Chapter 5

Trigonometric Functions

Curriculum Expectations

Trigonometric Functions

Connecting Graphs and Equations of Sinusoidal Functions

D2.1 describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numeric or graphical representation

D2.2 predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural gas consumption in Ontario from previous consumption)

Technology Notes

- The technology used in this chapter includes graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, and computer algebra systems, such as the CAS engine on the TI-Nspire™ CAS graphing calculator.

D2.3 make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0° to 360° and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining why the relationship is a function

D2.4 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in degrees, and determine and describe key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)

D2.5 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, where $f(x) = \sin x$ or $f(x) = \cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

Sample problem: Investigate the graph $f(x) = 2\sin(x - d) + 10$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

D2.6 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$

D2.7 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x) = \cos x$ to sketch $g(x) = 3\cos 2x - 1$, and state the domain and range of each function.

D2.8 represent a sinusoidal function with an equation, given its graph or its properties

Sample problem: A sinusoidal function has an amplitude of 2 units, a period of 180°, and a maximum at (0, 3). Represent the function with an equation in two different ways.

Solving Problems Involving Sinusoidal Functions

D3.1 collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials, measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data

Sample problem: Measure and record distance–time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.

D3.2 identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range

Sample problem: Using data from Statistics Canada, investigate to determine if there was a period of time over which changes in the population of Canadians aged 20–24 could be modelled using a sinusoidal function.

D3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles

Sample problem: Investigate, using graphing technology in degree mode, and explain how the function $h(t) = 5\sin(30(t + 3))$ approximately models the relationship between the height and the time of day for a tide with an amplitude of 5 m, if high tide is at midnight.

D3.4 predict the effects on a mathematical model (i.e., graph, equation) of an application involving periodic phenomena when the conditions in the application are varied (e.g., varying the conditions, such as speed and direction, when walking in a circle in front of a motion sensor)

Sample problem: The relationship between the height above the ground of a person riding a Ferris wheel and time can be modeled using a sinusoidal function. Describe the effect on this function if the platform from which the person enters the ride is raised by 1 m and if the Ferris wheel turns twice as fast.

D3.5 pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation

Sample problem: The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function $h(t) = 25\sin(3(t - 30)) + 27$, where $h(t)$ is the height, in metres, and t is the time, in seconds. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, the height after 30 s, and the time required to complete one revolution.

Chapter 5 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 5 Opener • 10–15 min	281			
Prerequisite Skills • 45–60 min	282–283	• G–1 Grid Paper • BLM 5–1 Prerequisite Skills		• grid paper
5.1 Modelling Periodic Behaviour • 60–75 min	284–293	• G–1 Grid Paper • BLM 5–2 Section 5.1 Practice		• grid paper • protractor, ruler, compass • graphing calculator • computer with Internet access • computer with *The Geometer's Sketchpad®*
5.2 The Sine Function and the Cosine Function • 60–75 min	294–301	• G–1 Grid Paper • BLM 5–3 Section 5.2 Tables for Investigate A • BLM 5–4 Graphs of Trigonometric and Reciprocal Functions • BLM 5–5 Section 5.2 Practice		• grid paper • protractor, ruler, compass • TI-83 Plus or TI-84 Plus graphing calculator • TI-Nspire™ CAS graphing calculator • computer with graphing software (optional)
Use Technology: Dynamically Unwrap the Unit Circle • 15–20 min	302–303			• graphing calculator

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
5.3 Investigate Transformations of Sine and Cosine Functions • 60–75 min	304–312	• G–1 Grid Paper • BLM 5–6 Section 5.3 Table for Investigate • BLM 5–7 Section 5.3 Practice	• BLM 5–8 Section 5.3 Achievement Check Rubric	• grid paper • graphing calculator • computer with graphing software (optional)
5.4 Graphing and Modelling With $y = a\sin[k(x - d)] + c$ and $y = a\cos[k(x - d)] + c$ • 60–75 min	313–321	• G–1 Grid Paper • G–2 Placemat • BLM 5–9 Section 5.4 Practice		• grid paper • graphing calculator • TI-Nspire™ CAS graphing calculator (optional) • computer with graphing software (optional)
5.5 Data Collecting and Modelling • 60–75 min	322–332	• BLM 5–10 Section 5.5 Practice		• graphing calculator • motion sensor • pendulum • computer with Internet access • TI-Nspire™ CAS graphing calculator (optional) • computer with graphing software (optional)
5.6 Use Sinusoidal Functions to Model Periodic Phenomena Not Involving Angles • 60–75 min	333–342	• G–1 Grid Paper • BLM 5–11 Section 5.6 Practice		• grid paper • graphing calculator • computer with Internet access • computer with graphing software (optional)
Use Technology: Create a Scatter Plot and a Function Using a TI-Nspire™ CAS Graphing Calculator • 15–20 min	343			• TI-Nspire™ CAS graphing calculator
Chapter 5 Review • 60–75 min	344–345	• G–1 Grid Paper • BLM 5–12 Chapter 5 Review		• grid paper • graphing calculator • computer with graphing software (optional)
Chapter 5 Problem Wrap-Up • 15–30 min	345		• BLM 5–13 Chapter 5 Problem Wrap-Up Rubric	• access to graphing technology
Chapter 5 Practice Test • 45–60 min	346–347	• G–1 Grid Paper	• BLM 5–14 Chapter 5 Practice Test	• grid paper • graphing calculator • computer with graphing software (optional)
Chapters 4 and 5 Review • 60–75 min	348–349	• G–1 Grid Paper		• grid paper • geometry set • graphing calculator • computer with graphing software (optional)
Chapter 5 Task: Modelling a Rotating Object • 75 min	350	• G–1 Grid Paper • BLM 5–16 BLM Answers	• BLM 5–15 Task: Modelling a Rotating Object Rubric	• string • large paper clip • tape measure • grid paper • access to graphing technology (optional)

Chapter 5 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–1	Grid Paper	Student Support
	BLM 5–1	Prerequisite Skills	Practice
5.1 Modelling Periodic Behaviour			
	G–1	Grid Paper	Student Support
	BLM 5–2	Section 5.1 Practice	Practice
5.2 The Sine Function and the Cosine Function			
	G–1	Grid Paper	Student Support
	BLM 5–3	Section 5.2 Tables for Investigate A	Student Support
	BLM 5–4	Graphs of Trigonometric and Reciprocal Functions	Student Support
	BLM 5–5	Section 5.2 Practice	Practice
Use Technology: Dynamically Unwrap the Unit Circle			
5.3 Investigate Transformations of Sine and Cosine Functions			
	G–1	Grid Paper	Student Support
	BLM 5–6	Section 5.3 Table for Investigate	Student Support
	BLM 5–7	Section 5.3 Practice	Practice
	BLM 5–8	Section 5.3 Achievement Check Rubric	Assessment
5.4 Graphing and Modelling With $y = a\sin[k(x - d)] + c$ and $y = a\cos[k(x - d)] + c$			
	G–1	Grid Paper	Student Support
	G–2	Placemat	Student Support
	BLM 5–9	Section 5.4 Practice	Practice
5.5 Data Collecting and Modelling			
	BLM 5–10	Section 5.5 Practice	Practice
5.6 Use Sinusoidal Functions to Model Periodic Phenomena Not Involving Angles			
	G–1	Grid Paper	Student Support
	BLM 5–11	Section 5.6 Practice	Practice
Use Technology: Create a Scatter Plot and a Function Using a TI-Nspire™ CAS Graphing Calculator			
Chapter 5 Review			
	G–1	Grid Paper	Student Support
	BLM 5–12	Chapter 5 Review	Practice
Chapter 5 Problem Wrap-Up			
	BLM 5–13	Chapter 5 Problem Wrap-Up Rubric	Assessment
Chapter 5 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 5–14	Chapter 5 Practice Test	Summative Assessment
Chapters 4 and 5 Review			
	G–1	Grid Paper	Student Support
Chapter 5 Task: Modelling a Rotating Object			
	G–1	Grid Paper	Student Support
	BLM 5–15	Task: Modelling a Rotating Object Rubric	Assessment
	BLM 5–16	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
282 to 283

Suggested Timing
45–60 min

Tools
- grid paper

Related Resources
- G–1 Grid Paper
- BLM 5–1 Prerequisite Skills

Assessment
You may wish to use **BLM 5–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of the topics.

Common Errors

- Students have forgotten how to determine the direction of a transformation, especially the horizontal translation.

R_x Have students check each transformation by calculating the coordinates of a point on the transformed function, and then comparing it to the graph.

Teaching Suggestions

- The intent of **question 2** is to review the unit circle and trigonometric ratios for special angles. Students can use patterns to answer the question to reduce the number of diagrams required.
- For questions involving transformations (**questions 5** to **16**), it is not intended that students tediously draw a large number of graphs on grid paper. Ask students for predictions, with reasons, and perhaps quick sketches on the board. Alternatively, consider having students work in small groups or pairs.
- **Questions 17** and **18** have particular relevance to determining the period of a sinusoidal function, especially a model that involves unusual numbers, such as the one for alternating electric current.

Chapter Problem

The Chapter Problem is introduced on page 283. The music-oriented theme of the Chapter Problem is well-suited to this chapter, providing a variety of contextual opportunities to apply and model with sinusoidal functions. The Chapter Problem is revisited in Section 5.1 (question 18), Section 5.2 (question 2), Section 5.3 (question 13), Section 5.4 (question 15), Section 5.5 (question 14), and Section 5.6 (question 17). These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 349. The Chapter Problem questions may be assigned in each section where they appear. Alternatively, you may wish to assign them all with the Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment. It is important to coach students throughout the chapter on what they will be expected to produce at the end.