Task

Student Text Page
280

Suggested Timing
75 min

Tools
- drinking straws
- scissors
- string (optional)

Related Resources
- BLM 4–16 Chapter 4 Task: Pyramids and Angles of Elevation Rubric

Ongoing Assessment

Use **BLM 4–16 Task: Pyramids and Angles of Elevation Rubric** to assess student achievement.

Pyramids and Angles of Elevation

Teaching Suggestions

- A "class size" pyramid can be assembled using metre sticks, and features can be pointed out before students build their own models.
- After building the pyramid, students can cut straws to add the height and slant height to the pyramid.
- When working through part c), students can use a piece of string to model the desired angle, and a protractor to see how it changes at the vertex of the angle changes. The string model leads nicely into part d).
- Students may have difficulty in developing an algebraic model directly. If so, have students try a numerical calculation first, and then develop a parallel algebraic model.
- Advise students that algebraic solutions, involving trignometric reasoning, are expected. The only measured value that they will need to use in their response is the side length, *s*, of each triangular face.

Hints for Evaluating a Response

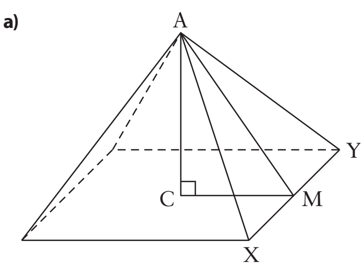
Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- Does the student provide diagrams that "look right?" Are lengths and angles accurately drawn to scale?
- Are diagrams properly labelled using accepted mathematical form?
- Does the student show a development that leads to the answer?
- Does the development include sufficient steps for a reader to follow?

Level 3 Sample Response

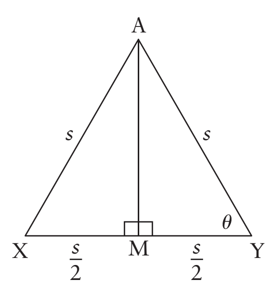
Students will use an actual value rather than the variable *s* throughout their response.

a)



Let *s* be the length of each straw, in centimetres.
Find the height from the apex to the centre, AC.

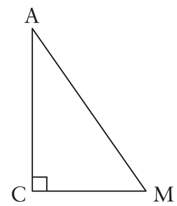
Consider one of the faces that forms a side.



This is an equilateral triangle, with $\theta = 60°$.

$$\sin\theta = \frac{AM}{s}$$
$$\sin 60° = \frac{AM}{s}$$
$$\frac{\sqrt{3}}{2} = \frac{AM}{s}$$
$$AM = \frac{\sqrt{3}}{2}s$$

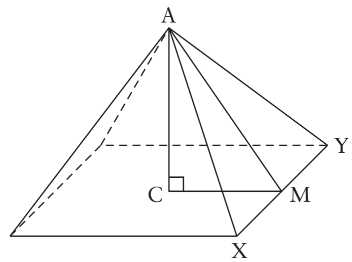
Now consider a triangle with a right angle at the centre of the base.



Since $CM = \frac{s}{2}$, use the Pythagorean theorem.

$$AC^2 = AM^2 - CM^2$$
$$AC^2 = \left(\frac{\sqrt{3}}{2}s\right)^2 - \left(\frac{s}{2}\right)^2$$
$$= \frac{s^2}{2}$$
$$AC = \frac{s}{\sqrt{2}}$$

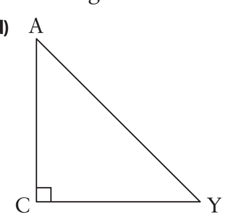
b) Standing at point M, the angle of elevation will be equal to ∠AMC.



$$\tan\angle AMC = \frac{AC}{CM}$$
$$= \frac{\left(\frac{s}{\sqrt{2}}\right)}{\left(\frac{s}{2}\right)}$$
$$= \frac{2}{\sqrt{2}}$$
$$= \sqrt{2}$$
$$\angle AMC = \tan^{-1}\sqrt{2}$$
$$= 54.7°$$

c) I predict that the angle will be smaller. The height of the pyramid, AC, is the same. The length of the base, CM, is shorter than the base CY because point Y is farther from the centre C than M is from the centre. The angle calculation using the tangent ratio involves a smaller number, hence the angle will be smaller.

d)

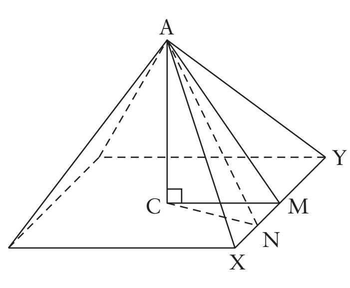


$$CY^2 = AY^2 - AC^2$$
$$= s^2 - \left(\frac{s}{\sqrt{2}}\right)^2$$
$$= \frac{s^2}{2}$$
$$CY = \frac{s}{\sqrt{2}}$$

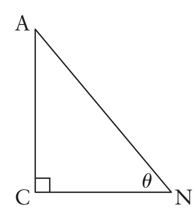
$$\tan\angle AYC = \frac{AC}{CY}$$
$$= \frac{\left(\frac{s}{\sqrt{2}}\right)}{\left(\frac{s}{\sqrt{2}}\right)}$$
$$= 1$$
$$\angle AYC = \tan^{-1}1$$
$$= 45°$$

Since $45° < 54.7°$, my prediction was true.

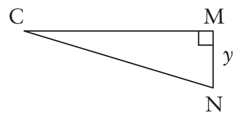
e) Let N be any point on the edge of the base.



Let θ be the angle at point N in △ACN. Find θ.



First, find the length CN using △NCM. Let y be the distance from the M to N, in centimetres.



$$CN^2 = CM^2 + MN^2$$
$$= \left(\frac{s}{2}\right)^2 + y^2$$
$$CN = \sqrt{\frac{s^2}{4} + y^2}$$

Use △ACN and the Pythagorean theorem.

$$\tan\theta = \frac{AC}{CN}$$
$$= \frac{\left(\frac{s}{\sqrt{2}}\right)}{\sqrt{\frac{s^2}{4} + y^2}}$$
$$\theta = \tan^{-1}\frac{\left(\frac{s}{\sqrt{2}}\right)}{\sqrt{\frac{s^2}{4} + y^2}}$$

The angle of elevation at any point N along the base is $\theta = \tan^{-1}\frac{\left(\frac{s}{\sqrt{2}}\right)}{\sqrt{\frac{s^2}{4} + y^2}}$.

This expression can by simplified to $\theta = \tan^{-1}\frac{\sqrt{2}s}{\sqrt{s^2 + 4y^2}}$.

Level 3 Notes

Look for the following:

- Diagram(s) are mostly labelled and mostly correct
- Statements indicate what is being calculated or derived
- Logical development in the calculation or derivation
- Sufficient detail to allow the reader to follow the argument
- Answers are mostly correct
- Use of equal signs, symbols, and other accepted notation is mostly correct

What Distinguishes Level 2

- Minor errors in the development leading to an answer that seems to make sense, but is not completely correct
- Sloppy mathematical form: missing equalities, incorrect notation, or non-standard symbols
- Gaps in logical reasoning, or missing steps in a calculation or derivation

What Distinguishes Level 4

- Completely correct answers or solutions
- Clear, orderly presentation that is easy to follow
- Reasoning indicates a mastery of the concepts involved
- Diagram(s) drawn with a geometry set or using technology
- Possible confirmation of the results using dynamic geometry software