Chapter 6

Discrete Functions

Curriculum Expectations

Discrete Functions

Representing Sequences

C1.1 make connections between sequences and discrete functions, represent sequences using function notation, and distinguish between a discrete function and a continuous function [e.g., $f(x) = 2x$, where the domain is the set of natural numbers, is a discrete linear function and its graph is a set of equally spaced points; $f(x) = 2x$, where the domain is the set of real numbers, is a continuous linear function and its graph is a straight line]

C1.2 determine and describe (e.g., in words; using flow charts) a recursive procedure for generating a sequence, given the initial terms (e.g., 1, 3, 6, 10, 15, 21, …), and represent sequences as discrete functions in a variety of ways (e.g., tables of values, graphs)

C1.3 connect the formula for the nth term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula

C1.4 represent a sequence algebraically using a recursion formula, function notation, or the formula for the nth term [e.g., represent 2, 4, 8, 16, 32, 64, … as $t_1 = 2$; $t_n = 2t_{n-1}$, as $f(n) = 2^n$, or as $t_n = 2^n$, or represent $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$ as $t_1 = \frac{1}{2}$; $t_n = t_{n-1} + \frac{1}{n(n+1)}$, as $f(n) = \frac{n}{n+1}$, or as $t_n = \frac{n}{n+1}$, where n is a natural number], and describe the information that can be obtained by inspecting each representation (e.g., function notation or the formula for the nth term may show the type of function; a recursion formula shows the relationship between terms)

Sample problem: Represent the sequence 0, 3, 8, 15, 24, 35, … using a recursion formula, function notation, and the formula for the nth term. Explain why this sequence can be described as a discrete quadratic function. Explore how to identify a sequence as a discrete quadratic function by inspecting the recursion formula.

C1.5 determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal's triangle, and represent the patterns in a variety of ways (e.g., tables of values, algebraic notation)

C1.6 determine, through investigation, and describe the relationship between Pascal's triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents [e.g., $(1 + x)^4$, $(2x - 1)^5$, $(2x - y)^6$, $(x^2 + 1)^5$]

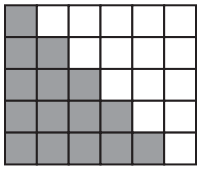
Investigating Arithmetic and Geometric Sequences and Series

C2.1 identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation

C2.2 determine the formula for the general term of an arithmetic sequence (i.e., $t_n = a + (n - 1)d$] or geometric sequence (i.e., $t_n = ar^{n-1}$), through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate any term in a sequence

C2.3 determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate the sum of a given number of consecutive terms

Sample problem: Given the following array built with grey and white connecting cubes, investigate how different ways of determining the total number of grey cubes can be used to evaluate the sum of the arithmetic series $1 + 2 + 3 + 4 + 5$. Extend the series, use patterning to make generalizations for finding the sum, and test the generalizations for other arithmetic series.



C2.4 solve problems involving arithmetic and geometric sequences and series, including those arising from real-world applications

Technology Notes

The technology used in this chapter includes graphing calculators, specifically the TI-83 Plus/TI-84 Plus series, *The Geometer's Sketchpad®*, spreadsheet software, and the TI-Nspire™ CAS graphing calculator.

Chapter 6 Planning Chart

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 6 Opener • 10–15 min	351			
Prerequisite Skills • 45–60 min	352–353	• G–3 Four Quadrant Grids • BLM 6–1 Prerequisite Skills • BLM 6–2 Exploring Patterns and Sequences		• grid paper • graphing calculator (optional)
6.1 Sequences as Discrete Functions • 55–75 min	354–363	• G–1 Grid Paper • T–1 Microsoft® *Excel* • T–2 *The Geometer's Sketchpad®* 4 • BLM 6–3 Section 6.1 Investigate • BLM 6–4 Section 6.1 Practice		• square dot paper • ruler • computer with *The Geometer's Sketchpad®* • graphing calculator • grid paper • computer with spreadsheet software • isometric dot paper
Use Technology: Use a TI-Nspire™ CAS Graphing Calculator to Write Terms in a Sequence • 10–15 min	364			• TI-Nspire™ CAS graphing calculator
6.2 Recursive Procedures • 50–75 min	365–372	• G–1 Grid Paper • BLM 6–5 Section 6.2 Practice		• grid paper • isometric dot paper • computer with *The Geometer's Sketchpad®* (optional) • computer with spreadsheet software (optional)
6.3 Pascal's Triangle and Expanding Binomial Powers • 60–75 min	373–379	• BLM 6–6 Section 6.3 Practice		• graphing calculator with computer algebra system (CAS) (optional)
6.4 Arithmetic Sequences • 60–75 min	380–387	• G–1 Grid Paper • T–1 Microsoft® *Excel* • BLM 6–7 Section 6.4 Practice		• grid paper • computer with spreadsheet software • TI-Nspire™ CAS graphing calculator (optional)
6.5 Geometric Sequences • 60–75 min	388–394	• G–1 Grid Paper • BLM 6–8 Section 6.5 Practice		• grid paper • large grid paper sheets and markers
6.6 Arithmetic Series • 60–75 min	395–401	• BLM 6–9 Section 6.6 Practice		• graphing calculator
6.7 Geometric Series • 60–75 min	402–409	• G–1 Grid Paper • BLM 6–10 Section 6.7 Practice	• BLM 6–11 Section 6.7 Achievement Check Rubric	• grid paper • counters • computer with spreadsheet software • graphing calculator
Chapter 6 Review • 60–75 min	410–411	• G–1 Grid Paper • BLM 6–12 Chapter 6 Review		• grid paper • graphing calculator
Chapter 6 Problem Wrap-Up • 15–30 min	411		• BLM 6–13 Chapter 6 Problem Wrap-Up Rubric	• chart paper and markers
Chapter 6 Practice Test • 45–60 min	412–413	• G–1 Grid Paper	• BLM 6–14 Chapter 6 Practice Test	• grid paper • graphing calculator
Chapter 6 Task: Mathematics in Media Studies • 75 min	414	• G–1 Grid Paper • BLM 6–16 BLM Answers	• BLM 6–15 Task: Mathematics in Media Studies Rubric	• grid paper • computer with graphing software (optional)

Chapter 6 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	G–3	Four Quadrant Grids	Student Support
	BLM 6–1	Prerequisite Skills	Practice
	BLM 6–2	Exploring Patterns and Sequences	Student Support
6.1 Sequences as Discrete Functions			
	G–1	Grid Paper	Student Support
	T–1	Microsoft® *Excel*	Student Support
	T–2	*The Geometers Sketchpad*® 4	Student Support
	BLM 6–3	Section 6.1 Investigate	Student Support
	BLM 6–4	Section 6.1 Practice	Practice
Use Technology: Use a TI-Nspire™ CAS Graphing Calculator to Write Terms in a Sequence			
6.2 Recursive Procedures			
	G–1	Grid Paper	Student Support
	BLM 6–5	Section 6.2 Practice	Practice
6.3 Pascal's Triangle and Expanding Binomial Powers			
	BLM 6–6	Section 6.3 Practice	Practice
6.4 Arithmetic Sequences			
	G–1	Grid Paper	Student Support
	T–1	Microsoft® *Excel*	Student Support
	BLM 6–7	Section 6.4 Practice	Practice
6.5 Geometric Sequences			
	G–1	Grid Paper	Student Support
	BLM 6–8	Section 6.5 Practice	Practice
6.6 Arithmetic Series			
	BLM 6–9	Section 6.6 Practice	Practice
6.7 Geometric Series			
	G–1	Grid Paper	Student Support
	BLM 6–10	Section 6.7 Practice	Practice
	BLM 6–11	Section 6.7 Achievement Check Rubric	Assessment
Chapter 6 Review			
	G–1	Grid Paper	Student Support
	BLM 6–12	Chapter 6 Review	Practice
Chapter 6 Problem Wrap-Up			
	BLM 6–13	Chapter 6 Problem Wrap-Up Rubric	Assessment
Chapter 6 Practice Test			
	G–1	Grid Paper	Student Support
	BLM 6–14	Chapter 6 Practice Test	Summative Assessment
Chapter 6 Task: Mathematics in Media Studies			
	G–1	Grid Paper	Student Support
	BLM 6–15	Task: Mathematics in Media Studies Rubric	Assessment
	BLM 6–16	BLM Answers	Answers

Prerequisite Skills

Student Text Pages
352 to 353

Suggested Timing
45–60 min

Tools
- grid paper
- graphing calculator (optional)

Related Resources
- G–3 Four Quadrant Grids
- BLM 6–1 Prerequisite Skills
- BLM 6–2 Exploring Patterns and Sequences

Assessment
You may wish to use **BLM 6–1 Prerequisite Skills** as a diagnostic assessment. Refer students to the Skills Appendix for examples and further practice of topics.

Common Errors
- Students have difficulty visualizing the patterns in the sequences in question 1.

R_x Provide manipulatives and encourage students to construct the next few diagrams in the sequences.

- Students need additional instruction before completing the questions because they find it difficult to recall the properties of several different types of functions.

R_x Provide exemplars of linear, quadratic, and exponential function graphs for them to look at before they attempt their own graphs.

Teaching Suggestions

- Introduce the chapter by having students work in small groups to explore a variety of patterns and sequences. **BLM 6–2 Exploring Patterns and Sequences** could be used for this activity. Then have students work individually or with a partner on the investigation. Identify areas of strength and weakness and incorporate these into the lessons in this chapter.

Chapter Problem

The Chapter Problem is introduced on page 353. To engage student interest in the problem, present a set of fractal pictures to the class or show the PowerPoint® presentation found on the McGraw-Hill Ryerson Web site. Ask students to write a description of one of the fractals that they find particularly interesting and share these descriptions with the class. Students will revisit fractals throughout this chapter and will have opportunities to investigate a variety of sequences that arise from the study of specific fractals. The Chapter Problem is revisited in Section 6.1 (question 12), Section 6.2 (question 15), Section 6.3 (question 11), Section 6.5 (question 12), and Section 6.7 (question 13). These questions are designed to help students move toward the Chapter Problem Wrap-Up on page 411. You may wish to assign the Chapter Problem questions as they appear in each section or assign the set of questions at the end of the chapter. This problem lends itself well to independent research on the topic.