# 2.2 Skills You Need: Operations With Rational Expressions

**Student Text Pages**
88 to 96

**Suggested Timing**
50–140 min

**Tools**
- graphing calculator

**Related Resources**
- BLM 2–4 Section 2.2 Practice

**Differentiated Instruction**

- Construct a **decision tree** to outline the steps for multiplying or dividing rational expressions. Construct another **decision tree** to outline the steps for adding or subtracting rational expressions.
- Use **Think-Pair-Share** to complete the Communicate Your Understanding questions.

**Common Errors**

- Students may try to cancel in the following way:

$$\frac{2x^2 + 5x + 2}{2x^2 + 5x - 12} = \frac{\cancel{2x^2} + \cancel{5x} + 2}{\cancel{2x^2} + \cancel{5x} - 12}$$
$$= -\frac{2}{12}$$
$$= -\frac{1}{6}$$

$R_x$ Have students substitute a value for $x$ in the original expression to show that it does not equal their simplified expression.

## Teaching Suggestions

- This section comes from an expectation that is described as being "as needed." Students will simplify many types of expressions—addition, subtraction, multiplication, and division. You are encouraged to pick and choose from this section as needed. As a prerequisite skill, students should be proficient at factoring algebraic expressions.
- Keep in mind that although these skills may not have a specific application in this course, they will be useful in grade 12 Advanced Functions and Calculus & Vectors.
- In **Example 2** part **b**), students are introduced to the idea of a restriction coming from the numerator of an expression. Review this idea carefully.
- You may wish to preface **Example 3** by reminding students of how to add and subtract fractions without variables in them.
- **Example 4** can sometimes be intimidating to students. Make connections to the methods in **Example 4** and those in **Example 3**.
- For **Example 5**, remind students of the relationship between speed, distance, and time.
- Be sure to read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.
- In **question C3**, a common mistake is finding a common denominator but not multiplying both the top and bottom of each fraction by the same number. Although algebraic methods should be encouraged to explain this, numerical methods could also be used to show that the two expressions are not equal.

**Communicate Your Understanding Responses (page 93)**

**C1** Change the operation between the two rational expressions from division to multiplication and then write the reciprocal of the second fraction. Divide out the factors that are common to the numerator of either rational expression and the denominator of either rational expression. The restrictions on the variable are $x \neq -8$, $x \neq -5$, $x \neq -4$, $x \neq 6$, and $x \neq 7$.

**C2** Answers may vary. Sample answer: $\frac{(x + 5)(x + 4)}{(x - 2)(x - 1)} \times \frac{x - 1}{x + 4}$

**C3** Answers may vary. Sample answer: The student could have found the lowest common denominator, 12, and then added the expressions in the numerator without applying the distributive law.

**C4** Find the lowest common denominator for the two rational expressions, apply the distributive law, and simplify the numerator of the new rational expression. The restrictions on the variable are $x \neq -3$ and $x \neq 1$.

## Practise, Connect and Apply, Extend

- **Question 11** relates to a commonly seen simplification problem (when the two denominators differ by a factor of $-1$).
- **Question 12** requires the use of reasoning skills to connect concepts learned in this chapter with past mathematical knowledge to represent both the volume and surface area of the open-topped box as functions in terms of $x$. Students will select tools to write a simplified expression for the ratio of volume to surface area.
- Let students know that **question 15** assumes that if swimmers swim with the current, their actual speed along the ground increases by adding the current

speed. Likewise, if they swim against the current, their ground speed decreases by the same amount.

- **Question 15** uses reasoning and connecting skills to determine expressions that represent the times to complete both swims. Students reflect on a situation involving currents and communicate their interpretations.
- **Question 18** could be used as a catalyst to discuss the history of mathematical thought.
- In **question 20**, let students know that they should start with small steps at the very bottom. For example,

$$1+\cfrac{1}{0+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{\frac{7}{6}}}}}}}}}} = 1+\cfrac{1}{0+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{\frac{13}{7}}}}}}}}}$$

- Use **BLM 2–4 Section 2.2 Practice** for remediation or extra practice.

**Ongoing Assessment**

Achievement Check, question 17, on student text page 96.

**Achievement Check, question 17, student text page 96**

This performance task is designed to assess the specific expectations covered in Sections 2.1 and 2.2. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

**Sample Solution**

**a)** $A = \dfrac{x + 4}{x^2 + 9x + 20} = \dfrac{x + 4}{(x + 4)(x + 5)} = \dfrac{1}{x + 5}$

Restrictions: $x \neq -5, x \neq -4$

$B = \dfrac{3x^2 - 9x}{x^2 + 3x - 18} = \dfrac{3x\,(x - 3)}{(x - 3)(x + 6)} = \dfrac{3x}{x + 6}$

Restrictions: $x \neq -6, x \neq 3$

**b)** The two expressions are not equivalent because neither the original nor simplified expressions are equivalent.

**c)** Answers may vary. Sample answer: An expression that appears to be equivalent to $A$ is $\dfrac{x - 2}{x^2 + 3x - 10}$ because it simplifies to $\dfrac{1}{x + 5}$, but $x \neq -5, x \neq 2$.

An expression that appears to be equivalent to $B$ is $\dfrac{3x^2 - 3x}{x^2 + 5x - 6}$ because it simplifies to $\dfrac{3x}{x + 6}$, but $x \neq -6, x \neq 1$.

**d)** Use the simplified form found in part a) to perform the indicated operation.

**i)** $A + B = \dfrac{1}{x + 5} + \dfrac{3x}{x + 6}$

$= \dfrac{(x + 6) + 3x(x + 5)}{(x + 5)(x + 6)}$

$= \dfrac{3x^2 + 16x + 6}{x^2 + 11x + 30}, x \neq -6, x \neq -5, x \neq -4, x \neq 3$

**ii)** $AB = \dfrac{1}{x + 5} \times \dfrac{3x}{x + 6}$

$= \dfrac{3x}{x^2 + 11x + 30}, x \neq -6, x \neq -5, x \neq -4, x \neq 3$

**iii)** $B \div A = \frac{3x}{x+6} \div \frac{1}{x+5}$

$= \frac{3x}{x+6} \times \frac{x+5}{1}$

$= \frac{3x^2 + 15x}{x+6}, x \neq -6, x \neq -5, x \neq -4, x \neq 3$

## Level 3 Notes

Look for the following:

- Simplification of expressions is mostly correct
- Most restrictions are stated and correct
- Understanding of how to identify whether expressions are equivalent is mostly evident
- Understanding of how to determine other equivalent expressions is mostly evident
- Operations with rational expressions in part d) are mostly correct
- Justification and explanations of solutions, where required, are mostly valid

## What Distinguishes Level 2

- Simplification of expressions is somewhat correct
- Some restrictions are stated and correct
- Understanding of how to identify whether expressions are equivalent is somewhat evident
- Understanding of how to determine other equivalent expressions is somewhat evident
- Operations with rational expressions in part d) are somewhat correct
- Justification and explanations of solutions, where required, are somewhat valid

## What Distinguishes Level 4

- Simplification of expressions is accurate, with only minor errors
- All restrictions are stated and accurate
- Understanding of how to identify whether expressions are equivalent is highly evident
- Understanding of how to determine other equivalent expressions is highly evident and supported with justification
- Operations with rational expressions in part d) are accurate, with only minor errors
- Justification and explanations of solutions, where required, are clearly valid

### Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectation | Selected Questions |
|---|---|
| Problem Solving | 10, 12 |
| Reasoning and Proving | 8–15, 17–20 |
| Reflecting | 10, 12, 13, 15, 17, 20 |
| Selecting Tools and Computational Strategies | 12, 16–20 |
| Connecting | 1–20 |
| Representing | 10, 12, 13, 15, 17 |
| Communicating | 15–18, 20 |