Chapters 4 to 5 Review

Student Text Pages
348 to 349

Suggested Timing
60–75 min

Tools
- grid paper
- geometry set
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper

Using the Chapters 4 and 5 Review

Each question reviews different skills and concepts. The students might work independently to complete the Chapters 4 and 5 Review, then in pairs to compare solutions. Alternatively, the Chapters 4 and 5 Review could be assigned for reinforcing skills and concepts in preparation for a specific chapter Practice Test.

This is an opportunity for students to assess themselves by completing selected questions and checking their answers against the answers in the back of the student textbook. They can then revisit any questions with which they had difficulty.

Task

Student Text Page
350

Suggested Timing
75 min

Tools
- string
- large paper clip
- tape measure
- grid paper
- access to graphing technology (optional)

Related Resources
- G–1 Grid Paper
- BLM 5–15 Task: Modelling a Rotating Object Rubric

Ongoing Assessment

Use **BLM 5–15 Task: Modelling a Rotating Object Rubric** to assess student achievement.

Modelling a Rotating Object

Teaching Suggestions

- The weight at the end of the string must be sufficient to keep the string taut during the rotation. The paper clip shown usually works well and provides convenient loops for tying the string.
- A reasonable estimate for the period can be obtained by timing a number of revolutions. Ten revolutions is usually sufficient. Ensure that students do not attempt to time just one revolution.
- The answer to part **i**) can be investigated quickly using a graphing calculator or spreadsheet. If an interactive whiteboard (such as the SMART Board™) is available, consider doing this as a class exercise when discussing the results of the task.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

- Does the student demonstrate situational awareness, i.e., a sense of the position of the paper clip in relation to the modelling function?
- Does the student show an understanding of the role of phase shift in determining the value of the function at the starting time and position?
- Does the student understand why reversing the direction of rotation affects the equation for height versus angle, but not for height versus time?
- Does the student understand why reversing the direction of rotation affects the equations for distance versus angle and distance versus time?
- Does the student demonstrate a comprehension of why the rotational distance travelled is a linear and not a sinusoidal function?

Level 3 Sample Response

a) The height from the pivot point of my shoulder to the floor is 128 cm.
The distance from the pivot point to the end of the paper clip is 96 cm.
The distance from my shoulder to the wall is 149 cm.

b) Twelve rotations are made in 15 s. Therefore, the period is $\frac{5}{4}$ s.

c) At rest, the angle is 0° and one period is 360°.
minimum: $-96 + 128 = 32$
maximum: $96 + 128 = 224$
The amplitude is 96.
The equation for the height of the paper clip, h, relative to the angle of rotation, x, is $h = -96 \sin(x + 90) + 128$.

d) $\text{period} = \frac{360°}{k}$

$$\frac{5}{4} = \frac{360°}{k}$$

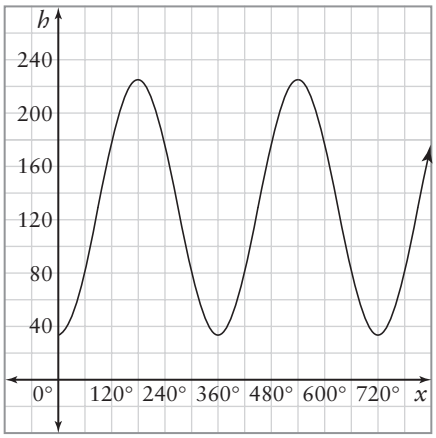
$$k = 288$$

The equation for the height of the paper clip relative to time, t, is $h = -96\cos 288t + 128$.

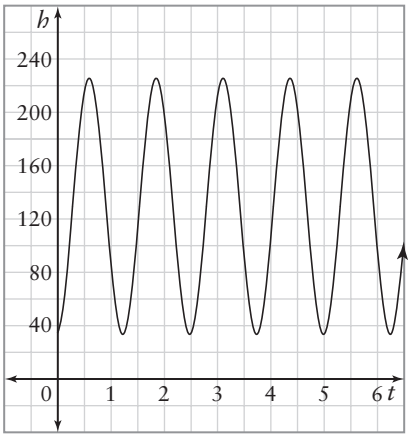
e) At rest, the distance from the paper clip to the wall is 149 cm.
minimum (at the front): $-96 + 149 = 53$
maximum (at the back): $96 + 149 = 245$
The amplitude is 96.
The equation for the distance from the paper clip to the wall, d, relative to the angle of rotation is $d = 96 \sin x + 149$.

f) The equation for the distance from the paper clip to the wall relative to time is $d = 96 \sin 288t + 149$.

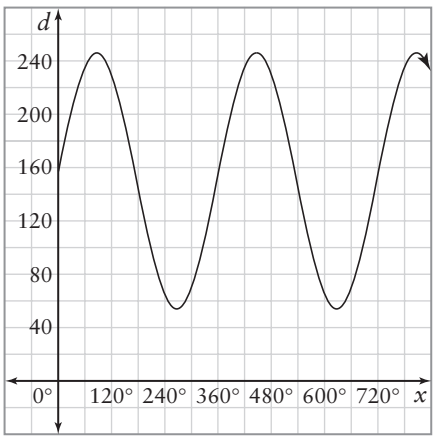
g) $h = -96 \sin(x + 90) + 128$



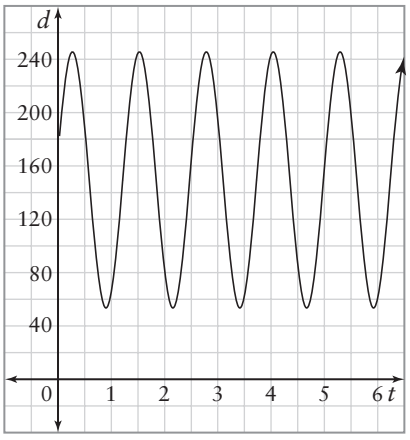
$h = -96 \cos 288t + 128$



$d = 96 \sin x + 149$



$d = 96 \sin 288t + 149$



h) When the direction is reversed, the angle becomes negative.

For part c), the equation $h = -96 \sin(x + 90) + 128$ becomes $h = -96 \sin(2x + 90) + 128$, or $h = -96 \sin(90 - x) + 128$.

For part d), there is no change in the height of the paper with time. The paper clip still travels from the bottom to the reference line, to the top, to the reference line again, and then to the bottom.

For part e), the paper clip now travels from the bottom, out to the front (minimum), to the top, to the back (maximum), and then to the bottom.
The angle becomes negative.
The equation $d = 96 \sin x + 149$ becomes $d = 96 \sin(-x) + 149$.

For part f), the paper clip now travels from the bottom, out to the front (minimum) first as time changes.
The equation $d = 96 \sin t + 149$ becomes $d = -96 \sin 288t + 149$.

i) When the string is taut, the path of the paper clip is a circle.
The rotational distance travelled by the paper clip for one rotation (360°) is $2\pi \times 96$ cm.

The rotational distance, D, travelled by an angle of rotation x is $D = \frac{x}{360} \times 2\pi \times 96$, or $D = \frac{8\pi}{15}x$.

The equation in part c) becomes a linear equation.

Level 3 Notes

Look for the following:

- Values for the measured distances are reasonable
- Sinusoidal modelling functions are written in correct form
- Each graph is properly labelled, indicating an awareness that the student is not modelling the same relations, despite similarities in form
- There is a match between the description of the motion, the graphs, and the modelling functions
- Proper maxima and minima are predicted by the measurements made
- An explanation of why reversing the direction of rotation affects some of the functions and graphs but not others
- An explanation of why the graph of rotational distance travelled will be linear rather than sinusoidal

What Distinguishes Level 2

- Phase shift is miscalculated, i.e., it does not result in the correct initial position
- The resulting equation has a maximum or minimum different from the one predicted
- The graph does not match the physical motion of the paper clip
- The student's prediction of the changes in the equations due to a reversal of the direction of rotation is incorrect

What Distinguishes Level 4

- Provides an explanation of how a constant speed was assured
- The student is aware of the fact that the arm and string must be kept straight, and how this was accomplished
- The correlation of the graph to the physical motion is discussed
- An estimate of the error in the period of rotation based on the number of rotations measured is provided
- An estimate of the error in the maximum and minimum due to the centre of rotation and the shoulder socket not really being fixed points is provided