What Distinguishes Level 4

- Understanding of method to find the diver's height and time in the air is clearly evident
- Use of quadratic formula to solve for t is accurate
- Understanding of the values represented by b and c in the quadratic equation is clearly evident
- Description of how the diver can change the b value in the quadratic equation is accurate and supported with a high degree of justification
- Understanding of how to determine how much longer the diver is in the air is clearly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15–17, 23, 24
Reasoning and Proving	7–9, 11–24
Reflecting	7, 15–17, 20–24
Selecting Tools and Computational Strategies	2, 7–9, 11–24
Connecting	1–24
Representing	9
Communicating	7, 20

Determine a Quadratic Equation Given Its Roots

Student Text Pages
52 to 59

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 1–7 Section 1.6 Practice
- BLM 1–8 Section 1.6 Achievement Check Rubric

Teaching Suggestions

- Allow 15 to 25 min for students to work on the **Investigate**. Allow more time if Method 2 is also done. Discuss student responses to step 5.
- In **Example 1**, stress to students that the value of a in $y = a(x - s)(x - t)$ determines a specific function in the family.
- In **Example 2**, be sure students understand that they need to develop the family of functions, and then substitute the given point to find the required value of a to determine the exact equation.
- **Example 3** uses representing skills to create a sketch and determine a model for the function. The skills of connecting, selecting tools, and representing have to be used. Reflecting on the situation and reasoning through how to solve the problem are necessary to solve the problem of finding the required maximum height of the tunnel.
- Extend **Example 3** by finding information on other parabolic tunnels or similar structures, such as parabolic arches, suspension bridges, and reflectors.
- Take time to discuss **Communicate Your Understanding** question C2. Provide specific examples to ensure students are clear on the answer.

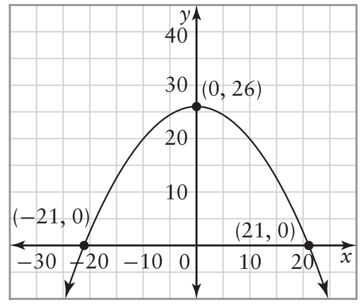
Differentiated Instruction

- Use a **gallery walk** to find the equation of a family of quadratic functions. Each group starts with given *x*-intercepts and graphs one quadratic function with those intercepts on large graph paper. The groups rotate, adding a quadratic function from the same family to each graph. Groups return to their original station and determine the exact equation of each function on their graph. Groups can use graphing calculators to check their equations. The completed posters can remain posted in the classroom throughout the chapter.

Investigate Answers (pages 52–53)

Method 1

1. x-intercepts: $(-21, 0)$, $(21, 0)$; vertex: $(0, 26)$



2. $y = a(x + 21)(x - 21)$

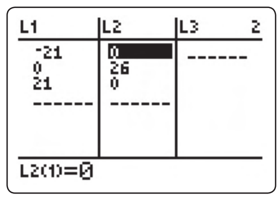
3. Substitute $x = 0$ and $y = 26$ into $y = a(x + 21)(x - 21)$. The value for a is $-\dfrac{26}{441}$.

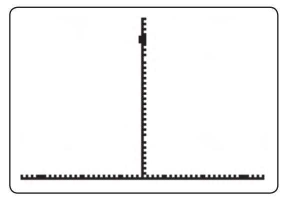
4. a) $y = -\dfrac{26}{441}(x + 21)(x - 21)$ **b)** $y = -\dfrac{26}{441}x^2 + 26$

5. Answers may vary. Sample answer: No. It is not possible to find the equation of a quadratic function given its zeros. You need to have the coordinates of one other point on the quadratic function.

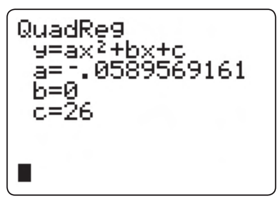
Method 2

1.



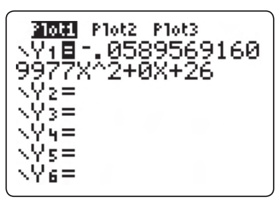


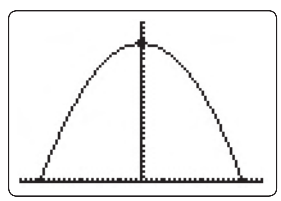
2.



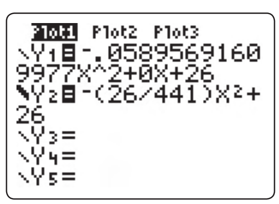
The equation of the quadratic function is $y = -0.058\,956\,916\,1x^2 + 26$.

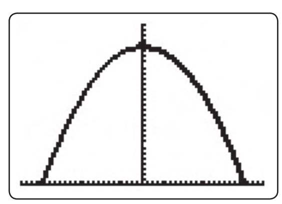
3.





4. a) The intercept form of the function is $y = -0.058\,956\,916\,1(x + 21)(x - 21)$.





b) Answers may vary. Sample answer: The second parabola is the same as the first parabola.

5. Answers may vary. Sample answer: No. It is not possible to determine the equation of a quadratic function given its zeros. You need to have the coordinates of one other point on the quadratic function.

Communicate Your Understanding Responses (page 57)

C1 Answers may vary. Sample answer: Substitute the x-intercepts x_1 and x_2 and point (x, y) into the equation $y = a(x - x_1)(x - x_2)$ to determine the value of a. Then, substitute the x-intercepts x_1 and x_2 and the value of a into the equation $y = a(x - x_1)(x - x_2)$.

C2 Answers may vary. Sample answer: Ronnie is correct. The vertex is a point on the quadratic function and can be used with the x-intercepts to find the exact equation of the quadratic function.

C3 Answers may vary. Sample answer: Mona is not correct. If Mona uses the binomial $(2x + 1)$ to determine the equation of the quadratic function with x-intercepts $x = -\frac{1}{2}$ and $x = \frac{3}{2}$ and a point (3, 5) on the function, the value for a is $\frac{5}{21}$. If Mona uses the binomial $\left(x + \frac{1}{2}\right)$ to determine the equation of the quadratic function with x-intercepts $x = -\frac{1}{2}$ and $x = \frac{3}{2}$ and a point (3, 5) on the function, the value for a is $\frac{10}{21}$. The graphs of the two quadratic functions will be different.

Common Errors

- Students put the y-axis at the origin of the kick and thus do not know how to proceed in question 7.

R_x Remind students that the question asks for the y-axis to include the vertex. This will help them see the symmetry of the parabola.

- Students do not understand the concept of a quadratic function having only one x-intercept in question 11.

R_x Remind students that when a quadratic function has only one x-intercept, the point (x-intercept, 0) is the vertex of the function.

Practise, Connect and Apply, Extend

- For **question 5**, be sure students understand how to write the two zeros from the given form. Review the technique to expand an expression such as $(x - 1 - \sqrt{11})(x - 1 + \sqrt{11})$.
- It may be beneficial for students to do **question 6** before **question 5**.
- **Question 7** gives students the opportunity to use their reflecting and reasoning skills to solve the problem of the maximum height reached by the ball and the horizontal distance travelled to reach the maximum height. Students will connect concepts that they have learned previously to develop two equations of the quadratic function that model the parabolic path of the ball. They will then use their communicating skills to outline the similarities and differences between the two functions.
- In **question 9**, students can enter the three points into the graphing calculator in **L1** and **L2**, and then use quadratic regression to verify the equation of each quadratic function.
- **Question 15** allows students to use their reasoning and reflecting skills to solve the problem of sketching the required function and determining the function that models the arch. They will use their representing skills to sketch a graph of the function and use their connecting skills from work learned previously to find the maximum height of the arch.
- Have students compare their sketches for **question 15**, part **a**) before completing the rest of the question.
- If time permits, use **question 19** for a class discussion question.
- Use **BLM 1–7 Section 1.6** Practice for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 18, on student text page 59.

Achievement Check, question 18, student text page 59

This performance task is designed to assess the specific expectations covered in Sections 1.3, 1.4, and 1.6. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

Provide students with **BLM 1–8 Section 1.6 Achievement Check Rubric** to help them understand what is expected.

a) The factored form of a quadratic function with zeros r and s is $y = a(x - r)(x - s)$. Substitute the zeros $r = -2$ and $s = 6$ and the point (3, 15) to obtain the value of a.

$15 = a(3 + 2)(3 - 6)$
$15 = a(-15)$
$a = -1$

The equation of the quadratic function in factored form is $y = -(x + 2)(x - 6)$.

b) Expand to write the equation in standard form.

$$\begin{aligned} y &= -(x + 2)(x - 6) \\ &= -(x^2 - 6x + 2x - 12) \\ &= -(x^2 - 4x - 12) \\ &= -x^2 + 4x + 12 \end{aligned}$$

The standard form of the quadratic equation is $y = -x^2 + 4x + 12$.

c) Complete the square to convert the standard form to vertex form.

$$\begin{aligned} y &= -x^2 + 4x + 12 \\ &= -(x^2 - 4x) + 12 \\ &= -(x^2 - 4x + 4 - 4) + 12 \\ &= -(x^2 - 4x + 4) + 4 + 12 \\ &= -(x^2 - 4x + 4) + 16 \\ &= -(x - 2)^2 + 16 \end{aligned}$$

The vertex form of the quadratic equation is $y = -(x - 2)^2 + 16$. The vertex is (2, 16).

d) Use the $y = -x^2 + 4x$ portion of the function to find the x-coordinate of the vertex, which is the same as the x-coordinate of the vertex of $y = -x^2 + 4x + 12$.

Substitute $y = 0$.

$0 = -x(x - 4)$

$x = 0$ or $x = 4$

The x-coordinate of the vertex is $x = 2$, the average of 0 and 4.

Substitute $x = 2$ into $y = -x^2 + 4x + 12$.

$y = -(2)^2 + 4(2) + 12 = 16$

This vertex (2, 16) is the same as the vertex found in part c).

e) The equation of a quadratic function passing through $(3, -30)$ with zeros -2 and 6 is $y = a(x + 2)(x - 6)$. Substitute $(3, -30)$ to solve for a.

$$\begin{aligned} -30 &= a(3 + 2)(3 - 6) \\ -30 &= a(5)(-3) \\ -30 &= -15a \\ 2 &= a \end{aligned}$$

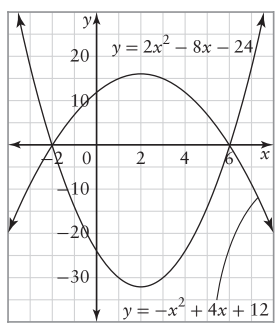
The equation of this second quadratic function in factored form is $y = 2(x + 2)(x - 6)$.

Expand to write the equation in standard form.

$$\begin{aligned} y &= 2(x + 2)(x - 6) \\ &= 2(x^2 + 2x - 6x - 12) \\ &= 2(x^2 - 4x - 12) \\ &= 2x^2 - 8x - 24 \end{aligned}$$

The standard form of the equation is $y = 2x^2 - 8x - 24$.

f)



The two parabolas have the same x-intercepts, -2 and 6. The parabola for the equation in part a) passes through the point (3, 15), and the parabola for the equation in part e) passes through the point $(3, -30)$.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7, 13, 15–17, 19
Reasoning and Proving	7, 8, 10, 11, 13–19
Reflecting	7, 13–17, 19
Selecting Tools and Computational Strategies	1, 3, 5, 6, 8
Connecting	1–19
Representing	1, 3, 5–7, 14–16, 18
Communicating	7, 9, 10, 17–19