Student Text Page
74

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 1–13 Task: Laser Beams Rubric

Ongoing Assessment

Use **BLM 1–13 Task: Laser Beams Rubric** to assess student achievement.

Laser Beams

Teaching Suggestions

- Before beginning the task, review with students the key concepts needed to solve a linear-quadratic system, including the concept of a tangent to a curve.
- This task can either be used as an in-class or a take-home assignment.
- If you are doing this task in class, pair weaker students with stronger ones.
- If needed, break down the task so that teams work with the shape of the parabolic roof and one of the colours. Each team can create a large graph that can be taped to the wall. If possible, each group can develop a graph on an acetate that can then be overlaid with other groups' acetates. Students should all work on the same size of grid paper and use the same scale.
- Be sure to have students check their work using a graphing calculator.

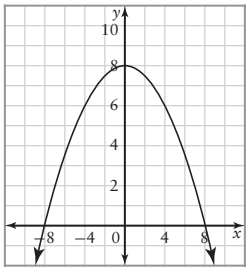
Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- Are the graphs properly constructed and fully labelled?
- Are the points of intersection properly determined?
- Are the heights on the tower of the different coloured lights properly determined?
- Is the equation of the path of a fifth laser that is tangent to the edge of the roof at the vertex properly determined?
- Is the equation of the laser light not intersecting at the common point properly adjusted so that it intersects at the common point?

Level 3 Sample Response

a)



b) To find the point(s) of intersection of the blue light and the edge of the roof, solve the linear-quadratic system $y = -\frac{1}{8}x^2 + 8$ and $6x + 8y - 73 = 0$.

Substitute $y = -\frac{1}{8}x^2 + 8$ into $6x + 8y - 73 = 0$.

$$6x + 8\left(-\frac{1}{8}x^2 + 8\right) - 73 = 0$$
$$6x - x^2 + 64 - 73 = 0$$
$$6x - x^2 - 9 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$
$$x = 3$$

Substitute $x = 3$ into $6x + 8y - 73 = 0$.

$$\begin{aligned} 6(3) + 8y - 73 &= 0 \\ 18 + 8y - 73 &= 0 \\ 8y - 55 &= 0 \\ 8y &= 55 \\ y &= \frac{55}{8}, \text{ or } 6.875 \end{aligned}$$

There is one point of intersection at (3, 6.875).

To find the point(s) of intersection of the green light and the edge of the roof, solve the linear-quadratic system $y = -\frac{1}{8}x^2 + 8$ and $x + 2y - 17 = 0$.

Substitute $y = -\frac{1}{8}x^2 + 8$ into $x + 2y - 17 = 0$.

$$\begin{aligned} x + 2\left(-\frac{1}{8}x^2 + 8\right) - 17 &= 0 \\ x - \frac{1}{4}x^2 + 16 - 17 &= 0 \\ x - \frac{1}{4}x^2 - 1 &= 0 \\ -\frac{1}{4}x^2 + x - 1 &= 0 \\ x^2 - 4x + 4 &= 0 \\ (x - 2)^2 &= 0 \\ x &= 2 \end{aligned}$$

Substitute $x = 2$ into $x + 2y - 17 = 0$.

$$\begin{aligned} 2 + 2y - 17 &= 0 \\ 2y - 15 &= 0 \\ 2y &= 15 \\ y &= \frac{15}{2}, \text{ or } 7.5 \end{aligned}$$

There is one point of intersection at (2, 7.5).

To find the point(s) of intersection of the orange light and the edge of the roof, solve the linear-quadratic system $y = -\frac{1}{8}x^2 + 8$ and $x + y - 10 = 0$.

Substitute $y = -\frac{1}{8}x^2 + 8$ into $x + y - 10 = 0$.

$$\begin{aligned} x + \left(-\frac{1}{8}x^2 + 8\right) - 10 &= 0 \\ x - \frac{1}{8}x^2 - 2 &= 0 \\ -\frac{1}{8}x^2 + x - 2 &= 0 \\ x^2 - 8x + 16 &= 0 \\ (x - 4)^2 &= 0 \\ x &= 4 \end{aligned}$$

Substitute $x = 4$ into $x + y - 10 = 0$.

$$\begin{aligned} 4 + y - 10 &= 0 \\ y - 6 &= 0 \\ y &= 6 \end{aligned}$$

There is one point of intersection at (4, 6).

To find the point(s) of intersection of the red light and the edge of the roof, solve the linear-quadratic system $y = -\frac{1}{8}x^2 + 8$ and $2x + 8y - 67 = 0$.

Substitute $y = -\frac{1}{8}x^2 + 8$ into $2x + 8y - 67 = 0$.

$$\begin{aligned} 2x + 8\left(-\frac{1}{8}x^2 + 8\right) - 67 &= 0 \\ 2x - x^2 + 64 - 67 &= 0 \\ 2x - x^2 - 3 &= 0 \\ x^2 - 2x + 3 &= 0 \end{aligned}$$

Discriminant $= (-2)^2 - 4(1)(3) = 4 - 12 = -8$

Since the discriminant is less than zero, there are no points of intersection.

c) Other than the red light, each colour light intersects the edge of the roof at one point. This means each line joining the light source to the edge of the roof is tangent to the roof.

d) Substitute $x = 9$ into the equation for each light.

Blue light: $6x + 8y - 73 = 0$

$6(9) + 8y - 73 = 0$

$54 + 8y - 73 = 0$

$8y - 19 = 0$

$8y = 19$

$y = \frac{19}{8}$

The height of the blue light source on the tower is 2.375 m.

Green light: $x + 2y - 17 = 0$

$9 + 2y - 17 = 0$

$2y - 8 = 0$

$2y = 8$

$y = 4$

The height of the green light source on the tower is 4 m.

Orange light: $x + y - 10 = 0$

$9 + y - 10 = 0$

$y - 1 = 0$

$y = 1$

The height of the orange light source on the tower is 1 m.

Red light: $2x + 8y - 67 = 0$

$2(9) + 8y - 67 = 0$

$18 + 8y - 67 = 0$

$8y - 49 = 0$

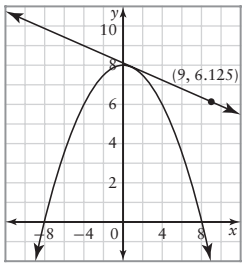
$8y = 49$

$y = \frac{49}{8}$

The height of the red light source on the tower is 6.125 m.

e) The vertex is at (0, 8). Since the parabola is symmetrical about $x = 0$, the tangent will be horizontal, with a slope of zero.
The equation for the path of the laser beam would be $y = 8$.
So, this light source should be located at a height of 8 m on the tower.

f) The laser light that does not share the property is the red light.
It is located at (9, 6.125) on the tower.



Find the slope of the line from (9, 6.125) to the edge of the roof. Let m represent the slope.
Use the two point form.

$y - y_1 = m(x - x_1)$

$y - 6.125 = m(x - 9)$

$y = m(x - 9) + 6.125$

To find the point of intersection of the red light and the edge of the roof, solve the linear-quadratic system $y = -\frac{1}{8}x^2 + 8$ and $y = m(x - 9) + 6.125$.

$-\frac{1}{8}x^2 + 8 = m(x - 9) + 6.125$

$-\frac{1}{8}x^2 + 8 = mx - 9m + 6.125$

$-\frac{1}{8}x^2 - mx + 9m + 1.875 = 0$

$x^2 + 8mx - 72m - 15 = 0$

For one point of intersection, the discriminant must equal zero.

$(8m)^2 - 4(1)(-72m - 15) = 0$

$64m^2 + 288m + 60 = 0$

$16m^2 + 72m + 15 = 0$

$m = \frac{-72 \pm \sqrt{72^2 - 4(16)(15)}}{2(16)}$

$m \doteq -0.219$ or $m \doteq -4.281$

From the diagram, the slope of the line should be between 0 and 1, so the slope of the line is -0.219.

Substitute $m = -0.219$ into $y = m(x - 9) + 6.125$.

$y = -0.219(x - 9) + 6.125$

$= -0.219x + 1.971 + 6.125$

$= -0.219x + 8.096$

The equation of the tangent line is $y = -0.219x + 8.096$.

For the point of intersection, solve $x^2 + 8mx - 72m - 15 = 0$ using $m = -0.219$.

$x^2 - 1.752x + 0.768 = 0$

$$x = \frac{1.752 \pm \sqrt{(-1.752)^2 - 4(1)(0.768)}}{2(1)}$$

Since the discriminant $(-1.752)^2 - 4(1)(0.768) \doteq 0$, $x \doteq \frac{1.752}{2}$, or 0.876.

Substitute $x = 0.876$ into $y = -0.219x + 8.096$.

$y = -0.219(0.876) + 8.096 = 7.904$

To one decimal point, the red light intersects the edge of the roof at (0.9, 7.9).

Level 3 Notes

Look for the following:

- Draws graphs in proper format, including an appropriate scale and labelled axes
- All equations are properly graphed and clearly labelled
- Provides full solutions for the points of intersection and the height of each light source on the tower
- Understands that one colour light does not share a common point of intersection and its equation is identified
- Uses correct mathematical form to communicate the information found

What Distinguishes Level 2

- Graphs are somewhat labelled and a scale has been chosen
- Graphs most of the equations
- Finds most of the heights on the tower
- Only identifies that one colour light does not share a common point of intersection
- Uses minimal mathematical form to communicate the information found

What Distinguishes Level 4

- Creates detailed, fully labelled, and appropriately scaled graphs such that the space displays as much information as possible
- Includes a complete set of steps in all solutions, including points of intersection to the curve as well as the points of intersection with the equation of the tower
- Understands that one colour light does not share a common point of intersection and identifies how to make the needed adjustment so that it ultimately shares this common point
- Places the new equation on the graph and properly labels the line
- Uses excellent mathematical form to communicate the information found