Chapter 2

Transformations of Functions

Curriculum Expectations

Characteristics of Functions

Technology Notes

The technology used in this chapter includes graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, *The Geometer's Sketchpad®*, and the TI-Nspire™ CAS graphing calculator.

Representing Functions

A1.4 relate the process of determining the inverse of a function to their understanding of reverse processes (e.g., applying inverse operations)

A1.5 determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the graph of a function and the graph of its inverse (e.g., the graph of the inverse is the reflection of the graph of the function in the line $y = x$)

Sample problem: Given a graph and a table of values representing population over time, produce a table of values for the inverse and graph the inverse on a new set of axes.

A1.6 determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine whether or not the inverse relation is a function

Sample problem: Given the graph of $f(x) = x^2$, graph the inverse relation. Compare the domain and range of the function with the domain and range of the inverse relation, and investigate connections to the domain and range of the functions $g(x) = \sqrt{x}$ and $h(x) = -\sqrt{x}$.

A1.7 determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function [e.g., $f(x) = (x - 2)^2 - 5$], and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the algebraic representations of a function and its inverse (e.g., the inverse of a linear function involves applying the inverse operations in the reverse order)

Sample problem: Given the equations of several linear functions, graph the functions and their inverses, determine the equations of the inverses, and look for patterns that connect the equation of each linear function with the equation of the inverse.

A1.8 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, and describe these roles in terms of transformations on the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axis)

Sample problem: Investigate the graph $f(x) = 3(x - d)^2 + 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

A1.9 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x)$ to sketch $g(x)$, and state the domain and range of each function, for the following: $f(x) = \sqrt{x}$, $g(x) = \sqrt{x - 4}$; $f(x) = \frac{1}{x}$, $g(x) = -\frac{1}{x + 1}$.

Determining Equivalent Algebraic Expressions

A3.1 simplify polynomial expressions by adding, subtracting, and multiplying

Sample problem: Write and simplify an expression for the volume of a cube with edge length $2x + 1$.

A3.3 simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values

Sample problem: Simplify $\frac{2x}{4x^2 + 6x} - \frac{3}{2x + 3}$, and state the restrictions on the variable.

A3.4 determine if two given algebraic expressions are equivalent (i.e., by simplifying; by substituting values)

Sample problem: Determine if the expressions $\frac{2x^2 + 4x + 6}{x + 1}$ and $8x^2 - 2x(4x - 1) - 6$ are equivalent.

Chapter 2 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 2 Opener • 10–15 min	75			
Prerequisite Skills • 45–60 min	76–77	• G–3 Four Quadrant Grids • BLM 2–1 Prerequisite Skills		• grid paper
2.1 Functions and Equivalent Algebraic Expressions • 50 min	78–85	• G–1 Grid Paper • BLM 2–2 Section 2.1 Investigate • BLM 2–3 Section 2.1 Practice		• grid paper • graphing calculator
Use Technology: Graph Functions Using a TI-Nspire™ CAS Graphing Calculator • 10–15 min	86–87			• TI-Nspire™ CAS graphing calculator
2.2 Skills You Need: Operations With Rational Expressions • 50–140 min	88–96	• BLM 2–4 Section 2.2 Practice		• graphing calculator
2.3 Horizontal and Vertical Translations of Functions • 70 min	97–104	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 2–5 Section 2.3 Practice		• grid paper • graphing calculator • computer with *The Geometer's Sketchpad®*
2.4 Reflections of Functions • 70 min	105–112	• G–1 Grid Paper • G–2 Placemat • T–2 *The Geometer's Sketchpad®* 4 • BLM 2–6 Section 2.4 Practice		• grid paper • graphing calculator • computer with *The Geometer's Sketchpad®*
2.5 Stretches of Functions • 70 min	113–122	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 2–7 Section 2.5 Practice		• grid paper • graphing calculator • computer with *The Geometer's Sketchpad®*
Use Technology: Use *The Geometer's Sketchpad®* to Explore Transformations • 60 min	123–124	• T–2 *The Geometer's Sketchpad®* 4		• computer with *The Geometer's Sketchpad®* • Translations1.gsp • Translations2.gsp • Stretches1.gsp • Stretches2.gsp
2.6 Combinations of Transformations • 70 min	125–131	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 2–8 Section 2.6 Summary Table • BLM 2–9 Section 2.6 Practice		• grid paper • graphing calculator • computer with graphing software (optional)
2.7 Inverse of a Function • 70 min	132–141	• G–1 Grid Paper • BLM 2–10 Section 2.7 Investigate Inverses With a Mira • BLM 2–11 Section 2.7 Practice	• BLM 2–12 Section 2.7 Achievement Check Rubric	• grid paper • graphing calculator
Chapter 2 Review • 60–75 min	142–143	• G–1 Grid Paper • BLM 2–13 Chapter 2 Review		• grid paper • graphing calulator • computer with graphing software (optional)
Chapter 2 Problem Wrap-Up • 15–30 min	143		• BLM 2–14 Chapter 2 Problem Wrap-Up Rubric	• computer with Internet access
Chapter 2 Practice Test • 45–60 min	144–145	• G–1 Grid Paper	• BLM 2–15 Chapter 2 Practice Test	• grid paper • graphing calculator • computer with graphing software (optional)
Chapter 2 Task: Functions in Design • 75 min	146	• G–1 Grid Paper • BLM 2–17 BLM Answers	• BLM 2–16 Task: Functions in Design Rubric	• grid paper • graphing calculator • computer with graphing software (optional)

Chapter 2 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–3	Four Quadrant Grids	Student Support
	BLM 2–1	Prerequisite Skills	Practice
2.1 Functions and Equivalent Algebraic Expressions			
	G–1	Grid Paper	Student Support
	BLM 2–2	Section 2.1 Investigate	Student Support
	BLM 2–3	Section 2.1 Practice	Practice
Use Technology: Graph Functions Using a TI-Nspire™ CAS Graphing Calculator			
2.2 Skills You Need: Operations With Rational Expressions			
	BLM 2–4	Section 2.2 Practice	Practice
2.3 Horizontal and Vertical Translations of Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad® 4*	Technology
	BLM 2–5	Section 2.3 Practice	Practice
2.4 Reflections of Functions			
	G–1	Grid Paper	Student Support
	G–2	Placemat	Student Support
	T–2	*The Geometer's Sketchpad® 4*	Technology
	BLM 2–6	Section 2.4 Practice	Practice
2.5 Stretches of Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad® 4*	Technology
	BLM 2–7	Section 2.5 Practice	Practice
Use Technology: Use *The Geometer's Sketchpad®* to Explore Transformations			
	T–2	*The Geometer's Sketchpad® 4*	Technology
2.6 Combinations of Transformations			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad® 4*	Technology
	BLM 2–8	Section 2.6 Summary Table	Student Support
	BLM 2–9	Section 2.6 Practice	Practice
2.7 Inverse of a Function			
	G–1	Grid Paper	Student Support
	BLM 2–10	Section 2.7 Investigate Inverses With a Mira	Student Support
	BLM 2–11	Section 2.7 Practice	Practice
	BLM 2–12	Section 2.7 Achievement Check Rubric	Assessment
Chapter 2 Review			
	G–1	Grid Paper	Student Support
	BLM 2–13	Chapter 2 Review	Practice
Chapter 2 Problem Wrap-Up			
	BLM 2–14	Chapter 2 Problem Wrap-Up Rubric	Assessment
Chapter 2 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 2–15	Chapter 2 Practice Test	Summative Assessment
Chapter 2 Task: Functions in Design			
	G–1	Grid Paper	Student Support
	BLM 2–16	Task: Functions in Design Rubric	Assessment
	BLM 2–17	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
76 to 77

Suggested Timing
45–60 min

Tools
- grid paper

Related Resources
- G–3 Four Quadrant Grids
- BLM 2–1 Prerequisite Skills

Assessment
You may wish to use **BLM 2–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of the topics.

Teaching Suggestions

- Students could partner up to verify each other's solutions.
- Stress that factoring expressions is an important prerequisite skill for this section.

Chapter Problem

The Chapter Problem is introduced on page 77. You may use this context as an opportunity to show that mathematics shows up in many jobs and can be used to analyse complex situations. The Chapter Problem is revisited in Section 2.1 (question 10), Section 2.3 (question 15), Section 2.5 (question 9), and Section 2.7 (question 13). These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 143. The Chapter Problem questions may be assigned in each section where they appear. Alternatively, you may wish to assign them all with the Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment. It is important to coach students throughout the chapter on what they will be expected to produce at the end.

Common Errors

- Students may forget to multiply all terms by the value outside the brackets when using the distributive property.

R_x Post some worked examples around the class for students to refer to.

2.1 Functions and Equivalent Algebraic Expressions

Student Text Pages
78 to 85

Suggested Timing
50 min

Tools
- grid paper
- graphing calculator

Teaching Suggestions

- This is the first time that students are exposed to the idea of restrictions, so it is important for them to understand why they are there. For example, although the expression $\frac{x^2 + 5x + 6}{x + 3}$ simplifies to $x + 2$, they are not equal without restrictions. Since the original function is undefined when $x = -3$, the simplified expression must also be undefined at $x = -3$.
- For a numerical investigation, use **BLM 2–2 Section 2.1 Investigate**. In this investigation, familiar forms of quadratic equations are used to show that different forms of the same equation can give the same result.
- In **Example 1**, note that the second equation is graphed with a thicker line. This is an effective technique to show the two functions may be equivalent.
- For **Example 2**, review factoring methods if you have not already assigned factoring as a prerequisite skill.