Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	8–10, 16, 17
Reasoning and Proving	6–17
Reflecting	8–10, 15–17
Selecting Tools and Computational Strategies	1–6, 8, 10–14, 16
Connecting	1–17
Representing	1–6, 8, 10–14, 16
Communicating	1, 6–9, 16, 17

2.4 Reflections of Functions

Student Text Pages
105 to 112

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator
- computer with *The Geometer's Sketchpad*® 4

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- T–2 *The Geometer's Sketchpad*® 4
- BLM 2–6 Section 2.4 Practice

Teaching Suggestions

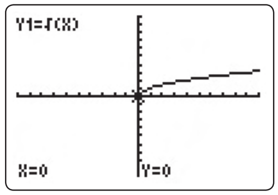
- In this section, it may be advantageous to use a graphing calculator projection unit or show graphing software using a data projector to quickly show the effect of transformations.
- The expectation covered in this section requires the use of technology to investigate the nature of transformations. In this section, a graphing calculator is used. An alternative investigation using *The Geometer's Sketchpad*® (GSP) is available in the **Use Technology** feature, part **B**, that follows Section 2.5. One advantage to this method is that in GSP you can exploit the nature of function notation. That is, once you define a function $f(x)$, you can transform that function by graphing $g(x) = f(-x)$ without actually typing in the new equation. This also allows you to instantly repeat the investigation with a new $f(x)$ and the rest of the transformations change to suit. If students have *The Geometer's Sketchpad*® on their home computers, this investigation can be assigned for homework to be consolidated during the next class. If needed, use **T–2 *The Geometer's Sketchpad*® 4** to support this activity.
- In **Example 1**, the function $f(x) = \sqrt{x} + 2$ is reflected by sketching the original function and then reflecting key points. Students should be encouraged to verify their answers using graphing technology wherever possible. Note that in part c), the use of double prime notation is introduced since there are two simultaneous reflections. This could be a source of discussion; that is, does it matter which of the two reflections occurs first? This could also be a reason to introduce the idea of reflection in a point (in this case, the origin). This idea is explored further in **question 11**.
- In **Example 2**, some reflections are shown and are to be described. Part **b**) is another opportunity to discuss the idea of reflection in a point.
- Be sure to read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.

Differentiated Instruction

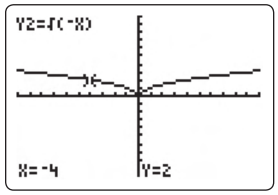
- Complete the Investigate using **co-operative task groups** of three students.
- Use a **placemat** divided into three parts to summarize the numerical, graphical, and algebraic representations of the reflections $g(x) = f(-x)$, $g(x) = -f(x)$, and $g(x) = -f(-x)$.
- Add the equations $g(x) = f(-x)$, $g(x) = -f(x)$, and $g(x) = -f(-x)$, and a description of these transformations, to the **what-so-what double entry chart** poster created in Section 2.3.
- Use **think-aloud** and have a group of students complete the Achievement Check (question 13) for the class.

Investigate Answers (page 105)

1.

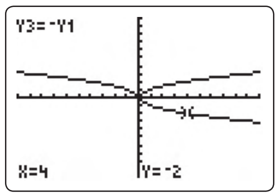


2.



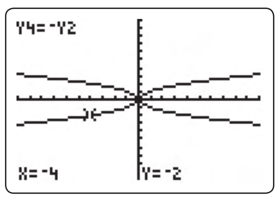
3. Answers may vary. Sample answer: The graph of **Y2** is a reflection of the graph of **Y1**. The graph of **Y1** can be reflected in the y-axis to create the graph of **Y2**.

4.



5. Answers may vary. Sample answer: The graph of **Y3** is a reflection of the graph of **Y1**. The graph of **Y1** can be reflected in the x-axis to create the graph of **Y3**.

6.



7. Answers may vary. Sample answer: The graph of **Y4** is a reflection of the graph of **Y1**. The graph of **Y1** can be reflected in the x-axis and then reflected in the y-axis to create the graph of **Y4**. The graph of **Y2** can be reflected in the x-axis to create the graph of **Y4**.

Communicate Your Understanding Responses (page 110)

C1 The function $f(x) = x$ will remain unchanged for a reflection in the x-axis followed by a reflection in the y-axis. The function $f(x) = x^2$ will remain unchanged for a reflection in the y-axis. The function $f(x) = \frac{1}{x}$ will remain unchanged for a reflection in the x-axis followed by a reflection in the y-axis. The function $f(x) = \sqrt{x}$ will remain unchanged for a reflection in the x-axis, followed by a reflection in the y-axis, followed by a reflection in the x-axis, followed by a reflection in the y-axis.

C2 **a)** Yes, $g(x)$ could be a reflection of f(x) in the x-axis.

b) No.

c) Yes, $g(x)$ could be a reflection of $f(x)$ in the y-axis.

C3 The student is not correct. Each point on each graph would have to be equidistant from the x-axis for the graph of $g(x)$ to be a reflection of the graph of $f(x)$ in the x-axis.

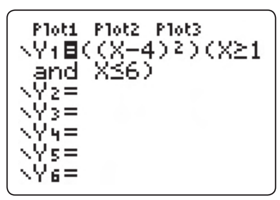
Common Errors

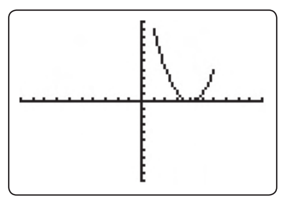
- Some students may confuse reflections in the *x*- and y-axes when trying to apply reflections algebraically.

R_x Have students remind themselves what happens to each coordinate when any point is reflected.

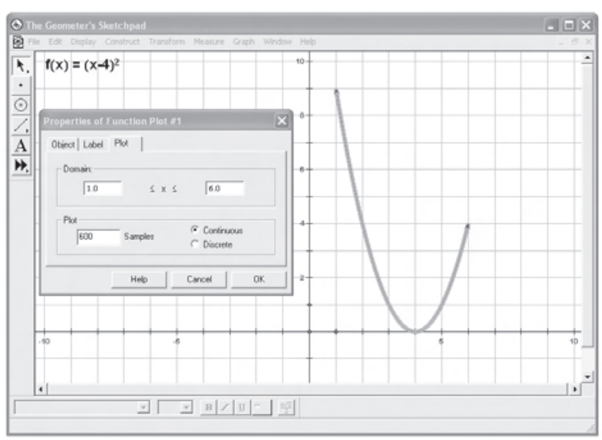
Practise, Connect and Apply, Extend

- In **question 6**, students should notice that the word *Teaz* in the logo is an ambigram. As an extension, have students look up other ambigrams and discuss the reflections.
- As a hint for **question 7**, refer students to specific examples given earlier in the section.
- In **question 8**, the idea of invariant points is addressed. Extend this discussion to indicate how the idea of invariant points could be used to identify types of transformations when only the graphs are seen.
- **Question 8** requires students to select tools and use connecting and representing skills to graph two different required reflections of the given function and to determine invariant points under the reflections. They will use their reasoning and reflecting skills to establish a function that might have an invariant point under the reflection $-f(-x)$, and will communicate the reason for the function that they have developed.
- For **question 9**, encourage students to test various reflections by checking if $f(-x)$, $-f(x)$, or $-f(-x)$ equals $g(x)$.
- Once students have investigated the idea of reflection in the origin in **question 11**, you may wish to consolidate the idea that the coordinates of each point are reversed and their signs changed.
- In **question 10**, students are asked to use graphing technology to create a logo with functions. The idea of domain and range must be used to use graphing technology properly. On the graphing calculator, to graph a function using a specific domain, use the form shown. Note that the inequality symbols and Boolean "ands" can be found by pressing 2nd [TEST].





In *The Geometer's Sketchpad*®, the domain of a function can be defined once the function is entered by right-clicking on the function itself (not the equation), choosing **Properties**, and then the **Plot** tab:



- **Question 12** gives students the opportunity to select tools and connect past mathematical knowledge in order to represent different required graphs. They will solve the problems of finding translations that can be applied to the given function that would have the same effect as reflections in the x- or y-axes by using their reasoning and reflecting skills. They are required to establish if this will work for any type of function and to communicate their findings.
- Another possible extension for **question 14** is to reflect in a point other than the origin.
- Use **BLM 2–6 Section 2.4 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 13, on student text page 112.

Achievement Check, question 13, student text page 112

This performance task is designed to assess the specific expectations covered in Sections 2.3 and 2.4. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Representing
- Reflecting
- Communicating

Sample Solution

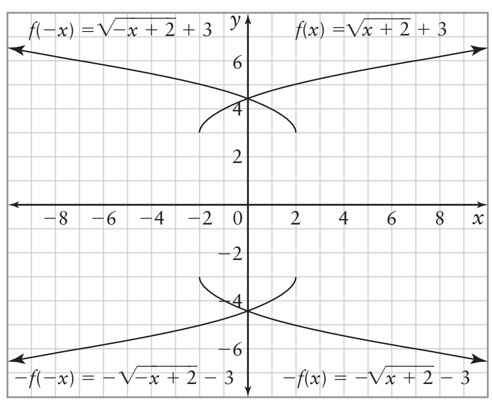
a) The base function is $g(x) = \sqrt{x}$. To obtain $f(x)$ translate $g(x)$ 2 units left and 3 units down.

b) $-f(x) = -\sqrt{x+2} - 3$; This represents a reflection in the x-axis of the graph of $f(x)$.

$f(-x) = \sqrt{-x+2} + 3$; This represents a reflection in the y-axis of the graph of $f(x)$.

$-f(-x) = -\sqrt{-x+2} - 3$; This represents a reflection in the y-axis and in the x-axis of the graph of $f(x)$.

c)



d)

Function	Domain	Range
$f(x) = \sqrt{x+2} + 3$	$\{x \in \mathbb{R}, x \geq -2\}$	$\{y \in \mathbb{R}, y \geq -3\}$
$-f(x) = -\sqrt{x+2} - 3$	$\{x \in \mathbb{R}, x \geq -2\}$	$\{y \in \mathbb{R}, y \leq -3\}$
$f(-x) = \sqrt{-x+2} + 3$	$\{x \in \mathbb{R}, x \leq 2\}$	$\{y \in \mathbb{R}, y \geq -3\}$
$-f(-x) = -\sqrt{-x+2} - 3$	$\{x \in \mathbb{R}, x \leq 2\}$	$\{y \in \mathbb{R}, y \leq -3\}$

e) There are no invariant points because there are no points that do not change under these transformations.

Level 3 Notes

Look for the following:

- Mostly accurate and fully labelled graphs
- Base function is correct
- Transformed equations are mostly correct
- Domain and range for each transformed function are mostly correct
- Understanding and justification of invariant point is mostly evident

What Distinguishes Level 2

- Somewhat accurate and partially labelled graphs
- Base function is correct
- Transformed equations are somewhat correct
- Domain and range for each transformed function are somewhat correct
- Understanding and justification of invariant point is mostly evident

What Distinguishes Level 4

- Accurate and fully labelled graphs
- Base function is correct
- Transformed equations are all correct
- Domain and range for each transformed function contain only minor errors
- Understanding and justification of invariant point is highly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7, 10, 12–14
Reasoning and Proving	5–8, 10–14
Reflecting	7, 8, 10–14
Selecting Tools and Computational Strategies	1–4, 8–10, 12–14
Connecting	1–14
Representing	1–4, 7–9, 12–14
Communicating	5, 7, 8, 11–13

2.5 Stretches of Functions

Student Text Pages
113 to 122

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator
- computer with *The Geometer's Sketchpad*®

Related Resources
- G–1 Grid Paper
- T–2 The *Geometer's Sketchpad*® 4
- BLM 2–7 Section 2.5 Practice

Teaching Suggestions

- In this section, it may be advantageous to use a graphing calculator projection unit or show graphing software using a data projector to quickly show the effect of transformations.
- The expectation covered in this section requires the use of technology to investigate the nature of transformations. In this section, a graphing calculator is used. An alternative investigation using *The Geometer's Sketchpad*® is available in the **Use Technology** feature, part **C**, that follows Section 2.5. The sketch clearly shows that when the stretch factor of, for example, 2 is used inside the function, the function is horizontally compressed by a factor of $\frac{1}{2}$.
  If students have *The Geometer's Sketchpad*® on their home computers, this investigation can be assigned for homework to be consolidated during the next class. If needed, use **T–2 *The Geometer's Sketchpad*® 4** to support this activity.
- When referring to stretches and compressions, in general, if the figure gets larger, then it has stretched; if it gets smaller, then it has compressed.