Chapter 3

Exponential Functions

Curriculum Expectations

Exponential Functions

Technology Notes
The technology used in this chapter includes graphing calculators, specifically the TI-83 Plus/TI-84 Plus series and the TI-Nspire™ CAS graphing calculator, as well as *The Geometer's Sketchpad®* and *Fathom™*.

Representing Exponential Functions

B1.1 graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ $(a > 0, a \neq 1)$, define this relation as the function $f(x) = a^x$, and explain why it is a function

B1.2 determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers)

Sample problem: The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$. What value would you assign to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.

B1.3 simplify algebraic expressions containing integer and rational exponents [e.g., $x^3 \div x^{\frac{1}{2}}$, $(x^6y^3)^{\frac{1}{3}}$], and evaluate numeric expressions containing integer and rational exponents and rational bases [e.g., 2^{-3}, $(-6)^3$, $4^{\frac{1}{2}}$, 1.01^{120}]

B1.4 determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x$ (a > 0, a ≠ 1), function machines]

Sample problem: Graph $f(x) = 2^x$, $g(x) = 3^x$, and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the y-intercepts.

Connecting Graphs and Equations of Exponential Functions

B2.1 distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations)

Sample problem: Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.

B2.2 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, and describe these roles in terms of transformations on the graph of $f(x) = a^x$ $(a > 0, a \neq 1)$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

Sample problem: Investigate the graph of $f(x) = 3^{x - d} - 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

B2.3 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graph of $f(x) = a^x$ $(a > 0, a \neq 1)$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x) = 3^x$ to sketch $g(x) = 3^{-(x + 1)} - 2$, and state the domain and range of each function.

B2.4 determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases [e.g., $f(x) = 9^x$ can be expressed as $f(x) = 3^{2x}$], and explain the connections between the equivalent forms in a variety of ways (e.g., comparing graphs; using transformations; using the exponent laws)

B2.5 represent an exponential function with an equation, given its graph or its properties

Sample problem: Write two equations to represent the same exponential function with a y-intercept of 5 and an asymptote at $y = 3$. Investigate whether other exponential functions have the same properties. Use transformations to explain your observations.

Solving Problems Involving Exponential Functions

B3.1 collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data

Sample problem: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.

B3.2 identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve)

Sample problem: Using data from Statistics Canada, investigate to determine if there was a period of time over which the increase in Canada's national debt could be modelled using an exponential function.

B3.3 solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations

Sample problem: The temperature of a cooling liquid over time can be modelled by the exponential function $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}}$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28 °C.

Chapter 3 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 3 Opener • 10–15 min	147			
Prerequisite Skills • 45–60 min	148–149	• G–3 Four Quadrant Grids • BLM 3–1 Prerequisite Skills		• grid paper
3.1 The Nature of Exponential Growth • 75 min	150–157	• G–1 Grid Paper • BLM 3–2 Section 3.1 Practice		• coloured tiles or linking cubes • grid paper • graphing calculator • computer with *The Geometer's Sketchpad®*
Use Technology: Use the Lists and Trace Features on a TI-Nspire™ CAS Graphing Calculator • 10–15 min	158–159			• TI-Nspire™ CAS graphing calculator
3.2 Exponential Decay: Connecting to Negative Exponents • 75–150 min	160–169	• BLM 3–3 Section 3.2 Practice		• graphing calculator • computer with spreadsheet software
3.3 Rational Exponents • 75 min	170–177	• BLM 3–4 Section 3.3 Practice		• graphing calculator
3.4 Properties of Exponential Functions • 75–150 min	178–187	• G–1 Grid Paper • G–2 Placement • T–2 *The Geometer's Sketchpad®* 4 • BLM 3–5 Section 3.4 Practice		• computer with *The Geometer's Sketchpad®* • graphing calculator • grid paper
3.5 Transformations of Exponential Functions • 75 min	188–198	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad®* 4 • BLM 3–6 Section 3.5 Practice	• BLM 3–7 Section 3.5 Achievement Check Rubric	• computer with *The Geometer's Sketchpad®* • graphing calculator • grid paper • TI-Nspire™ CAS graphing calculator (optional) • toothpicks or drinking straws (optional)
3.6 Making Connections: Tools and Strategies for Applying Exponential Models • 75 min	199–209	• T–3 *Fathom*™ • BLM 3–8 Section 3.6 Practice		• graphing calculator • computer with *Fathom*™ • computer with Internet access • temperature probe (optional)
Chapter 3 Review • 60–75 min	210–211	• G–1 Grid Paper • BLM 3–9 Chapter 3 Review		• graphing calculator (optional) • *The Geometer's Sketchpad®* (optional) • grid paper (optional)
Chapter 3 Problem Wrap-Up • 15–30 min	211		• BLM 3–10 Chapter 3 Problem Wrap-Up Rubric	• computer with Internet access (optional)
Chapter 3 Practice Test • 45 min	212–213	• G–1 Grid Paper	• BLM 3–11 Chapter 3 Practice Test	• grid paper • graphing calculator
Chapters 1 to 3 Review • 60–75 min	214–217	• G–1 Grid Paper		• grid paper • graphing calculator
Chapter 3 Task: Radioactive Isotopes • 75 min	218	• G–1 Grid Paper • BLM 3–13 BLM Answers	• BLM 3–12 Task: Radioactive Isotopes Rubric	• grid paper • graphing calculator • computer with graphing software (optional) • computer with Internet access (optional)

Chapter 3 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–3	Four Quadrant Grids	Student Support
	BLM 3–1	Prerequisite Skills	Practice
3.1 The Nature of Exponential Growth			
	G–1	Grid Paper	Student Support
	BLM 3–2	Section 3.1 Practice	Practice
Use Technology: Use the Lists and Trace Features on a TI-Nspire™ CAS Graphing Calculator			
3.2 Exponential Decay: Connecting to Negative Exponents			
	BLM 3–3	Section 3.2 Practice	Practice
3.3 Rational Exponents			
	BLM 3–4	Section 3.3 Practice	Practice
3.4 Properties of Exponential Functions			
	G–1	Grid Paper	Student Support
	G–2	Placement	Student Support
	T–2	*The Geometer's Sketchpad®* 4	Student Support
	BLM 3–5	Section 3.4 Practice	Practice
3.5 Transformations of Exponential Functions			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad®* 4	Student Support
	BLM 3–6	Section 3.5 Practice	Practice
	BLM 3–7	Section 3.5 Achievement Check Rubric	Assessment
3.6 Making Connections: Tools and Strategies for Applying Exponential Models			
	T–3	*Fathom™*	Student Support
	BLM 3–8	Section 3.6 Practice	Practice
Chapter 3 Review			
	G–1	Grid Paper	Student Support
	BLM 3–9	Chapter 3 Review	Practice
Chapter 3 Problem Wrap-Up			
	BLM 3–10	Chapter 3 Problem Wrap-Up Rubric	Assessment
Chapter 3 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 3–11	Chapter 3 Practice Test	Summative Assessment
Chapters 1 to 3 Review			
	G–1	Grid Paper	Student Support
Chapter 3 Task: Radioactive Isotopes			
	BLM 3–12	Task: Radioactive Isotopes Rubric	Assessment
	BLM 3–13	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
148 to 149

Suggested Timing
45–60 min

Tools
- grid paper

Related Resources
- G–3 Four Quadrant Grids
- BLM 3–1 Prerequisite Skills

Assessment
You may wish to use **BLM 3–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Differentiated Instruction

- Label **three corners** of the room with the title cards Product Rule, Quotient Rule, and Power of a Power Rule. Include the expressions $(x^a)(x^b)$, $\frac{x^a}{x^b}$, and $(x^a)^b$, and the method for simplifying each expression on the title cards (see question 1). Create cue cards using questions 3 and 4. Students move to the appropriate corner of the room after mentally determining the appropriate rule to use when simplifying the expression on each cue card.

Common Errors

- Some students mix up the product rule and the power of a power rule.

R_x Have students explore numerical examples and/or concrete instances involving each rule before moving to the general algebraic versions of the exponent rules.

Teaching Suggestions

- Have students work in pairs or small groups to review these topics and work through the questions.
- Parts **e**) and **f**) of **question 6** provide an opportunity to review the importance of brackets when evaluating exponents.
- **Question 11** reviews transformations of functions. Some students will mix up the effects of vertical and horizontal transformations. Suggest that students explore the effects of these transformations using graphing technology, or a table of values.

Chapter Problem

The Chapter Problem is introduced on page 149. The questions are related to various problems in astronomy, and have connections to physics (e.g., law of universal gravitation, Kepler's laws of planetary motion). The Chapter Problem is revisited in Section 3.2 (questions 18 to 20) and Section 3.3 (questions 12 and 13). These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 211. You may wish to assign the Chapter Problem questions in each section as they are encountered. Alternatively, you may wish to assign them all with the Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.