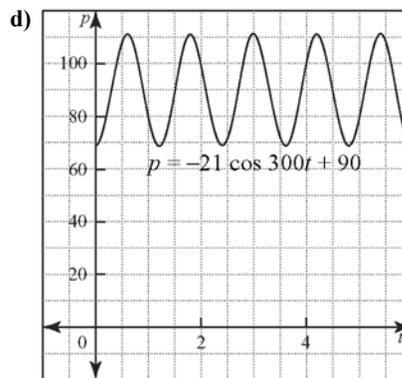
5. a) $h = 25 \cos 5\theta$ **b)** 11°, 61°, 83°
c)
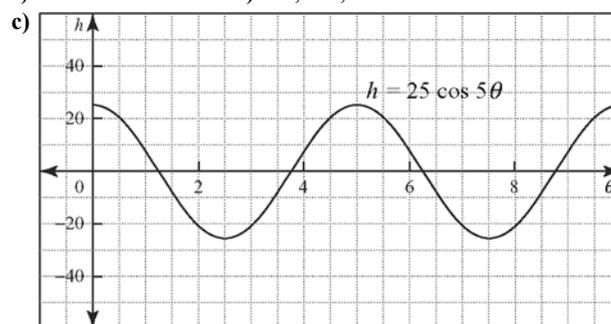
6. a) Answers may vary. Sample answer: 12 months, indicating that the number is cyclic every year.
b)
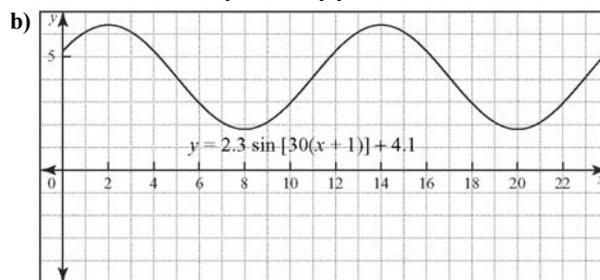
c) February **d)** August

5.6 Use Sinusoidal Functions to Model Periodic Phenomena Not Involving Angles

1. a) green: $y = \sin \dfrac{36}{53}x$, yellow: $y = \sin \dfrac{18}{29}x$,

red: $y = \sin \dfrac{1}{2}x$

b)
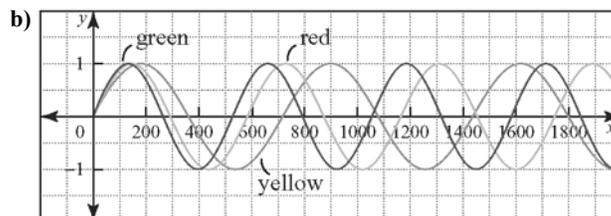
c) green: $3\dfrac{3}{4}$, yellow: $3\dfrac{1}{2}$, red: $2\dfrac{3}{4}$

d) Answers may vary. Sample answer: Since red goes through fewer cycles in a given space, its path does not get altered as much as the paths of other colours.

e) Answers may vary. Sample answer: Green will have a larger path change through a prism than yellow.

2. a) 2.5 m   **b)** 1 min   **c)** $y = 2.5 \cos t$

d)
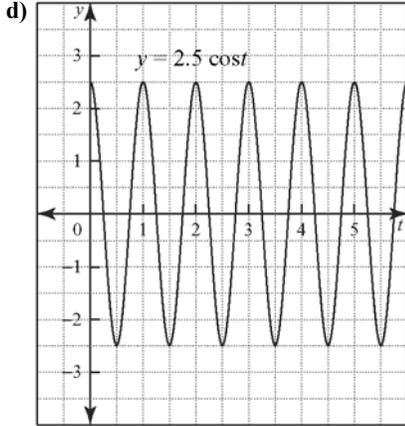
e) Answers may vary. Sample answer: The cosine function starts at a maximum, and we would start time ($t = 0$) when a wave is generated.

3. a) 365

b) maximum 16, minimum 8

c) 10 h

d) Answers may vary (based on the day of the year on which this question is assigned).

4. a) and c)
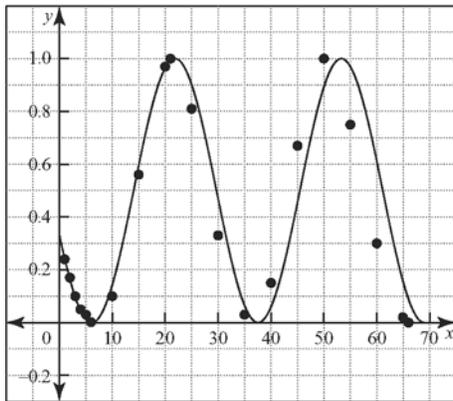
b) Answers may vary. Sample answer:
$y = 0.5 \sin [0.2(x - 14)] + 0.5$

d) Answers may vary. Sample answer: The curve fits the data well.

5. a) Answers may vary. Sample answer: They have the same period of 24.

b)
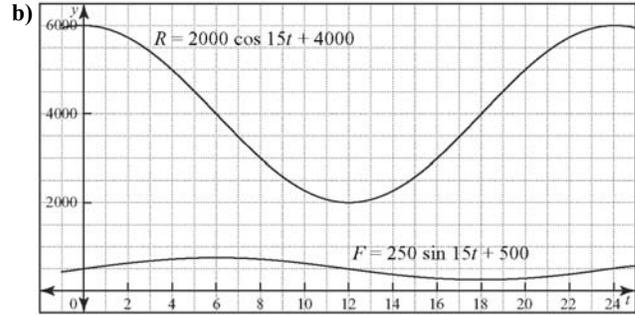
c) Answers may vary. Sample answer: As the foxes start to eat the rabbits, the number of rabbits starts to decrease, and the number of fox starts to increase. When there are too many foxes and not enough rabbits in the food supply, the number of foxes starts to decrease. With this decrease in the number of foxes, the number of rabbits then starts to increase. The cycle then repeats itself.

Chapter 5 Review

1. Answers may vary. Sample answer: The thermostat will change the temperature of the house on a regular basis, thus creating a cyclic pattern to the temperatures.

2. a) 120   **b)** 3   **c)** 11   **d)** 4   **e)** $y = 4 \sin 3x + 7$

3. Answers may vary. Sample answer: The period of the function changes, causing the sound to become distorted. The period of the function decreases as the car approaches and decreases as the car moves away.

4.
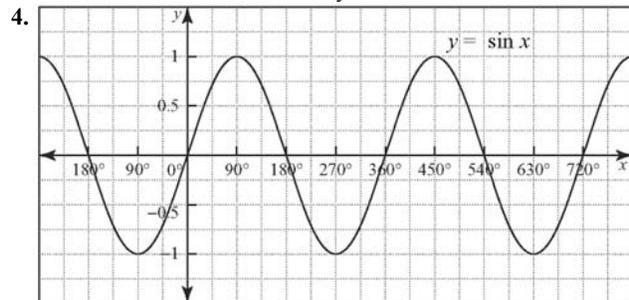
5. a) 2   **b)** 180°   **c)** 30° to the left   **d)** 2 units down

e)
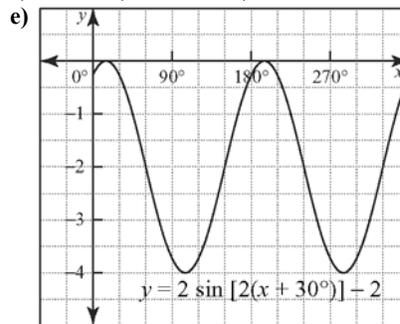
f) Answers may vary. Sample answer: The graph would shift an additional 30° to the left, but all other properties of the function would remain the same.

6. a) $y = 12 \sin [4(x - 15°)] + 8$

   b) $y = 12 \cos [4(x - 38°)] + 8$

   c) Answers may vary. Sample answer: If the two graphs overlap each other, then the two equations are equivalent.

7. a) period 12, amplitude 6, phase shift 2 to the right, vertical shift up 8

   b) 2 m   **c)** 14 m

   d) domain $\{t \in \mathbb{R}\}$, range $\{h \in \mathbb{R}, 2 \le h \le 14\}$

   e) Answers may vary. Only the vertical shift would change.

8. a)

Time		Calls
5 to 6 a.m.	0	25
6 to 7 a.m.	1	12
7 to 8 a.m.	2	13
8 to 9 a.m.	3	24
9 to 10 a.m.	4	38
10 to 11 a.m.	5	37
11 to 12 a.m.	6	25
12 to 1 p.m.	7	11
1 to 2 p.m.	8	13
2 to 3 p.m.	9	24
3 to 4 p.m.	10	39

   b) and **d)**
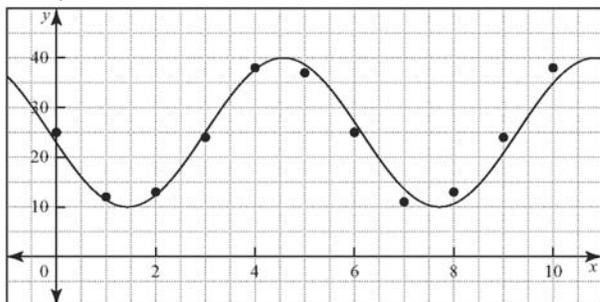
   c) $y = 15 \sin [60(t - 3)] + 25$

   e) Answers may vary. Sample answer: Between 37 and 38 calls.

9. a) $h = 0$ **b)** 220 m

   c) Answers may vary. Sample answer: You can use the formula for the circumference of a circle, since we know the radius of the circle is 35 cm.

   d) 35 cm

Chapter 5 Practice Test

   1. B   **2.** C   **3.** B   **4.** B   **5.** A   **6.** B   **7.** C   **8.** D

   9. Answers may vary. Sample answer: The rotation of Earth on its axis, the movement of Earth around the sun, and the tides on Earth are all examples of periodic motion. In all cases, the values being measured are repeated as the motion continues.

10. a) 4   **b)** 120°   **c)** 60° to the right

   d) up 1 unit   **e)** $y = 4 \sin [3(x - 30°)] + 1$

11. a) $y = 4 \cos 2x + 5$

   b)
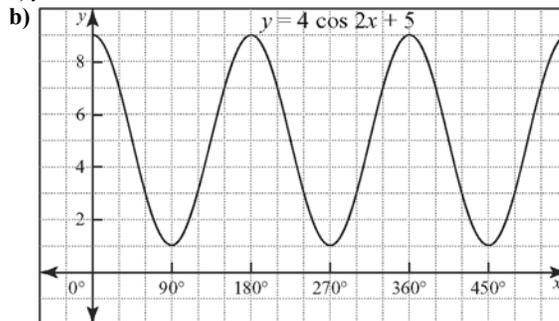
12. a) $\dfrac{1}{2}$ **b)** 90°   **c)** 30° to the right

   d) $\dfrac{3}{4}$ units up   **e)** minimum $\dfrac{1}{4}$, maximum $\dfrac{5}{4}$

   f) domain $\{x \in \mathbb{R}\}$, range $\left\{y \in \mathbb{R}, \dfrac{1}{4} \le y \le \dfrac{5}{4}\right\}$

13. a) and **c)**
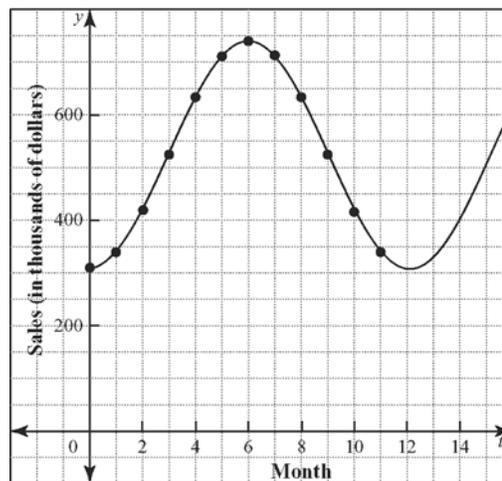
   b) $y = 215 \sin [30(x - 3)] + 525$

   d) Answers may vary. Sample answer: The function closely describes the values on the chart.

14. a) $h = 1.5 \sin 360t$ **b)** $v = 1.5 \cos 360t$

   c)