1.3 Maximum or Minimum of a Quadratic Function

Student Text Pages
25 to 32

Suggested Timing
75–90 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 1–4 Section 1.3 Practice

Teaching Suggestions

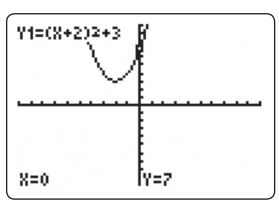
- **In Investigate A**, you may need to review methods available to graph quadratic relations. In step 1, the first relation is given in vertex form, so students can plot the vertex and then make a symmetric sketch of the parabola. For the second relation, students may need to create a table of values.
- Allow 20 min to complete **Investigate A**. A review on completing the square may be needed. You can pair weaker students with stronger ones to work through step 1 if time permits. Have students share their responses to step 5.
- Allow 20 min to complete **Investigate B**. Review the differences and similarities between functions of the form $y = ax^2 + bx + c$ when the only change to the family of curves is the value of c. This will help students understand why factoring the $ax^2 + bx$ part of the function provides so much information. Have students share their responses to step 7.
- In **Example 2**, remind students that once the partial factoring is done, the value for the x-coordinate of the vertex must be substituted into the original function to find the corresponding y-coordinate, which is the minimum.
- For **Example 3**, once students have worked out the solution using both methods, ask them which method they prefer and why.
- In **Example 4**, ask students to use partial factoring to find the maximum and compare it to the one obtained using technology. Students can use the TI-83 Plus or TI-84 Plus graphing calculator to find the zeros and maximum of a quadratic function that models a projectile. Refer to the Technology Appendix when using the **zero** and **maximum** operations. Then, ask students to comment on which technique they preferred, partial factoring or using the graphing calculator.
- Review the **Key Concepts** as a class so you can access student understanding of the concepts covered in this section.
- Have students expand $(x + b)^2$ to see why dividing by 2 and squaring are needed to answer **Communicate Your Understanding** question C1.

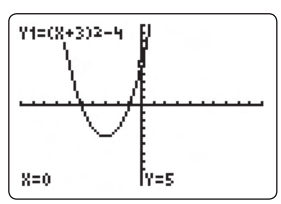
Differentiated Instruction

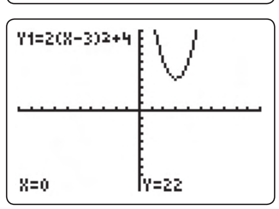
- Use **Think-Pair-Share** for Investigates A and B. Students should discuss their answers to the Investigates.
- Use a modified **jigsaw** to solve a variety of problems involving a minimum or a maximum. Put students into groups of four. Two students in each group use the complete the square method to solve the problem, and the other two students use the partial factoring method to solve the same problem. Each pair of students teaches their method to the other pair. The process can be repeated for a new problem. Students using Method 1 to solve the first problem should use Method 2 to solve the second problem to test their understanding of the method they just learned from their peers.
- Add the terms *zeros*, *vertex*, *maximum*, and *minimum* to the **word wall** constructed in Section 1.1.

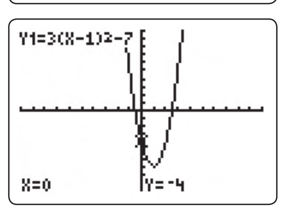
Investigate Answers (pages 25, 27)

Investigate A

1. a)


b)


c)


d)


2. Answers may vary. Sample answer: The graphs of each pair of functions are the same because the first equation can be written in the form of the second equation using algebraic operations.

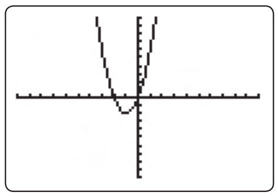
3. Answers may vary. Sample answer: The first equation in each pair can be rewritten in the form of the second equation by expanding and simplifying the expression on the right-hand side of the first equation.

4. Answers may vary. Sample answer: The second equation in each pair can be rewritten in the form of the first equation by completing the square of the expression on the right-hand side of the second equation.

5. Answers may vary. Sample answer: If the two quadratic functions have the same tables of values and their graphs look the same, then the two quadratic functions in different forms represent the same function. When using a graphing calculator, in addition to observing that the graphs look the same on the view screen, you should also check that the tables of values for both graphs are the same. This will ensure that the two quadratic functions are the same.

Investigate B

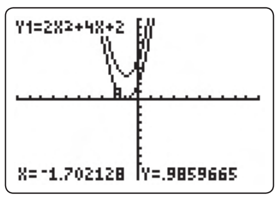
1.



2. The function has two x-intercepts.

3. The vertex of the parabola is at $(-1, -2)$.

4.



5. $g(x)$ has one x-intercept; $h(x)$ has no x-intercepts.

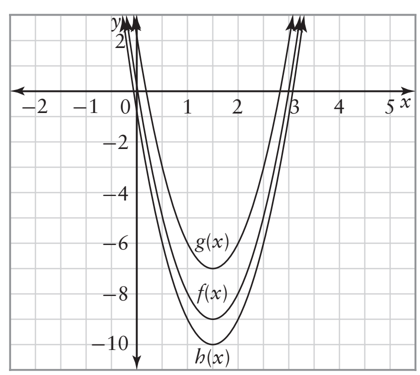
6. Answers may vary. Sample answer: Parabola $g(x) = 2x^2 + 4x + 2$ is the translation of parabola $f(x) = 2x^2 + 4x$ upward by 2 units. Parabola $h(x) = 2x^2 + 4x + 5$ is the translation of parabola $f(x)$ upward by 5 units. Since the vertex of parabola $f(x)$ is $(-1, -2)$, the vertex of parabola $g(x)$ is $(-1, 0)$ and the vertex of parabola $h(x)$ is $(-1, 3)$.

7. Answers may vary. Sample answer: Since the value of a in $f(x) = 2x^2 + 4x + k$ is positive, the parabola of the form $f(x) = 2x^2 + 4x + k$ has a minimum. From step 6, for $k = 0$, the minimum of $f(x) = 2x^2 + 4x$ is -2. To find the minimum of a parabola of the form $f(x) = 2x^2 + 4x + k$, add k to -2. The minimum of the parabola is $k - 2$.

Communicate Your Understanding Responses (page 31)

C1 Answers may vary. Sample answer: The coefficient of x is divided by 2 and the result is squared to make the first three terms a perfect square trinomial.

C2 Answers may vary. Sample answer: The three functions $f(x)$, $g(x)$, and $h(x)$ are in the same form: $y = 4x(x - 3) + k$, with $k = 0$ for $f(x)$, $k = 2$ for $g(x)$, and $k = -1$ for $h(x)$. So, the graphs of the three functions have the same parabolic shape, the parabolas open upward, and they have a minimum. Parabola $g(x)$ is the translation of parabola $f(x)$ upward by 2 units. Parabola $h(x)$ is the translation of parabola $f(x)$ downward by 1 unit. The x-coordinate of the vertex of the parabolas are the same: $x = \frac{3}{2}$.
The minimum of $f(x)$ is -9, of $g(x)$ is $-9 + 2 = -7$, and of $h(x)$ is $-9 - 1 = -10$.



C3 Answers may vary. Sample answer: Ryan should work with the function $y = 3x^2 - 9x$ and find the x-coordinate of the vertex, since the vertex of $y = 3x^2 - 9x - 17$ has the same x-coordinate. To find the x-intercepts, substitute $y = 0$.

$0 = 3x^2 - 9x$
$0 = 3x(x - 3)$
$3x = 0$ or $(x - 3) = 0$
$x = 0$ or $x = 3$

The average of these two x-intercepts gives the x-coordinate of the vertex of the functions $y = 3x^2 - 9x$ and $y = 3x^2 - 9x - 17$.

$\frac{0 + 3}{2} = \frac{3}{2}$

The x-coordinate of the vertex is $\frac{3}{2}$. By substituting $x = \frac{3}{2}$ into $y = 3x^2 - 9x - 17$, the vertex of the parabola is found to be $\left(\frac{3}{2}, -\frac{95}{4}\right)$.

Common Errors

- Students have difficulty with partial factoring in question 12 due to the fractional coefficient of n^2.

R_x Remind students that to perform common factoring using the coefficient of n^2 as the common factor, the resulting coefficient of n equals the original coefficient of n divided by the common factor.

- Students have difficulty determining the base dimension of the rectangle in the cross section of the eavestrough.

R_x Remind students that when x represents the length of each side to be bent up, the remaining cross sectional measurement is $30 - 2x$.

Practise, Connect and Apply, Extend

- In **question 2**, ask students to check their answers using partial factoring.
- In **question 3**, ask students to check their answers by completing the square or using technology.
- Remind students that in **questions 5** and **6**, they must use "Let …" statements to define the variables they will use in their functions.
- **Question 7** requires students to use reasoning and reflecting skills to solve the problem of finding the maximum height of the ball. Students will select tools and connect concepts they have learned in the past to solve this problem.
- Since **question 8** can be done by partial factoring, completing the square, or using technology, you may divide the class into three groups, each responsible for one of the three techniques, and have each group report to the class.
- Extend **question 11** by switching the sum from 10 to an odd number and setting the condition that the two numbers be whole numbers or integers. Then, ask students to write about their findings.
- **Question 12** gives students the opportunity to solve the problem using reasoning and reflecting skills. Students are required to formulate and then communicate conclusions that can be made from the function. Students will select tools to find the vertex, and use connecting skills to perform the algebra necessary to determine if there is a maximum or a minimum value at the vertex. Students will also represent the function with a graph.
- **Question 15** can be extended by giving a fixed initial velocity and the acceleration due to gravity, which is 9.81 m/s^2 on Earth, 3.77 m/s^2 on Mars, 1.62 m/s^2 on the moon, or even 274.13 m/s^2 on the Sun, for students to find the maximum height of the thrown ball and the time to reach this height.
- Use **BLM 1–4 Section 1.3 Practice** for remediation or extra practice.

Mathematical Process Expectations

Process Expectation	Selected Questions
Problem Solving	7–12, 14, 15
Reasoning and Proving	5–15
Reflecting	5–12, 14, 15
Selecting Tools and Computational Strategies	1, 2, 5–15
Connecting	1–15
Representing	12
Communicating	12, 13

Use Technology

Student Text Page
33

Suggested Timing
15–20 min

Tools
- TI-Nspire™ CAS graphing calculator

Use a TI-Nspire™ CAS Graphing Calculator to Find the Maximum or Minimum and the Zeros of a Quadratic Function

Teaching Suggestions

- It may be beneficial to pair struggling students with students who are more familiar with the operation of the TI-Nspire™ CAS graphing calculator.
- The "plot a point and drag" approach has been chosen to parallel *The Geometer's Sketchpad*® solutions presented in the chapter. You can also simply press (menu) and select the **Trace** operation, followed by **Graph Trace**. The cursor keys will move the tracing point along the graph. Whenever it reaches a maximum, minimum, or zero, an appropriate box will appear.

1.4 Skills You Need: Working With Radicals

Student Text Pages
34 to 40

Suggested Timing
75–90 min

Tools
- scientific calculator (optional)

Related Resources
- BLM 1–5 Section 1.4 Practice

Teaching Suggestions

- Allow 15 to 20 min for the **Investigate**. Once students have completed the investigation, have them share their responses to step 4.
- In **Example 1**, stress to students that the greatest perfect square factor must be removed from under the radical sign to put a radical in simplest form. For example, in part **c**), some students may use 9×20 for 180 and simplify the radical to $3\sqrt{20}$. Since 20 can be written as 4×5 and 4 is still a perfect square factor, $3\sqrt{20}$ is not in simplest form.
- Before students work through **Example 2**, parallel the addition and subtraction of radicals to simplifying like terms in a polynomial expression such as $3x + 2x$, where the coefficients 3 and 2 are added to become the coefficient of x.
- In **Example 2**, remind students to simplify all radicals to mixed radicals of the same radicand before adding and subtracting.
- In **Example 3**, remind students that once the radicands have been multiplied, the radical must be changed to its simplest form. For parts **d**) and **e**), parallel the simplification of expressions with radicals to simplifying an expression such as $(3x + 2)(2x - 5)$.
- In **Example 4**, remind students of the formula for the volume of a square-based pyramid if needed.
- Be sure to review the **Key Concepts** with the class to ensure that students understand all the concepts.
- Work through the **Communicate Your Understanding** questions with students. Ask for the correct answer to question C1 to see if students are comfortable enough with the material taught to simplify the expression mentally.