Chapter 4

Trigonometry

Curriculum Expectations

Trigonometric Functions

Technology Notes

The technology used in this chapter includes scientific calculators, graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, *The Geometer's Sketchpad®*, and the TI-Nspire™ CAS graphing calculator.

Determining and Applying Trigonometric Ratios

D1.1 determine the exact values of the sine, cosine, and tangent of the special angles: 0°, 30°, 45°, 60°, and 90°

D1.2 determine the values of the sine, cosine, and tangent of angles from 0° to 360°, through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to special angles)

D1.3 determine the measures of two angles from 0° to 360° for which the value of a given trigonometric ratio is the same

D1.4 define the secant, cosecant, and cotangent ratios for angles in a right triangle in terms of the sides of the triangle $\left(\text{e.g., } \sec A = \frac{hypotenuse}{adjacent}\right)$, and relate these ratios to the cosine, sine, and tangent ratios $\left(\text{e.g., } \sec A = \frac{1}{\cos A}\right)$

D1.5 prove simple trigonometric identities, using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$; the quotient identity $\tan x = \frac{\sin x}{\cos x}$; and the reciprocal identities $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$, and $\cot x = \frac{1}{\tan x}$

Sample problem: Prove that $1 - \cos^2 x = \sin x \cos x \tan x$.

D1.6 pose problems involving right triangles and oblique triangles in two-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)

D1.7 pose problems involving right triangles and oblique triangles in three-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law

Sample problem: Explain how a surveyor could find the height of a vertical cliff that is on the other side of a raging river, using a measuring tape, a theodolite, and some trigonometry. Determine what the surveyor might measure, and use hypothetical values for these data to calculate the height of the cliff.

Chapter 4 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 4 Opener • 10–15 min	219			
Prerequisite Skills • 45–60 min	220–221	• G–1 Grid Paper • BLM 4–1 Prerequisite Skills		• geometry set • grid paper • computer with *The Geometer's Sketchpad*® (optional)
4.1 Special Angles • 60–70 min	222–231	• G–1 Grid Paper • T–2 *The Geometer's Sketchpad*® 4 • BLM 4–2 Trigonometric Ratios of Special Angles • BLM 4–3 Section 4.1 Practice		• geometry set • grid paper • computer with *The Geometer's Sketchpad*®
4.2 Co-terminal and Related Angles • 60–70 min	232–240	• G–1 Grid Paper • BLM 4–2 Trigonometric Ratios of Special Angles • BLM 4–4 Investigate Related Angles • BLM 4–5 Section 4.2 Practice		• geometry set • grid paper • computer with *The Geometer's Sketchpad*® • graphing calculator
Use Technology: Use a Computer Algebra System to Find Exact Trigonometric Ratios and Angles • 10–15 min	241–242			• TI-Nspire™ CAS graphing calculator
4.3 Reciprocal Trigonometric Ratios • 60–70 min	243–248	• G–1 Grid Paper • BLM 4–6 Use a Computer Algebra System to Find Reciprocal Trigonometric Ratios • BLM 4–7 Section 4.3 Practice		• geometry set • grid paper • computer with *The Geometer's Sketchpad*® • computer algebra system (CAS) (optional)
4.4 Problems in Two Dimensions • 60–70 min	249–258	• G–1 Grid Paper • BLM 4–8 Section 4.4 Practice • BLM 4–10 Solving Triangles Decision Tree	• BLM 4–9 Section 4.4 Achievement Check Rubric	• geometry set • grid paper • computer with *The Geometer's Sketchpad*® (optional)
Use Technology: Use Geometry Software to Test for the Ambiguous Case • 10–15 min	259–260			• computer with *The Geometer's Sketchpad*®
4.5 Problems in Three Dimensions • 60–70 min	261–269	• BLM 4–11 Section 4.5 Practice		
4.6 Trigonometric Identities • 60–70 min	270–275	• BLM 4–2 Trigonometric Ratios of Special Angles • BLM 4–12 Section 4.6 Practice		• graphing calculator
Chapter 4 Review • 60–75 min	276–277	• G–1 Grid Paper • BLM 4–13 Chapter 4 Review		• grid paper • geometry set • computer with *The Geometer's Sketchpad*® (optional)
Chapter 4 Problem Wrap-Up • 15–30 min	277		• BLM 4–14 Chapter 4 Problem Wrap-Up Rubric	• computer with *The Geometer's Sketchpad*®
Chapter 4 Practice Test • 45–60 min	278–279	• G–1 Grid Paper	• BLM 4–15 Chapter 4 Practice Test	• grid paper
Chapter 4 Task: Pyramids and Angles of Elevation • 75 min	280	• BLM 4–17 BLM Answers	• BLM 4–16 Task: Pyramids and Angles of Elevation Rubric	• drinking straws • scissors • string (optional)

Chapter 4 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–1	Grid Paper	Student Support
	BLM 4–1	Prerequisite Skills	Practice
4.1 Special Angles			
	G–1	Grid Paper	Student Support
	T–2	*The Geometer's Sketchpad*® 4	Student Support
	BLM 4–2	Trigonometric Ratios of Special Angles	Student Support
	BLM 4–3	Section 4.1 Practice	Practice
4.2 Co-terminal and Related Angles			
	G–1	Grid Paper	Student Support
	BLM 4–2	Trigonometric Ratios of Special Angles	Student Support
	BLM 4–4	Investigate Related Angles	Student Support
	BLM 4–5	Section 4.2 Practice	Practice
Use Technology: Use a Computer Algebra System to Find Exact Trigonometric Ratios and Angles			
4.3 Reciprocal Trigonometric Ratios			
	G–1	Grid Paper	Student Support
	BLM 4–6	Use a Computer Algebra System to Find Reciprocal Trigonometric Ratios	Student Support
	BLM 4–7	Section 4.3 Practice	Practice
4.4 Problems in Two Dimensions			
	G–1	Grid Paper	Student Support
	BLM 4–8	Section 4.4 Practice	Practice
	BLM 4–9	Section 4.4 Achievement Check Rubric	Assessment
	BLM 4–10	Solving Triangles Decision Tree	Student Support
Use Technology: Use Geometry Software to Test for the Ambiguous Case			
4.5 Problems in Three Dimensions			
	BLM 4–11	Section 4.5 Practice	Practice
4.6 Trigonometric Identities			
	BLM 4–2	Trigonometric Ratios of Special Angles	Student Support
	BLM 4–12	Section 4.6 Practice	Practice
Chapter 4 Review			
	G–1	Grid Paper	Student Support
	BLM 4–13	Chapter 4 Review	Practice
Chapter 4 Problem Wrap-Up			
	BLM 4–14	Chapter 4 Problem Wrap-Up Rubric	Assessment
Chapter 4 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 4–15	Chapter 4 Practice Test	Summative Assessment
Chapter 4 Task: Pyramids and Angles of Elevation			
	BLM 4–16	Chapter 4 Task: Pyramids and Angles of Elevation Rubric	Assessment
	BLM 4–17	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
220–221

Suggested Timing
45–60 min

Tools
- geometry set
- grid paper
- computer with *The Geometer's Sketchpad*® (optional)

Related Resources
- G–1 Grid Paper
- BLM 4–1 Chapter 4 Prerequisite Skills

Assessment
You may wish to use **BLM 4–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Common Errors

- Some students tend to write down intermediate results when performing a calculation. This is most common when applying the sine law and the cosine law. This practice introduces cumulative rounding errors.

R_x Have students practise performing these calculations as a single series of operations. Reinforce this practice by having students demonstrate their skills to each other.

- The most common error in applying the cosine law is forgetting to take the square root.

R_x Ensure that students check for the square root as part of the series of operations.

- When applying the Pythagorean theorem, students incorrectly insert the measure of the hypotenuse as one of the side measures.

R_x Have students begin such a problem by labelling the hypotenuse on the diagram. If no diagram is given, have students begin by sketching a diagram.

Teaching Suggestions

- Scientific calculators vary in how calculations are entered. Remind students to review the operation of their own calculators to ensure that they are familiar with the keystroke sequences required, especially for long calculations such as those that apply the cosine law. There is less variation in the models of graphing calculators available to students. Also remind students to check that their calculators are set to degree mode.
- **Questions 1** and **2** require direct hands-on measurement of side lengths and angles. If you have made *The Geometer's Sketchpad*® (GSP) available to students outside of the classroom, you can have them use pencil and paper in class, and assign a short GSP exercise outside of class. This removes the need to book a computer lab while still providing the opportunity to explore a solution using technology.
- **Questions 3** to **13** provide several opportunities for students to practise the calculator skills that they will need throughout this chapter. Consider having students work in pairs or small groups for this exercise, taking care to include strong and weak students in the same group.
- Tool selection is a critical skill in successfully solving problems involving trigonometry. Ensure that students do not skip the "justify" part of a question.

Chapter Problem

The Chapter Problem is introduced on page 221. The orienteering theme of the Chapter Problem is well suited to this chapter, providing a variety of contextual opportunities. The problem is revisited in Section 4.1 (question 16), Section 4.2 (question 15), Section 4.3 (question 15), Section 4.4 (question 17), Section 4.5 (question 11), and Section 4.6 (question 14). These questions are designed to help students move toward the Chapter 4 Problem Wrap-Up on page 277. The Chapter Problem questions may be assigned in each section as they are encountered. Alternatively, you may wish to assign them all with the Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment. It is important to coach students throughout the chapter on what they will be expected to produce at the end.