1.5 Solving Quadratic Equations

Student Text Pages
43 to 51

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 1–6 Section 1.5 Practice

Teaching Suggestions

- Allow 15 to 20 min for students to complete the **Investigate** and another 5 min to discuss their results in step 6. If time permits, split the class into four groups, and have each group prepare a solution using one of the techniques. Each group can then present their solution to the class.
- In **Example 1**, students can use the **zero** operation on a TI-83 Plus or TI-84 Plus graphing calculator to find the x-intercepts. An alternative is to enter a second equation **Y2** = 0. Then press [2nd] [CALC] and select **5:intersect** to find where the function entered in **Y1** intersects the x-axis. Refer to the Technology Appendix when using the **zero** and **intersect** operations.
- Students should find that the results in **Examples 2** and **3** are the same, with the exception that **Example 3** allows for the exact solutions to be determined.
- In **Example 3**, part **a**), discuss with students how $\frac{-8 + 2\sqrt{6}}{-4}$ is simplified to $\frac{4 - \sqrt{6}}{2}$. Students need to see that there are two fractions.
- Using the TI-83 Plus or TI-84 Plus graphing calculator in **Examples 2** and **4**, students will be able to visualize the number of zeros for different quadratic equations given in the form $ax^2 + bx + c = 0$.
- Review the concept of discriminant in **Example 4**. Encourage students to copy into their notebooks the margin note on how the value of the discriminant affects the number of solutions.
- Read over the **Key Concepts** with the class and allow at least 10 min for discussion on the **Communicate Your Understanding** questions. Students may feel strongly about the ease of using one technique over the others.

Differentiated Instruction

- Use **four corners** to determine when to solve a quadratic equation by factoring, completing the square, using the quadratic formula, or graphing. Hold up a sample quadratic equation. Students move to the corner of the room that represents the best method for solving that particular equation.
- Use **what-so-what double entry** and create a T-chart to summarize key concepts. List possible discriminant values—greater than 0, less than 0, and equal to 0—on the left side and the number of solutions, along with a sample graph, on the right side.

Investigate Answers (page 43)

1. $x = -2$ or $x = 2$; there are two solutions
2. $x = -3$ or $x = 1$
3. $x = -3$ or $x = 1$
4. $x = -3$ or $x = 1$
5. Answers may vary. Sample answer: The equations in steps 1 to 4 all have two solutions. The two solutions to each equation in steps 2 to 4 are the same. Each of these two solutions is 1 greater than the two solutions in step 1.
6. Answers may vary. Sample answer:

$$\begin{aligned} a(x-h)^2 + k &= 0 \\ a(x-h)^2 &= -k \\ (x-h)^2 &= -\frac{k}{a} \\ x - h &= \pm\sqrt{-\frac{k}{a}} \\ x &= h \pm \sqrt{-\frac{k}{a}} \end{aligned}$$

To solve $2(x-3)^2 - 32 = 0$, substitute $a = 2$, $h = 3$, and $k = -32$ into $x = h \pm \sqrt{-\frac{k}{a}}$.

$$x = 3 \pm \sqrt{-\frac{-32}{2}} = 3 \pm \sqrt{16} = 3 \pm 4$$

$x = -1$ or $x = 7$

Communicate Your Understanding Responses (page 49)

C1 Answers may vary. Sample answer: Minh should first try to solve the quadratic equation by factoring. He could also use the quadratic formula to solve a quadratic equation that can be factored. If the quadratic equation expression does not factor, Minh could use the discriminant to determine the number of roots. If there are two real and distinct roots or one real and equal root, Minh could use the method of completing the square, or solve the quadratic equation using a graphing calculator. Minh could also use the quadratic formula to solve for the two roots. If there are no real roots, Minh could use the quadratic formula to explain why.

C2 Answers may vary. Sample answer: The easiest technique to use to solve a quadratic equation of the form $ax^2 + bx = 0$ is by factoring. There are only two terms in the quadratic equation and the equation can always be solved as follows:

$$ax^2 + bx = 0$$
$$ax\left(x + \frac{b}{a}\right) = 0$$
$$ax = 0 \text{ or } x + \frac{b}{a} = 0$$
$$x = 0 \text{ or } x = -\frac{b}{a}$$

C3 Answers may vary. Sample answer: Deepi can determine the number of x-intercepts that the quadratic function has by using the discriminant, the value under the radical sign in the quadratic formula. If the discriminant is greater than 0, there are two x-intercepts. If the discriminant is 0, there is one x-intercept. If the discriminant is less than 0, there are no x-intercepts.

Practise, Connect and Apply, Extend

- Have students check their answers to **question 1** using a graphing calculator.
- In **questions 3** and **5**, remind students that "exact answers" means irrational answers should be written as radicals. Students may use a graphing calculator to check the answers.
- In **question 4**, remind students that they do not need to find the roots.
- **Question 9** gives students the opportunity to select tools to represent a graph of two given functions on the same set of axes, and to use their reasoning and connecting skills to answer questions related to the intersection of the two graphs.
- In **question 11**, remind students that for a quadratic equation of the form $x^2 + bx + c = 0$, they are looking for two numbers that multiply to give c and add to give b.
- In **question 16**, remind students to use a "Let …" statement to identify what the variable they are using represents.
- Many students will solve **question 18** using the quadratic formula. Have them verify using a graphing calculator.
- **Question 20** allows students to select tools and use connecting skills with their mathematical knowledge to solve each quadratic equation. They will also use reflecting and reasoning skills to establish an understanding of the given statement and communicate this understanding.
- **Question 22** is the classic derivation of the quadratic formula. Pair weaker students with stronger students to see how this derivation is done.
- Use **BLM 1−6 Section 1.5 Practice** for remediation or extra practice.

Common Errors

- Students do not understand which value in the quadratic relation in question 21 can change.

R_x Point out that gravity and the height of the diving board will not change, so the values −4.9 and 10 are constants. Discuss the meaning of the value that can change, i.e., the initial upward velocity of the diver.

- Students do not know the procedure to obtain the area of the acute isosceles triangle in question 24.

R_x Remind students that the non-shaded area can be found by subtracting the area of the three shaded right triangles from the area of the square.

Ongoing Assessment

Achievement Check, question 21, on student text page 51.

Achievement Check, question 21, student text page 51

This performance task is designed to assess the specific expectations covered in Section 1.5. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Connecting
- Reflecting
- Communicating
- Selecting Tools and Computational Strategies

Sample Solution

a) At the start, $t = 0$.

$h(0) = -4.9(0)^2 + 3(0) + 10 = 10$

So, the diver started her dive from a height of 10 m.

b) Substitute $h = 0$ into the quadratic equation and solve for t.

$0 = -4.9t^2 + 3t + 10$

Use the quadratic formula with $a = -4.9$, $b = 3$, and $c = 10$.

$$t = \frac{-3 \pm \sqrt{3^2 - 4(-4.9)(10)}}{2(-4.9)}$$

$t \doteq -1.15$ or $t \doteq 1.77$

Since $t \geq 0$, the negative value is inadmissible.

The diver was in the air for approximately 1.8 s.

c) The value 10 in the equation $h(t) = -4.9t^2 + 3t + 10$ represents the height of the diver at the start. So, the value of $c = 10$ will never change if the diver always starts from the same height.

d) The only value that can change in the quadratic equation is $b = 3$. To change this value, the diver would have to jump up or down more quickly or more slowly; that is, change the speed of her dive.

e) If $b = 6$, the new equation would be $h(t) = -4.9t^2 + 6t + 10$.

Use the quadratic formula with $a = -4.9$, $b = 6$, and $c = 10$.

$$t = \frac{-6 \pm \sqrt{6^2 - 4(-4.9)(10)}}{2(-4.9)}$$

$t \doteq -0.94$ or $t \doteq 2.17$

The value -0.94 is inadmissible.

Subtract: $2.17 - 1.77 = 0.4$

The diver would be in the air for approximately 0.4 s longer.

Level 3 Notes

Look for the following:

- Understanding of method to find the diver's height and time in the air is mostly evident
- Use of quadratic formula to solve for t is mostly accurate
- Understanding of the values represented by b and c in the quadratic equation is mostly evident
- Description of how the diver can change the b value in the quadratic equation is mostly accurate and supported with some justification
- Understanding of how to determine how much longer the diver is in the air is mostly evident

What Distinguishes Level 2

- Understanding of method to find the diver's height and time in the air is somewhat evident
- Use of quadratic formula to solve for t is somewhat accurate
- Understanding of the values represented by b and c in the quadratic equation is somewhat evident
- Description of how the diver can change the b value in the quadratic equation is somewhat accurate and supported with little justification
- Understanding of how to determine how much longer the diver is in the air is somewhat evident

What Distinguishes Level 4

- Understanding of method to find the diver's height and time in the air is clearly evident
- Use of quadratic formula to solve for t is accurate
- Understanding of the values represented by b and c in the quadratic equation is clearly evident
- Description of how the diver can change the b value in the quadratic equation is accurate and supported with a high degree of justification
- Understanding of how to determine how much longer the diver is in the air is clearly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15–17, 23, 24
Reasoning and Proving	7–9, 11–24
Reflecting	7, 15–17, 20–24
Selecting Tools and Computational Strategies	2, 7–9, 11–24
Connecting	1–24
Representing	9
Communicating	7, 20

1.6 Determine a Quadratic Equation Given Its Roots

Student Text Pages
52 to 59

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 1–7 Section 1.6 Practice
- BLM 1–8 Section 1.6 Achievement Check Rubric

Teaching Suggestions

- Allow 15 to 25 min for students to work on the **Investigate**. Allow more time if Method 2 is also done. Discuss student responses to step 5.
- In **Example 1**, stress to students that the value of a in $y = a(x - s)(x - t)$ determines a specific function in the family.
- In **Example 2**, be sure students understand that they need to develop the family of functions, and then substitute the given point to find the required value of a to determine the exact equation.
- **Example 3** uses representing skills to create a sketch and determine a model for the function. The skills of connecting, selecting tools, and representing have to be used. Reflecting on the situation and reasoning through how to solve the problem are necessary to solve the problem of finding the required maximum height of the tunnel.
- Extend **Example 3** by finding information on other parabolic tunnels or similar structures, such as parabolic arches, suspension bridges, and reflectors.
- Take time to discuss **Communicate Your Understanding** question C2. Provide specific examples to ensure students are clear on the answer.