McGraw-Hill Ryerson

Functions 11

Teacher's Resource

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Overview of *Functions 11*

The McGraw-Hill Ryerson *Functions 11* program has six components.

Student Text

The student text introduces topics in real-world contexts. In each numbered section, **Investigate** activities encourage students to develop their own understanding of new concepts. **Examples** present solutions in a clear, step-by-step manner, and then the **Key Concepts** summarize the new principles. **Communicate Your Understanding** gives students an opportunity to reflect on the concepts of the numbered section, and helps you assess students' grasp of the new ideas and readiness to proceed with the exercises.

Practise questions are knowledge questions and assist students in building their understanding. **Connect and Apply** questions allow students to use what they have learned to solve problems and make connections among concepts. **Extend** questions are more challenging and thought-provoking. Answers to Practise, Connect and Apply, and Extend questions are provided at the back of the text. Most numbered sections conclude with a few **Math Contest** questions. **Chapter Tasks** are more involved problems. Sample solutions to the Chapter Tasks are provided in this Teacher's Resource.

A **Chapter Review** is provided at the end of each chapter. **Cumulative Reviews** are provided after Chapters 3, 5, and 7. A course review is provided that encompasses Chapters 1 to 7.

The text includes a number of items that can be used as assessment tools:

- **Communicate Your Understanding** questions assess student understanding of the concepts
- **Achievement Checks** provide opportunities for formative assessment using the four Achievement Chart Categories, Knowledge and Understanding, Thinking, Communication, and Application
- **Practice Tests** contain multiple choice, short response, and extended response questions to help model classroom testing practices
- **Chapter Problem Wrap-Ups** finish each chapter by providing a set of questions that involve all four Achievement Chart Categories
- **Chapter Tasks** are presented after each chapter and may combine concepts from the preceding chapters

Technology is integrated throughout the program and includes the use of scientific calculators, graphing calculators, computer algebra systems, spreadsheet programs, dynamic geometry software, and the Internet.

Teacher's Resource

This Teacher's Resource provides the following teaching and assessment suggestions:

- **Teaching Suggestions** for all the sections
- **Practice** and chapter-specific blackline masters
- Answers to the **Investigate** questions
- Responses for the **Communicate Your Understanding** questions
- Responses for the **Chapter Problem Wrap-Up** and **Chapter Tasks**
- Students' **Common Errors** and suggested remedies
- Solutions and rubrics for the **Achievement Check** questions
- **Differentiated Instruction** strategies for students

Computerized Assessment Bank CD-ROM

The Computerized Assessment Bank CD-ROM (CAB) contains over 1000 questions based on the material presented in the student text, and allows you to create and modify tests. Questions are connected to the chapters in the student text. The question types include: True/False, Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated to the corresponding Achievement Chart Category, specific curriculum expectation, and curriculum strand from the Ontario Mathematics MHF3U Curriculum.

Solutions CD

The Solutions CD provides worked-through solutions for all questions in the numbered sections of the student text, except for Achievement Check questions, which are in the Teacher's Resource. In addition, the Solutions CD provides complete solutions for questions in the Review, Practice Test, and Cumulative Review features.

Study Guide

The Study Guide provides additional information related to the Key Concepts sections in the student text. The guide includes practice questions for all sections of the text, a set of challenge questions for each chapter, and a practice exam that covers the entire course. Answers for all questions and full solutions for a selection of questions are provided at the back of the guide.

Web site

In addition to our McGraw-Hill Ryerson Web site, you can access the password protected site to obtain ready-made files for *The Geometer's Sketchpad*® activities in the text, files to support the student text activities, further support material for differentiated learners, and supplemental activities.

To access this site go to:
http://www.mcgrawhill.ca/books/functions11
username: functions11U
password: teacher11F

Structure of the Teacher's Resource

The teaching notes for each chapter have the following structure:

Chapter Opener

The following items are included in the Chapter Opener:

- **Specific Expectations** that apply to the chapter
- **Planning Chart** that provides an overview of each chapter at a glance, and specifies:
  Student Text Pages references and **Suggested Timing** for numbered sections
  Related blackline masters available on the Teacher's Resource CD-ROM
  Assessment blackline masters for each section of the chapter
  Special tools and/or technology tools that may be needed
- A Chapter Blackline Masters Checklist provides a list of the masters available on the Teacher's Resource CD-ROM and their purpose.

Prerequisite Skills

The following items are included in the margin:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The key items in this section include:
- **Teaching Suggestions** for how to use the Prerequisite Skills
- **Assessment** ideas on how to ascertain that students are ready for this chapter
- **Common Errors** and remedies to help you anticipate and deal with common errors that may occur
- Introduction to a **Chapter Problem** that may include questions designed to help students move toward the **Chapter Problem Wrap-Up** at the end of the chapter

Numbered Sections

The following items are listed in the margin:
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The **Teaching Suggestions** include the following key elements:
- **Student Text Pages** references and **Suggested Timing**
- **Teaching Suggestions** give insights or point out connections on how to present the material from the text
- **Investigate Answers** and **Communicate Your Understanding Responses** let you know the expected outcomes of these activities
- Notes for the **Practise, Connect and Apply**, and **Extend** questions in the text provide comments on specific questions to anticipate any difficulties, and hints on how to help students answer the questions
- **Achievement Check Answers**
- **Common Errors** and remedies give you ideas on how to help students who make typical mistakes
- **Ongoing Assessment** refers you to the **Achievement Check Rubric** to assess student achievement
- **Differentiated Instruction** items provide suggestions for alternative ways to approach some key topics for students
- **Mathematical Process Integration** chart lists questions that provide good opportunities for students to use the processes

End of Chapter Items

The **Chapter Review** and **Cumulative Reviews** (at the end of Chapters 3, 5, and 7) include the following items:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The **Chapter Problem Wrap-Up** includes the following elements:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice and remediation, assessment, or enhancement
- **Using the Chapter Problem** includes teaching suggestions specific to the problem
- **Summative Assessment** refers you to the **Chapter Problem Rubric** to assess student achievement
- **Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 response

The **Practice Test** has the following key features:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

- **Study Guide** directs students who have difficulty with specific questions to appropriate examples to review
- **Summative Assessment** refers you to the **Chapter Test** to assess student performance

A **Chapter Task** occurs at the end of each chapter and includes:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** useful for extra practice or remediation, assessment, or enhancement
- **Teaching Suggestions** with steps for you to follow, a list of questions you can use to help students begin the Task, and a list of questions you should consider when assessing students' responses
- **Ongoing Assessment** refers you to the **Chapter Task Rubric** to assess student achievement
- **Level 3 Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 answer

The **Teacher's Resource CD-ROM** provides various blackline masters, including:
- Generic Masters
- Technology Masters
- Practice Masters
- Assessment Masters
- Chapter-specific Masters

Program Philosophy

This McGraw-Hill Ryerson *Functions 11* program is designed to:
- provide full support in teaching the Ontario MHF3U mathematics curriculum
- support and extend students' progress from concrete to representational and abstract thinking
- offer a diversity of options that collectively deliver student and teacher success

Approaches to Teaching Mathematics

> *Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.*
> *Ontario Ministry of Education and Training, 2004*

Students learn best by using a concrete, discovery-oriented approach to develop concepts. Once these concepts have been developed, a connectionist approach helps students consolidate their learning.

Transmission-Oriented
- teaching involves "delivering" the curriculum
- focuses on procedures and routines
- emphasizes clear explanations and practice
- "chalk-and-talk"

Connectionist-Oriented
- teaching involves helping students develop and apply their own conceptual understandings
- focuses on different models and methods and the connections among them
- emphasizes "problematic" challenges and teacher-student dialogue

Discovery-Oriented
- teaching involves helping students learn by "doing"
- focuses on applying strategies to practical problems and using concrete materials
- emphasizes student-determined pacing
- "hands-on"

This variety of approaches can be seen in the *Functions 11* program design.

Feature	Teaching Style(s) Supported
Chapter Problem	connectionist
Investigate, Reflect	discovery, connectionist
Examples	transmission, connectionist
Key Concepts	connectionist, transmission
Communicate Your Understanding	connectionist, discovery
Practise	transmission
Connect and Apply	connectionist, transmission
Extend	connectionist, transmission
Review	transmission, connectionist
Task	discovery, connectionist

Instructional Practice

The resources available in today's classroom offer opportunities and challenges. Indeed, the principal challenge—one that many teachers of mathematics are reluctant to confront—is to teach successfully to the opportunities available.

Grouping

Instructional practice that incorporates a variety of grouping approaches enhances the richness of learning for students.

Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004

At one end of the scale, individual work provides an opportunity for students to work on their own, at their own pace. At the other extreme, class discussion of problems and ideas creates a synergistic learning environment. In between, carefully selected groups bring cooperative learning into play.

Manipulatives and Materials

Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.

Ontario Ministry of Education and Training, 2003

Although many teachers feel unsure about teaching with manipulatives and other concrete materials, many students find them a powerful way to learn. The *Functions 11* program supports the use of manipulatives, where appropriate, and helps you adapt to this kind of teaching. The Teaching Suggestions sections in the Teacher Resource provide suggestions for developing student understanding using semi-concrete materials, such as diagrams and charts.

Technology

In the *Functions 11* program, graphing calculator instructions are provided in parallel with conventional calculations.

Special computer software designed for the classroom and licensed by the Ministry of Education for use in Ontario classrooms, such as *The Geometer's Sketchpad*®, provide powerful tools for teaching and learning. The *Functions 11* program supports the use of such software as an enhancement to the classroom experience. Multiple solutions for worked-through examples in the text allow you to enjoy wide flexibility in lesson planning. As a result, you can plan activities using a combination of tools.

Materials to support using technology are available on the McGraw-Hill Ryerson Web site. The Internet provides great opportunities for enhancing learning. Recommended Web sites for your reference are provided via links on the McGraw-Hill Ryerson Web site at *http://www.mcgrawhill.ca/books/functions11*. For students, the *Functions 11* page on the McGraw-Hill Ryerson Web site at *http://www.mcgrawhill.ca* lists safe and reliable Web site links for students to explore as part of the *Functions 11* program.

The Geometer's Sketchpad® is licensed in Ontario for use by students at home. Consider providing each student with a copy of the software to install on a home computer so that it can be used as a tool for homework. Ensure that students without home computers have an alternative, pairing these students with students who are willing to share their home computers, or providing the software on computers in the school library.

Literacy

Effective mathematics classrooms show students that math is everywhere in their world. For example, students should see that their work in graphing can be used in Science class. The written work they produce explaining their answers is also a language arts product. When connections such as these are made, students begin to see that math is not an isolated subject, but rather a vital part of everyday life.

Connections

Connections give students help to understand a symbol, a phrase, or a new word. They also provide suggestions for connecting mathematics to literacy, by connecting terms in mathematics to vocabulary used in other contexts. This feature provides one more way for students to feel successful in mathematics.

Writing and Mathematics

Being able to communicate ideas clearly is an important part of the *Functions 11* program. Students are asked to write about the mathematics they are learning, and communicate their understanding about what they are learning.

Take time to discuss the importance of being able to communicate understanding. The students' responses are meant to communicate with you and are assessed as part of the mathematics work.

Problem Solving

> *Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.*
>
> *National Council of Teachers of Mathematics, 2000*

Problem solving is an integral part of mathematics learning. The National Council of Teachers of Mathematics recommends that problem solving be the focus of all aspects of mathematics teaching because it encompasses skills and functions, which are an important part of everyday life.

> *NCTM Problem-Solving Standard*
> Instructional programs should enable all students to—
> - Build new mathematical knowledge through problem solving
> - Solve problems that arise in mathematics and in other contexts
> - Apply and adapt a variety of appropriate strategies to solve problems
> - Monitor and reflect on the process of mathematical problem solving

McGraw-Hill Ryerson has made the problem-based learning approach the focus of its program. In *Functions 11*, a variety of problem-solving opportunities are provided for students:

- The **Mathematical Processes Expectations** (Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating) are embedded throughout the student textbook. The Teacher's Resource identifies which questions provide good opportunities to use the mathematical processes for each numbered section.
- **Math Contest** questions are included at the end of most numbered section exercises to give students more opportunities to solve non-routine problems.
- Each chapter begins with an investigation of a real-life problem. The **Chapter Problem** is then revisited throughout the chapter through **Chapter Problem** questions, and ends with the **Chapter Problem Wrap-Up**.
- At the end of each chapter, students are presented with a **Chapter Task** where the solution path is not readily apparent and where solving the problem requires more than just applying a familiar procedure. These tasks require students to apply what they have learned in the current chapter and previous chapters to solve broad-based problems.
- An additional Task for each chapter can be found on the Teacher's Resource Web site. Go to *http://www.mcgrawhill.ca/books/functions11* and follow the links.

Mathematical Processes

The seven expectations presented at the start of the mathematics curriculum in Ontario describe the mathematical processes that students need to learn and apply as they investigate mathematical concepts, solve problems, and communicate their understanding. Although the seven processes are categorized, they are interconnected and are integrated into student learning in all areas of the *Functions 11* program.

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Communication	Communicating
Application	Selecting tools and computational strategies Connecting
Thinking	Problem solving Reasoning and proving Reflecting

Problem Solving

Problem solving is the basis of the *Functions 11* program. Students can achieve the expectations by using this essential process, and it is an integral part of the mathematics curriculum in Ontario. Useful problem-solving strategies include: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; solving a simpler problem; working backward; using logical reasoning.

Reasoning and Proving

Critical thinking is an essential part of mathematics. As the students investigate mathematical concepts in *Functions 11*, they learn to: make generalizations based on specific findings; use counter-examples to disprove conjectures; use deductive reasoning.

Reflecting

Students are given opportunities to regularly and consciously reflect on their thought processes as they work through the problems in *Functions 11*. As they reflect, they learn to: recognize when the technique they are using is not helpful; make a conscious decision to switch to a different strategy; rethink the problem; search for related knowledge; determine the reasonableness of an answer.

Selecting Tools and Computational Strategies

Students are given many opportunities to use a variety of manipulatives, electronic tools, and computational strategies in the *Functions 11* program. The student text provides examples of the use of various types of technology to perform mathematical tasks, investigate mathematical ideas, and solve problems.

Connecting

Functions 11 is designed to give students many opportunities to make connections between concepts, skills, mathematical strands, and other subject areas. These connections help them see that mathematics is much more than a series of isolated skills and concepts. Connecting mathematics to their everyday lives also helps students see that mathematics is useful and relevant outside the classroom.

Representing

Throughout the *Functions 11* program, students represent mathematical ideas in various forms: numeric, geometric, graphical, algebraic, pictorial, and concrete representations, as well as representation using technology. Encourage students encouraged to use more than one representation for a single problem, seeing the connections between them.

Communicating

Students use many different ways of communicating mathematical ideas in the *Functions 11* program, including: oral, visual, writing, numbers, symbols, pictures, graphs, diagrams, and words. The process of communication helps students reflect on and clarify ideas, relationships, and mathematical arguments.

Using Mathematical Processes

Encourage students to use the mathematical processes in their work by prompting them with questions such as the following:

- *How can you tell whether your answer is correct/reasonable?* This promotes reasoning and reflection.
- *Why did you choose this method?* This promotes reflection, reasoning, selecting tools and computational strategies, and communication.
- *Could you have solved the problem another way?* This promotes reasoning, reflection, selecting tools and computational strategies, representing, and communication.
- *In what context have you solved a problem like this before?* This promotes connecting.

Use **Assessment Masters A1** through **A7** to share with students evidence that you may be looking for when you are assessing them on the mathematical processes expectations.

You can also encourage students to use a Think-Pair-Share approach to problem solving (see the **Assessment** section in this Program Overview). They will benefit greatly from brainstorming ideas and comparing methods of approach. A useful life skill is willingness to try different methods of solving a problem, learning from methods that perhaps do not reach the final goal, and being able to change their approach to reach the solution.

Technology

Technology is a major focus in several of the chapters of *Functions 11*, providing students with hands-on experience in creating and transforming graphs of functions, using graphing calculators and/or *The Geometer's Sketchpad*®. If at all possible, a classroom environment should be in place in which students are encouraged to reach for and apply technology whenever they feel the situation calls for it. In such an environment, the ongoing use of technology becomes another tool in the student's problem-solving tool kit.

Technology Teaching Suggestions

- If you are using an interactive white board (such as the Smartboard®) or have a projector, you can demonstrate the calculator steps as part of the lesson.
- If this is the first time that students have used the **TI-Nspire™ CAS Graphing Calculator**, you may need to take some time to let students familiarize themselves with the keyboard and screen, and show them how to open documents and pages. Refer to the McGraw-Hill Ryerson Web site for step-by-step instructions for all calculator functions needed.

Types of Programs

Several software programs are used or suggested in *Functions 11*: *The Geometer's Sketchpad*®; *Fathom*™; Microsoft® *Excel*.

Technology BLMs are available, providing students with step-by-step directions on how to use technology to explore the mathematical concepts of the text:

- T–1 Microsoft® *Excel*
- T–2 *The Geometer's Sketchpad*® 4
- T–3 *Fathom*™

The **Technology Appendix**, on pages 496–516, of the student text provides clear step-by-step instruction in the basic functions of the TI-83/84Plus graphing calculator and the basic features of *The Geometer's Sketchpad*®.

Support for the TI-Nspire CAS graphing calculator is provided on the McGraw-Hill Ryerson Web site. A document showing step-by-step instructions for all calculator functions needed for *Functions 11* is posted there and can be downloaded by teachers or students.

Assessment

The main purpose of assessment is to improve student learning. Assessment data help you determine the instructional needs of your students during the learning process and some data are used for reporting.

Assessment can be used diagnostically to determine prior knowledge, formatively to inform instructional planning, and in a summative manner to determine how well the students have achieved the expectations at the end of a learning cycle.

Diagnostic Assessment

Assessment for diagnostic purposes can determine where individual students will need support and will help to determine how the classroom time needs to be spent. *Functions 11* provides you with diagnostic support at the start of the text and the beginning of every chapter.

- The **Prerequisite Skills** section at the beginning of each chapter provides connections to essential concepts and skills needed for the upcoming chapter.
- For students needing additional support, **Practice Masters** are provided in this Teacher's Resource that both develop conceptual understanding and improve procedural efficiency.

Diagnostic support is also provided at the start of every section.

- Each numbered section of the text begins with an introduction to facilitate open discussion in the classroom.
- Each activity starts with a question that stimulates prior knowledge and allows you to monitor students' readiness.

Formative Assessment

Formative assessment tools are provided throughout the text and Teacher's Resource. Formative assessment allows you to determine students' strengths and weaknesses and guide your class towards improvement. *Functions 11* provides blackline masters for student use that complement the text in areas where formative assessment indicates that students need support.

The **Chapter Opener**, visual, and the introduction to the **Chapter Problem** at the beginning of each chapter in the student book provide opportunities for you to do a rough formative assessment of student awareness of the chapter content.

Within each lesson:
- **Key Concepts** can be used as a focus for classroom discussion to determine the students' readiness to continue.
- **Communicate Your Understanding** questions allow you to determine if students have developed the needed conceptual understanding and/or skills.
- **Practise, and Connect and Apply** questions offer an opportunity for you to monitor students' procedural skills, their application of procedures, their ability to communicate their understanding of concepts, and their ability to solve problems related to the section's Key Concepts.
- **Achievement Check** questions allow students to demonstrate their knowledge and understanding and their ability to apply, think of, and communicate what they have learned.
- **Chapter Problem** questions provide opportunities to verify that students are developing the skills and understanding they need to complete the **Chapter Problem Wrap-Up** questions.
- **Extend** questions are more challenging and thought-provoking, and are aimed at Level 3 and 4 performance.
- **Chapter Reviews** and **Cumulative Reviews** provide an opportunity to assess Knowledge/Understanding, Thinking, Communication, and Application.

Summative Assessment

Summative data is used for both planning and evaluation.
- A **Practice Test** at the end of each chapter assesses students' achievement in Knowledge/Understanding, Thinking, Communication, and Application.
- The **Chapter Problem** uses an open-ended question format to evaluate students' understanding of Knowledge and Understanding, Thinking, Communication, and Application.
- **Chapter Tasks** include open-ended questions. Rubrics are presented at the end of each chapter.

Portfolio Assessment

Student-selected portfolios:
- Help you assess students' growth and mathematical understanding
- Provide insight into students' self-awareness about their own progress
- Help parents understand their child's growth

Functions 11 has many components that provide ideal portfolio items, such as:
- Students' responses to the **Chapter Problem Wrap-Up** assignments
- Responses to **Communicate Your Understanding** questions, which allow students to explore their initial understanding of concepts
- Answers to **Achievement Check** questions, which are designed to show students' mastery of specific expectations
- **Chapter Task** assignments

Assessment Masters

Functions 11 provides chapter-specific blackline masters, such as Chapter Tests, Chapter Problem Wrap-Up rubrics, and Task rubrics. In addition, the program includes generic assessment blackline masters. These BLMs will help you to effectively monitor student progress and evaluate instructional needs.

Generic Assessment BLM	Type	Purpose
A–1 **Problem Solving**	**Checklist**	**Assess** students' problem solving skills.
A–2 **Reasoning and Proving**	**Checklist**	**Assess** students' reasoning and proving skills.

Generic Assessment BLM	Type	Purpose
A–3 **Reflecting**	**Checklist**	**Assess** students' understanding of an expectation.
A–4 **Selecting Tools and Computational Strategies**	**Checklist**	**Assess** students' ability to select the appropriate tool(s) and strategies for solving a problem.
A–5 **Connecting**	**Checklist**	**Assess** students' ability to make connections of concepts learned to problem situations, to connect prior learning with current concepts.
A–6 **Representing**	**Checklist**	**Assess** students' ability to represent the problem and/or solution using an appropriate representation.
A–7 **Communicating**	**Checklist**	**Assess** students' ability to explain and communicate effectively their learning.
A-8 **Learning Skills Descriptors**	**Checklist**	**Diagnose** and report on overall learning and progress.
A-9 **Study Skills for Success in Math**	**Checklist**	**A** list of good study habits that you can share with students.

Reaching all Students

Functions 11 accommodates a broad range of needs and learning styles. This Teacher's Resource provides support in addressing multiple intelligences and learning styles through **Differentiated Instruction** items in the margin which provide suggestions for alternative ways to approach key topics.

Accommodations for Students with Language Difficulties

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Give concise, step-by-step directions • Teach students to look for cue words, highlight these words • Use visual models • Use visual representations to accompany word problems • Encourage students to look for common patterns in word problems	• Provide reference charts with operations and formulae • Post reference charts with math vocabulary • Reinforce learning with visual aids and manipulatives • Using a visual format, post strategies for problem solving • Use a peer tutor or buddy system	• Read instructions/ word problems to students on tests • Extend time lines

Accommodations for Students with Visual/Perceptual/Spatial/Motor Difficulties

Instructional	Environmental	Assessment
• Reduce copying • Provide worksheets • Provide concrete examples • Provide a math journal • Encourage and teach self-talk strategies • Chunk learning and tasks	• visual bombardment • a work carrel or work area that is not visually distracting • rest periods and breaks	• Extend time lines • Provide consumable tests • Reduce the number of questions required to indicate competency • Provide a scribe for lengthy written answers

Accommodations for Students with Memory Difficulties

Instructional	Environmental	Assessment
• Regularly review concepts • Activate prior knowledge • Teach mnemonic strategies (e.g., FOIL, CAST) • Teach visualization strategies • Colour-code steps • Teach math concepts related to daily living	• Provide reference charts with commonly used facts, formulae, and steps for problem-solving • Allow use of a calculator • Use games and computer programs for drill and repetition	• Allow use of formula sheets • Allow use of other reference charts as appropriate • Allow use of calculators • Extend time lines • Present one concept-type of question at a time

Accommodations for Enrichment

Instructional	Environmental	Assessment
• Structure learning activities to develop higher-order thinking skills • Provide open-ended questions • Value learner's interests and learning style, and allow for student input • Encourage students to link learning to wider applications • Encourage learners to reflect on the process of their own learning • Encourage and reward creativity • Avoid repetitive tasks and activities	• Encourage a stimulating environment that invites exploration of mathematical concepts • Display pictures of role models who excel in mathematics • Provide access to computer programs that extend learning	• Reduce the number of questions • Allow for opportunities to use non-traditional formats • Pose questions that require higher-level thinking skills • Reward creativity

Accommodations for ESL Students

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Explain colloquial expressions and figurative speech • Review comparative forms of adjectives	• Display reference charts with mathematical terms and language • Encourage personal math dictionaries with math terms and formulae	• Allow access to personal math dictionaries • Read instructions to students and clarify terms • Allow additional time

Differentiated Instruction ideas and different learning strategies are recommended as alternative ways to approach some key topics. By addressing these topics in a new or different way, you can provide students with the opportunity to learn in a manner that may increase their chances of success.

Types of Strategies

Many different types of cooperative learning strategies are suggested in the Differentiated Instruction margin items in this Teacher's Resource.

Think-Pair-Share

Students individually think about a concept and then pick a partner to share their ideas with. For example, students might work on the Communicate Your Understanding questions, and then discuss the concepts with a partner. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Jigsaw

Individual group members research a specific part of the information for a project. They then share what they have learned so that the entire group gets information about all areas being studied.

Another way of using the Jigsaw method is to assign "home" and "expert" groups. For example, to explore the effects of k in exponential function $y = k(x^n)$, you might have a home group of four in which each member is responsible for researching one of $k > 1$, $0 < k < 1$, $-1 < k < 0$, and $k < -1$. Individual members then move to expert groups which include all of the students responsible for researching one of the possible intervals for k. Each expert group discusses and comes to a consensus. Individual members then return to their home group and teach other members about their results.

Placemat

In groups of four, students individually complete their section of a placemat (BLM provided on the *Functions 11: Teacher's Resource* CD-ROM). The group then pools their responses and completes the centre portion of the placemat with group responses. Use for pre-assessment, review, or to summarize a topic.

Concept Attainment

Based on a list of examples and non-examples of a concept, students identify and define the concept. Then, they determine the critical attributes of the concepts and apply their defined concept to generate their own examples and non-examples.

Think Aloud
Students work through a problem in front of the class, verbalizing their thinking throughout. This method can help develop process thinking in students.

Decision Tree
Students use a graphic organizer flow chart to identify key decisions and consequences.

Carousel
Students at different stations display and explain topics or concepts to other classmates who rotate through the stations, usually in order.

Timed Retell
Students sit in pairs facing each other. After some preparation time, Student A has 30 s to tell what she or he knows about the topic to Student B. Student B then retells the talk for about 30 s and adds additional information. Both students then write a summary of the talk.

Frayer Model
Students complete four quadrants for a specified topic: definition, facts/characteristics, examples, and non-examples. Variation: Give students a completed model and ask them to identify the topic/concept.

Inside/Outside Circle
Students face each other in pairs, forming two concentric circles. Students take turns giving information to their partner, then the outside circle rotates one person to the right. Students then share information with their new partners. The process continues until the students in the outside circle return to their starting point.

Four Corners
Students move from one corner of the room based on preference or opinion. (Example: To differentiate a particular function, students choose from algebraic model using pencil and paper, algebraic model using first principles, algebraic model using technology, dynamic model (GSP).)

Graffiti
There are two different methods of graffiti. In the first method, students in groups take turns adding information to a sheet of paper passed around the group (for example, adding lines to a solution). In the second method, groups or pairs of students move around the room and add information to questions posted on chart paper around the room.

Gallery Walk
Groups move from station to station, read what is there, and add information, eventually returning to their original station.

Anticipation Guide
Prior to a unit of study, students form conjectures about the material they are about to learn. Return to the conjectures at the end of the unit to compare.

What-So What Double Entry
Use a T-chart to organize information. On the left side, list information. On the right side, list how information is used.

Word Wall/Information Wall
Progressively place terminology, formulas, or information on a classroom wall during a unit of study. Actively re-group the information as the unit progresses to emphasize important concepts.

K-W-L Instructional Strategy
Students create a chart with three columns: K, W, and L. They fill in columns 1 and 2 based on their prior knowledge of a concept. They complete column 3 after reading, researching or completing a unit about the concept.

Curriculum Correlation between McGraw-Hill Ryerson *Functions 11* and The Ontario Curriculum *Functions, Grade 11, University Preparation* (MCR3U)

This course introduces the mathematical concept of the function by extending students' experiences with linear and quadratic relations. Students will investigate properties of discrete and continuous functions, including trigonometric and exponential functions; represent functions numerically, algebraically, and graphically; solve problems involving applications of functions; investigate inverse functions; and develop facility in determining equivalent algebraic expressions. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Prerequisite: Principles of Mathematics, Grade 10, Academic

Mathematical process expectations.

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

Problem Solving	• develop, select, apply, compare and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., use inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical idea and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representation to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

The mathematical process expectations are integrated throughout *McGraw-Hill Ryerson Functions 11.*

A. Characteristics of Functions

Overall Expectations

By the end of this course, students will:

A1. demonstrate an understanding of functions, their representations, and their inverses, and make connections between the algebraic and graphical representations of functions using transformations;

A2. determine the zeros and the maximum or minimum of a quadratic function, and solve problems involving quadratic functions, including those arising from real-world applications;

A3. demonstrate an understanding of equivalence as it relates to simplifying polynomial, radical, and rational expressions.

Specific Expectations

	Chapter/Section	Pages
1. Representing Functions		
By the end of this course students will:		
A1.1 explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., identifying a one-to-one or many-to-one mapping; using the vertical line test) ***Sample problem:*** Investigate, using numeric and graphical representations, whether the relation $x = y^2$ is a function, and justify your reasoning.	**1.1, 1.2**	1–24
A1.2 represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions [e.g., evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$];	**1.2**	16–24
A1.3 explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$; describe the domain and range of a function appropriately (e.g., for $y = x^2 + 1$, the domain is the set of real numbers, and the range is $y \geq 1$); and explain any restrictions on the domain and range in contexts arising from real-world applications ***Sample problem:*** A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?	**1.1, 1.2, 1.3**	1–34
A1.4 relate the process of determining the inverse of a function to their understanding of reverse processes (e.g., applying inverse operations);	**2.7**	132–141
A1.5 determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the graph of a function and the graph of its inverse (e.g., the graph of the inverse is the reflection of the graph of the function in the line $y = x$) ***Sample problem:*** Given a graph and a table of values representing population over time, produce a table of values for the inverse and graph the inverse on a new set of axes.	**2.7**	132–141
A1.6 determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine whether or not the inverse relation is a function ***Sample problem***: Given the graph of $f(x) = x^2$, graph the inverse relation. Compare the domain and range of the function with the domain and range of the inverse relation, and investigate connections to the domain and range of the functions $g(x) = \sqrt{x}$, and $h(x) = -\sqrt{x}$.	**2.7**	132–141
A1.7 determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function [e.g., $f(x) = (x - 2)2 - 5$], and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the algebraic representations of a function and its inverse (e.g., the inverse of a linear function involves applying the inverse operations in the reverse order) ***Sample problem***: Given the equations of several linear functions, graph the functions and their inverses, determine the equations of the inverses, and look for patterns that connect the equation of each linear function with the equation of the inverse.	**1.7**	60–69
A1.8 determine, through investigation using technology, and describe the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$ in terms of transformations on the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) ***Sample problem***: Investigate the graph $f(x) = 3(x - d)^2 + 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.	**2.3, 2.4, 2.5, 2.6**	97–141

	Chapter/Section	Pages
A1.9 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$, and state the domain and range of the transformed functions ***Sample problem***: Transform the graph of $f(x)$ to sketch $g(x)$, and state the domain and range of each function, **for** the following: $f(x) = \sqrt{x}$, $g(x) = \sqrt{x} - 4$, $f(x) = \frac{1}{x}$, $g(x) = -\frac{1}{x} + 1$.	**2.3, 2.4, 2.5, 2.6**	97–141
2. Solving Problems Involving Quadratic Functions		
By the end of this course, students will:		
A2.1 determine the number of zeros (i.e., x-intercepts) of a quadratic function, using a variety of strategies (e.g., inspecting graphs; factoring; calculating the discriminant) ***Sample problem***: Investigate, using graphing technology and algebraic techniques, the transformations that affect the number of zeros for a given quadratic function.	**1.5**	43–51
A2.2 determine the maximum or minimum value of a quadratic function whose equation is given in the form $f(x) = ax^2 + bx + c$, using an algebraic method (e.g., completing the square; factoring to determine the zeros and averaging the zeros) ***Sample problem:*** Explain how partially factoring $f(x) = 3x^2 - 6x + 5$ into the form $f(x) = 3x(x - 2) + 5$ helps you determine the minimum of the function.	**1.3, 1.5**	25–33, 43–51
A2.3 solve problems involving quadratic functions arising from real-world applications and represented using function notation ***Sample problem:*** The profit, $P(x)$,of a video company, in thousands of dollars, is given by $P(x) = -5x^2 + 550x - 5000$, where x is the amount spent on advertising, in thousands of dollars. Determine the maximum profit that the company can make, and the amounts spent on advertising that will result in a profit and that will result in a profit of at least \$4 000 000.	**1.3**	25–33
A2.4 determine, through investigation, the transformational relationship among the family of quadratic functions that have the same zeros, and determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function ***Sample problem:*** Determine the equation of the quadratic function that passes through (2, 5) if the roots of the corresponding quadratic equation are $1 + \sqrt{5}$ and $1 - \sqrt{5}$.	**1.6**	52–59
A2.5 solve problems involving the intersection of a linear function and a quadratic function graphically and algebraically (e.g., determining the time when two identical cylindrical water tanks contain equal volumes of water, if one tank is being filled at a constant rate and the other is being emptied through a hole in the bottom) ***Sample problem***: Determine, through investigation, the equations of the lines that have a slope of 2 and that intersect the quadratic function $f(x) = x(6 - x)$ once; twice; never.	**1.7**	60–69
3. Determining Equivalent Algebraic Expressions		
By the end of this course, students will:		
A3.1 simplify polynomial expressions by adding, subtracting, and multiplying ***Sample problem***: Write and simplify an expression for the volume of a cube with edge length $2x + 1$.	**2.1**	78–85
A3.2 verify, through investigation with and without technology, that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, $a = 0$, $b = 0$, and use this relationship to simplify radicals (e.g., $\sqrt{24}$) and radical expressions obtained by adding, subtracting, and multiplying [e.g., $(2 + \sqrt{6})(3 - \sqrt{12})$]	**1.4**	34–40
A3.3 simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values ***Sample problem:*** Simplify $\frac{2x}{4x^2 + 6x} - \frac{3}{2x + 3}$, and state the restrictions on the variable.	**2.2**	88–96
A3.4 determine if two given algebraic expressions are equivalent (i.e., by simplifying; by substituting values) ***Sample problem***: Determine if the expressions $\frac{2x^2 - 4x - 6}{x + 1}$ and $8x^2 - 2x(4x - 1) - 6$ are equivalent.	**2.1**	78–85

B. Exponential Functions

Overall Expectations

By the end of this course, students will:

B1. evaluate powers with rational exponents, simplify expressions containing exponents, and describe properties of exponential functions represented in a variety of ways;

B2. make connections between the numeric, graphical, and algebraic representations of exponential functions;

B3. identify and represent exponential functions, and solve problems involving exponential functions, including those arising from real-world applications.

Specific Expectations

	Chapter/Section	Pages
1. Representing Exponential Functions		
By the end of this course students will:		
B1.1 graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ ($a > 0$, a $\neq 1$), define this relation as the function $f(x) = a^x$, and explain why it is a function	3.4	178–187
B1.2 determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers) ***Sample problem***: The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$.What value would you assign to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.	3.3	170–177
B1.3 simplify algebraic expressions containing integer and rational exponents [e.g., $(x^{\frac{1}{3}}) \div (x^{\frac{1}{2}})$, $(x^6y^3)^{\frac{1}{3}}$], and evaluate numerical expressions containing integer and rational exponents and rational bases [e.g. , 2^{-3}, $(-6)^3$, $4^{\frac{1}{2}}$, 1.01^{120}];	3.2, 3.3	160–177
B1.4 determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x(a > 0, a \neq 1)$, function machines] ***Sample problem***: Graph $f(x) = 2^x$, $g(x) = 3^x$, and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the y-intercepts.	3.4, 3.5	178–198
2. Connecting Graphs and Equations of Exponential Functions		
By the end of this course, students will:		
B2.1 distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations) [Sample problem: Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.	3.1	150–159
B2.2 determine, through investigation using technology, and describe the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$ in terms of transformations on the graph of $f(x) = ax(a > 0$, $a \neq 1)$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) ***Sample problem***: Investigate the graph $f(x) = 3^{x-d} - 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.	3.5	188–198
B2.3 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graph of $f(x) = ax$, $a > 0$, $a \neq 1$, and state the domain and range of the transformed functions ***Sample problem***: Transform the graph of $f(x) = 3x$ to sketch $g(x) = 3^{-(x+1)} - 2$, and state the domain and range of each function.	3.5	188–198
B2.4 determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases [e.g., $f(x) = 9^x$ can be expressed as $f(x) = 3^{2x}$], and explain the connections between the equivalent forms in a variety of ways (e.g., comparing graphs; using transformations; using the exponent laws)	3.5	188–198

	Chapter/Section	Pages
B2.5 represent an exponential function with an equation, given its graph or its properties ***Sample problem***: Write two equations to represent the same exponential function with a y-intercept of 5 and an asymptote at $y = 3$. Investigate whether other exponential functions have the same properties. Use transformations to explain your observations.).	**3.1, 3.2,** **3.4, 3.5**	148–169 178–198
3. Solving Problems Involving Exponential Functions		
By the end of this course, students will:		
B3.1 collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data ***Sample problem***: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.	**3.6**	199–209
B3.2 identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve) ***Sample problem***: Using data from Statistics Canada, investigate to determine if there was a period of time over which the increase in Canada's national debt could be modelled using an exponential function.)	**3.1, 3.2**	148–169
B3.3 solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations ***Sample problem***: The temperature of a cooling liquid over time can be modelled by the exponential function $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28 °C.	**3.6**	199–209

C. Discrete Functions

Overall Expectations

By the end of this course, students will:

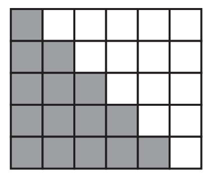
C1. demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle;

C2. demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems;

C3. make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities.

Specific Expectations

	Chapter/Section	Pages
1. Representing Sequences		
By the end of this course students will:		
C1.1 make connections between sequences and discrete functions, represent sequence using function notation, and distinguish between a discrete function and a continuous function [e.g., $f(x) = 2x$, where the domain is the set of natural numbers, is a discrete linear function, and its graph is a set of equally spaced points; $f(x) = 2x$, where the domain is the set of real numbers, is a continuous linear function and its graph is a straight line]	**6.1**	354–364
C1.2 determine and describe (e.g., in words; using flow charts) a recursive procedure for generating a sequence, given the initial terms (e.g., 1, 3, 6, 10, 15, 21, ...), and represent sequences as discrete functions in variety of ways (e.g., tables of values, graphs)	**6.1, 6.2**	354–372
C1.3 connect the formula for the nth term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula;	**6.1, 6.2**	354–372

	Chapter/Section	Pages
C1.4 represent a sequence algebraically using a recursion formula, function notation, or the formula for the nth term [e.g., represent 2, 4, 8, 16, 32, 64, ... as $t_1 = 2$; $t_n = 2t_{n-1}$, as $f(n) = 2^n$, or as $t_n = 2^n$, or represent $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$ as $t_1 = \frac{1}{2}$; $t_n = t_{n-1} + \frac{1}{n(n-1)}$, as $f(n) = \frac{n}{n+1}$, or as $t_n = \frac{n}{n+1}$, where n is a natural number], and describe the information that can be obtained by inspecting each representation (e.g., function notation or the formula for the nth term may show the type of function; a recursion formula shows the relationship between terms) ***Sample Problem***: Represent the sequence 0, 3, 8,15, 24, 35, ... using a recursion formula, function notation, and the formula for the nth term. Explain why this sequence can be described as a discrete quadratic function. Explore how to identify a sequence as a discrete quadratic function by inspecting the recursion formula.)	6.1	354–364
C1.5 determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal's triangle, and represent the patterns in a variety of ways (e.g., tables of values, algebraic notation)	6.3	373–379
C1.6 determine, through investigation, and describe the relationship between Pascal's triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents [e.g., $(1 + x)^4$, $(2x - 1)^5$, $(2x - y)^6$, $(x^2 + 1)^5$]	6.3	373–379
2. Investigating Arithmetic and Geometric Sequences and Series		
By the end of this course, students will:		
C2.1 identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation	6.4, 6.5	380–394
C2.2 determine the formula for the general term of an arithmetic sequence [i.e., $t_n = a + (n - 1)d$] or geometric sequence (i.e., $t_n = ar^{n-1}$), through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate any term in a sequence	6.4, 6.5	380–394
C2.3 determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate the sum of a given number of consecutive terms ***Sample problem***: Given the array built with grey and white connecting cubes, investigate how different ways of determining the total number of grey cubes can be used to evaluate the sum of the arithmetic series 1 + 2 + 3 + 4 + 5. Extend the series, use patterning to make generalizations for finding the sum, and test the generalizations for other arithmetic series. 	6.6, 6.7	395–409
C2.4 solve problems involving arithmetic and geometric sequences and series, including those arising from real-world applications	6.4, 6.5, 6.6, 6.7	380–409
3. Solving Problems Involving Financial Applications		
By the end of this course, students will:		
C3.1 make and describe connections between simple interest, arithmetic sequences, and linear growth, through investigation with technology (e.g., use a spreadsheet or graphing calculator to make simple interest calculations, determine first differences in the amounts over time, and graph amount versus time) ***Sample problem***: Describe an investment that could be represented by the function $f(x) = 500(1.05^x)$.	7.1	418–425
C3.2 make and describe connections between compound interest, geometric sequences, and exponential growth, through investigation with technology (e.g., use a spreadsheet to make compound interest calculations, determine finite differences in the amounts over time, and graph amount versus time) ***Sample problem***: Describe an investment that could be represented by the function $f(x) = 500(1.05)^x$.	7.2	426–435
C3.3 solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), the principal, P (also referred to as present value, PV), or the interest rate per compounding period, i, using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$] ***Sample problem***: Two investments are available, one at 6% compounded annually and the other at 6% compounded monthly. Investigate graphically the growth of each investment, and determine the interest earned from depositing $1000 in each investment for 10 years.	7.2, 7.3	426–443
C3.4 determine, through investigation using technology (e.g., scientific calculator; the TVM solver in a graphing calculator; online tools), the number of compounding periods, n, using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]; describe strategies (e.g., guessing and checking; using the power of a power rule for exponents; using graphs) for calculating this number and solve related problems	7.2, 7.3	426–443
C3.5 explain the meaning of the term *annuity*, and determine the relationships between ordinary simple annuities (i.e., annuities in which payments are made at the *end* of each period and compounding and payment periods are the same), geometric series, and exponential growth, through investigation with technology (e.g., use a spreadsheet to determine and graph the future value of an ordinary simple annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary annuity are related to the terms of a geometric series)	7.4	444–455

	Chapter/Section	Pages
C3.6 determine, through investigation using technology (e.g., the TVM Solver in a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary annuities in situations where the compounding period and the payment period are the same (e.g., long-term savings plans, loans) ***Sample problem***: Compare the amounts at age 65 that would result from making an annual deposit of \$1000 starting at age 20, or from making an annual deposit of \$3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?	7.4, 7.5	444–465
C3.7 solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary annuity in situations where the compounding period and the payment period are the same (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).	7.5	456–465

D. Trigonometric Functions

Overall Expectations

By the end of this course, students will:

D1. determine the values of the trigonometric ratios for angles less than 360º; prove simple trigonometric identities; and solve problems using the primary trigonometric ratios, the sine law, and the cosine law;

D2. demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions;

D3. identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including those arising from real-world applications.

Specific Expectations

	Chapter/Section	Pages
1. Determining and Applying Trigonometric Ratios		
By the end of this course students will:		
D1.1 determine the exact values of the sine, cosine, and tangent of the special angles: 0º, 30º, 45º, 60º, and 90º	**4.1**	222–231
D1.2 determine the values of the sine, cosine, and tangent of angles from 0º to 360º, through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to special angles)	**4.1**	222–231
D1.3 determine the measures of two angles from 0º to 360º for which the value of a given trigonometric ratio is the same;.	**4.2**	232–242
D1.4 define the secant, cosecant, and cotangent ratios for angles in a right triangle in terms of the sides of the triangle (e.g., $\sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$), and relate these ratios to the cosine, sine, and tangent ratios (e.g., $\sec A = \frac{1}{\cos A}$)	**4.3**	243–248
D1.5 prove simple trigonometric identities, using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$; the quotient identity $\tan x = \frac{\sin x}{\cos x}$; and the reciprocal identities $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$, and $\cot x = \frac{1}{\tan x}$. ***Sample problem***: Prove that $1 - \cos^2 x = \sin x \cos x \tan x$.	**4.6**	270–275
D1.6 pose and solve problems involving right triangles and oblique triangles in two-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case);	**4.4**	249–260
D1.7 pose and solve problems involving right triangles and oblique triangles in three-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law ***Sample problem***: Explain how a surveyor could find the height of a vertical cliff that is on the other side of a raging river, using a measuring tape, a theodolite, and some trigonometry. Determine what the surveyor might measure, and use hypothetical values for these data to calculate the height of the cliff.	**4.5**	261–270
2. Connecting Graphs and Equations of Sinusoidal Functions		
By the end of this course, students will:		
D2.1 describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numerical or graphical representation	**5.1**	294–293

	Chapter/Section	Pages
D2.2 predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural-gas consumption in Ontario from previous consumption)	5.1	284–293
D2.3 make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining why the relationship is a function	5.2	294–303
D2.4 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in degrees, and determine and describe key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)	5.2	294–303
D2.5 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, where $f(x) = \sin x$ or $f(x) = \cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions) ***Sample problem***: Investigate the graph $f(x) = 2\sin(x - d) + 10$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.	5.3	304–312
D2.6 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$	5.3, 5.4	304–321
D2.7 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions ***Sample problem***: Transform the graph of $f(x) = \cos x$ to sketch $g(x) = 3\cos 2x - 1$, and state the domain and range of each function.	5.4	313–321
D2.8 represent a sinusoidal function with an equation, given its graph or its properties ***Sample problem***: A sinusoidal function has an amplitude of 2 units, a period of 180º, and a maximum at (0, 3). Represent the function with an equation in two different ways.	5.4	313–321
3. Solving Problems Involving Sinusoidal Functions		
By the end of this course, students will:		
D3.1 collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data ***Sample problem***: Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data	5.5	322–332
D3.2 identify sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range ***Sample problem***: Using data from Statistics Canada, investigate to determine if there was a period of time over which changes in the population of Canadians aged 20–24 could be modelled using a sinusoidal function.	5.5	322–332
D3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles ***Sample problem***: Investigate, using graphing technology in degree mode, and explain how the function $h(t) = 5\sin(30(t + 3))$ approximately models the relationship between the height and the time of day for a tide with an amplitude of 5 m, if high tide is at midnight.	5.6	333–342
D3.4 predict the effects on a mathematical model (i.e., graph, equation) of an application involving sinusoidal functions when the conditions in the application are varied (e.g., varying the conditions, such as speed and direction, when walking in a circle in front of a motion sensor) ***Sample problem***: The relationship between the height above the ground of a person riding a Ferris wheel and time can be modelled using a sinusoidal function. Describe the effect on this function if the platform from which the person enters the ride is raised by 1 m and if the Ferris wheel turns twice as fast.	5.5, 5.6	322–343
D3.5 pose and solve problems based on applications involving a sinusoidal function by using a given graph or a graph generated with technology from its equation ***Sample problem***: The height above the ground of a rider on a Ferris wheel can be modelled by the sine function $h(t) = 25\sin 3(t - 30) + 27$, where $h(t)$ is the height, in metres, and t is the time, in seconds. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, the height after 30 s, and the time required to complete one revolution.	5.5, 5.6	322–343