1.7 Solve Linear-Quadratic Systems

Student Text Pages
60 to 69

Suggested Timing
75 min

Tools
- grid paper
- graphing calculator
- TI-Nspire™ CAS graphing calculator (optional)

Related Resources
- G–1 Grid Paper
- BLM 1–9 Section 1.7 Practice

Differentiated Instruction
- Use **Think-Pair-Share** to complete Investigates A and B.
- Add the terms *tangent line* and *secant* to the **word wall** constructed for this chapter.
- Use **graffiti** to practise solving problems involving a linear-quadratic system. Arrange students into groups of three or four. Each student is given a different problem and completes the first step of their solution. Each page is then passed to the next student, who completes the second step of the solution. The pages are passed around the group until all of the assigned problems are complete. Use **think-aloud** and have each group share one completed solution with the whole class.

Teaching Suggestions

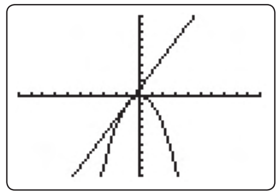
- Allow 10 to 15 min to complete **Investigate A**. Discuss the answers for step 4.
- Allow 20 to 25 min to complete **Investigate B**. Have students work in pairs where appropriate. A discussion on the results of step 5 may be needed before students work on step 6.
- In some cases, using either or both Investigates may be appropriate. **Investigate A** is open ended and students may work with a partner. For struggling students, use the following changes to **Investigate A**:
  1. a) Graph the functions $y = -x + 7$ and $y = x^2 + 5$.
     b) At how many points do the graphs intersect?
  2. a) Graph the functions $y = 2x + 4$ and $y = x^2 + 5$.
     b) At how many points do the graphs intersect?
  3. a) Graph the functions $y = 3x - 4$ and $y = x^2 + 5$.
     b) At how many points do the graphs intersect?
  4. a) Graph the functions $y = x^2 + 5$ and $x = 1$.
     b) At how many points do the graphs intersect?
  5. **Reflect** Given a line and a parabola, at how many points might they intersect?
- In **Example 1**, Method 1, have students first try factoring out the common factor -0.48 instead of dividing both sides by -0.24 to determine the intersecting points. Students will still get the correct answer.
- **Example 1** can be extended in several ways. Students can investigate using trigonometry to find the banner angle or using the distance formula to find the length of the banner. Students can use symmetry to investigate where the banner can be connected.
- Either use Method 2 in **Example 1** as a check on the work in Method 1, or have the class split into two groups, with half of the class working on Method 1 while the other half is working on Method 2. Then, discuss which method is easier.
- Remind students of the three possible values of a discriminant and their meanings before doing **Example 2**.
- Be sure all students understand the difference between a secant line and a tangent line before they work through **Example 3**.
- In **Example 4**, explain why the position needs to be -100. Use a line diagram of the motion with a clearly chosen positive direction.
- Review the **Key Concepts** and take 5 to 10 min to discuss the **Communicate Your Understanding** questions before students attempt the **Practise** questions.

Investigate Answers (pages 60–61)

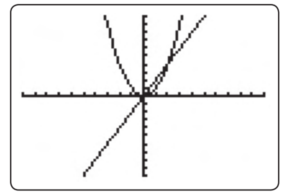
Investigate A

1. Answers may vary. Sample answer:

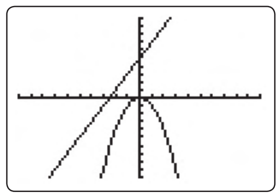
A line and a parabola may intersect at one point:



A line and a parabola may intersect at two points:



A line and a parabola may not intersect at all:



2. The line $y = 2x + 1$ and the parabola $y = -x^2$ intersect at one point: $(-1, -1)$.

$$-x^2 = 2x + 1$$
$$x^2 + 2x + 1 = 0$$
$$(x + 1)^2 = 0$$
$$x = -1$$
$$y = -1$$

The line $y = 2x$ and the parabola $y = x^2$ intersect at two points: (0, 0) and (2, 4).

$$x^2 = 2x$$
$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \text{ or } x = 2$$
$$y = 0 \text{ or } y = 4$$

The line $y = 2x + 5$ and the parabola $y = -x^2$ do not intersect.

$$-x^2 = 2x + 5$$
$$x^2 + 2x + 5 = 0$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4 - 20}}{2}$$
$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$\sqrt{-16}$ is not a real number.

3. Answers may vary. Sample answer:

For the line and parabola intersecting at one point, the discriminant is $2^2 - 4(1)(1) = 0$.

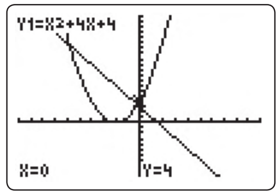
For the line and parabola intersecting at two points, the discriminant is $(-2)^2 - 4(1)(0) = 4$.

For the line and parabola not intersecting, the discriminant is $2^2 - 4(1)(5) = -16$.

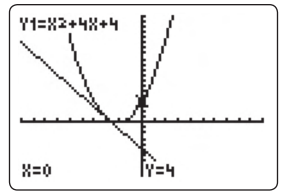
4. Answers may vary. Sample answer: If the discriminant is equal to 0, the line and parabola will intersect at one point. If the discriminant is greater than 0, the line and parabola will intersect at two points. If the discriminant is less than 0, the line and parabola do not intersect.

Investigate B

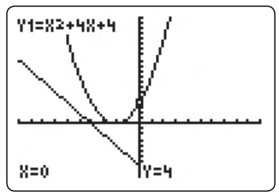
1. $y = -2x + k$
2. $x^2 + 6x + 4 - k = 0$
3. $6^2 - 4(1)(4 + k) = 36 - 4(4 - k) = 36 - 16 + 4k = 20 + 4k$
4. a) If $20 + 4k > 0$, $4k > -20$, or $k > -5$; two points of intersection
   b) If $20 + 4k = 0$, $4k = -20$, or $k = -5$; one point of intersection
   c) If $20 + 4k < 0$, $4k < -20$, or $k < -5$; no points of intersection
5. a) $y = -2x + 3$ b) $y = -2x - 5$ c) $y = -2x - 8$
6. a)



b)



c)



Communicate Your Understanding Responses (page 67)

C1 Answers may vary. Sample answer: Tell Larissa to calculate the discriminant for several quadratic functions and then use a graphing calculator to graph the quadratic functions to determine the number of zeros that each function has. Then tell her to summarize her results to determine the relationship between the value of the discriminant and the number of zeros of a quadratic function.

C2 Answers may vary. Sample answer: This is a good idea. The maximum number of times that a line can intersect a parabola is twice. If Randy substitutes the values for x into the linear function, he will get two points of intersection. If he substitutes the two values for x into the quadratic equation, he may find more than two points of intersection for the linear-quadratic system.

C3 Answers may vary. Sample answer: Using an algebraic method to determine the points of intersection of a linear-quadratic system will give an exact solution. Using a graphical method often gives an approximate solution. It may take longer to solve the linear-quadratic system using an algebraic method than using a graphical method. With a graphing calculator, the system can be solved very quickly.

Practise, Connect and Apply, Extend

- Have students verify the solutions to **questions 1** and **6** using a graphing calculator.
- **Question 8** gives students the opportunity to use their reasoning skills to determine how to find the intersection of the paths of the space probe and the asteroid. They will select the most efficient tools and use connecting skills to help them determine if there is a point of intersection, and then communicate their results.
- Have students explain the meaning of the intersection of the two paths in **question 8**. Ask students what other piece of information would be needed to determine if there would be a collision. (The answer is time—the two objects can be in the same location at different times without a collision.)
- **Question 12** uses reflecting and reasoning skills to solve the problem of finding the y-intercept of the line associated with the longest support beam. Students will select tools and connect knowledge of algebraic skills learned previously to solve this problem.
- Remind students that in **question 12**, "touches at one point" means "is tangent to the curve."

Common Errors

- Students do not remember the interpretations of the values of the discriminant for questions 10 and 11.

R_x Refer students back to the marginal note for discriminant in Section 1.5 before attempting these questions.

- Students think they need to do something with the variable t to account for the fact that the second parachutist left 5 s after the first in question 15.

R_x Point out that (t – 5) in the second equation already accounts for the 5-s difference between the two parachutists.

- A sketch for the linear-quadratic system in **question 13** may help students understand why $x = 2$ is not a tangent.
- Have students compare their sketches for **question 14**, part **a**) before completing the rest of the question.
- In **question 16**, point out that the second equation is $y = 7$.
- Use **BLM 1–9 Section 1.7 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 69.

Achievement Check, question 17, student text page 69

This performance task is designed to assess the specific expectations covered in Section 1.7. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) Solve the linear-quadratic system $y = -0.0044x^2 + 21.3$ and $y = 0.0263x + 1.82$.

$$\begin{aligned} -0.0044x^2 + 21.3 &= 0.0263x + 1.82 \\ 0 &= 0.0044x^2 - 21.3 + 0.0263x + 1.82 \\ 0 &= 0.0044x^2 + 0.0263x - 19.48 \\ x &= \frac{-0.0263 \pm \sqrt{(0.0263)^2 - 4(0.0044)(-19.48)}}{2(0.0044)} \end{aligned}$$

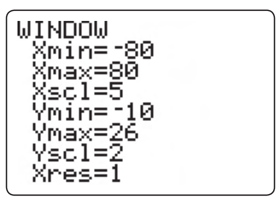
$x \doteq 63.62$ or $x \doteq -69.59$

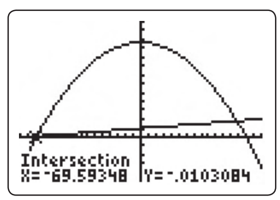
The corresponding y-values are

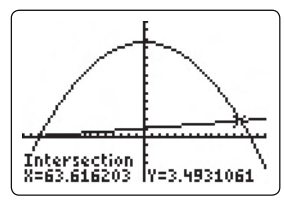
- for $x = -69.59$, $y = 0.0263(-69.59) + 1.82 \doteq -0.01$
- for $x = 63.62$, $y = 0.0263(63.62) + 1.82 \doteq 3.49$

To one decimal place, the points of intersection are $(-69.6, 0)$ and $(63.6, 3.5)$.

b) Window settings: First point of intersection: Second point of intersection:







c) Use the distance formula to find the length of the bridge:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For $(x_1, y_1) = (-69.6, 0)$ and $(x_2, y_2) = (63.6, 3.5)$,

$d = \sqrt{(63.6 + 69.6)^2 + (3.5 - 0)^2} \doteq 133.2$

The length of the bridge is approximately 133.2 m.

d) First find the length of the bridge that spans the Humber River.

Solve the linear-quadratic system $y = -0.0044x^2 + 21.3$ and $y = 0$.

$$\begin{aligned} -0.0044x^2 + 21.3 &= 0 \\ 0.0044x^2 &= 21.3 \\ x^2 &= \frac{21.3}{0.0044} \\ x^2 &= \pm\sqrt{\frac{21.3}{0.0044}} \end{aligned}$$

$x \doteq 69.6$ or $x \doteq -69.6$

The corresponding y-values are 0. To one decimal place, the points of intersection are $(69.6, 0)$ and $(-69.6, 0)$.

$2 \times 69.6 = 139.2$

The length of this bridge is approximately 139.2 m.

The difference of the lengths of the two bridges is: 139.2 m − 133.2 m = 6.0 m

Thus, the walkway of the North Bay bridge is approximately 6 m shorter than that of the Humber River bridge.

Level 3 Notes

Look for the following:

- Understanding of method to solve for the points of intersection of a linear-quadratic system is mostly evident
- Use of quadratic formula to solve for x is mostly accurate
- Understanding of method to find the length of each bridge is mostly evident
- Use of the distance formula is mostly accurate
- Understanding of method to determine which bridge is longer is mostly evident
- Justification of results is mostly evident
- Most calculations are accurate with few minor errors

What Distinguishes Level 2

- Understanding of method to solve for the points of intersection of a linear-quadratic system is somewhat evident
- Use of quadratic formula to solve for x is somewhat accurate
- Understanding of method to find the length of each bridge is somewhat evident
- Use of the distance formula is somewhat accurate
- Understanding of method to determine which bridge is longer is somewhat evident
- Justification of results is somewhat evident
- Some calculations are accurate

What Distinguishes Level 4

- Understanding of method to solve for the points of intersection of a linear-quadratic system is clearly evident
- Use of quadratic formula to solve for x is highly accurate
- Understanding of how to find the length of each bridge is clearly evident
- Use of the distance formula is highly accurate
- Understanding of how to determine which bridge is longer is clearly evident
- Justification of results is clearly evident
- Calculations are mostly accurate and are accompanied by correct units

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	12
Reasoning and Proving	8, 10–18
Reflecting	12, 13, 15–17
Selecting Tools and Computational Strategies	2, 3, 6–8, 10–12, 14–18
Connecting	1–18
Representing	14, 17
Communicating	8, 13, 17