What Distinguishes Level 2

- Somewhat accurate and partially labelled graphs
- Base function is correct
- Transformed equations are somewhat correct
- Domain and range for each transformed function are somewhat correct
- Understanding and justification of invariant point is mostly evident

What Distinguishes Level 4

- Accurate and fully labelled graphs
- Base function is correct
- Transformed equations are all correct
- Domain and range for each transformed function contain only minor errors
- Understanding and justification of invariant point is highly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7, 10, 12–14
Reasoning and Proving	5–8, 10–14
Reflecting	7, 8, 10–14
Selecting Tools and Computational Strategies	1–4, 8–10, 12–14
Connecting	1–14
Representing	1–4, 7–9, 12–14
Communicating	5, 7, 8, 11–13

Stretches of Functions

Student Text Pages
113 to 122

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator
- computer with *The Geometer's Sketchpad®*

Related Resources
- G–1 Grid Paper
- T–2 The *Geometer's Sketchpad®* 4
- BLM 2–7 Section 2.5 Practice

Teaching Suggestions

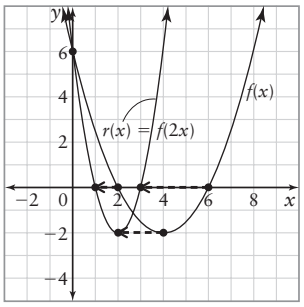
- In this section, it may be advantageous to use a graphing calculator projection unit or show graphing software using a data projector to quickly show the effect of transformations.
- The expectation covered in this section requires the use of technology to investigate the nature of transformations. In this section, a graphing calculator is used. An alternative investigation using *The Geometer's Sketchpad®* is available in the **Use Technology** feature, part **C**, that follows Section 2.5. The sketch clearly shows that when the stretch factor of, for example, 2 is used inside the function, the function is horizontally compressed by a factor of $\frac{1}{2}$.

  If students have *The Geometer's Sketchpad®* on their home computers, this investigation can be assigned for homework to be consolidated during the next class. If needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- When referring to stretches and compressions, in general, if the figure gets larger, then it has stretched; if it gets smaller, then it has compressed.

Differentiated Instruction

- Use **Think-Pair-Share** to complete the Investigate and Communicate Your Understanding questions.
- Add the equations $g(x) = af(x)$ and $g(x) = f(kx)$ and a description of their transformations to the **what-so-what double entry chart** poster created in Section 2.3.

- **Example 1** part **b)** illustrates the effect of horizontal stretches on a function. This is particularly hard to show as graphing points will seem to show a vertical stretch. Refer to the diagram shown in the solution to clarify. Remember that for every vertical stretch, there will be a corresponding horizontal stretch that will have the same effect. Use *The Geometer's Sketchpad*® sketch from the alternative investigation mentioned above to show the effects dynamically.
- Using the method shown in **Example 1**, you may wish to use a slightly more complex function (say, $f(x) = 0.5(x - 2)(x - 6)$) that better shows the effects of horizontal stretches, including the effects on intercepts.

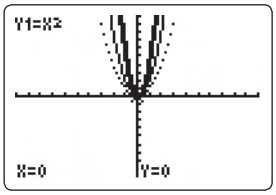


- **Example 2** shows how the same transformation can be described by different (but related) vertical and horizontal stretches.
- Read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.

Investigate Answers (pages 113–114)

A

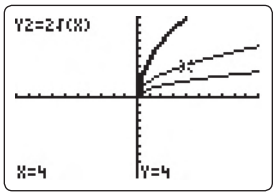
1.



2. Answers may vary. Sample answer: The equation of $g(x)$ is the same as the equation of $f(x)$ with the x^2-variable multiplied by 2. The equation of $h(x)$ is the same as the equation of $f(x)$ with the x^2-variable multiplied by 5. The graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched vertically by a factor of 5.

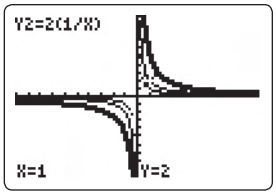
3. Answers may vary. Sample answer:

Set 2:



The equation of $g(x)$ is the same as the equation of $f(x)$ with the square root function multiplied by 2. The equation of $h(x)$ is the same as the equation of $f(x)$ with the square root function multiplied by 5. The graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched vertically by a factor of 5.

Set 3:



The equation of $g(x)$ is the same as the equation of $f(x)$ with the rational function multiplied by 2. The equation of $h(x)$ is the same as the equation of $f(x)$ with the rational function multiplied by 5. The graph of $g(x)$ is the graph of $f(x)$ stretched vertically by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched vertically by a factor of 5.

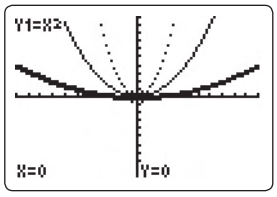
4. a) Answers may vary. Sample answer: The value of a in $g(x) = af(x)$ stretches the graph vertically by a factor of a if $a > 0$.

b) The point (0, 0) is an invariant point.

c) The domain and range for the function do not change when the graph of the function is stretched vertically by a factor of a, $a > 0$.

B

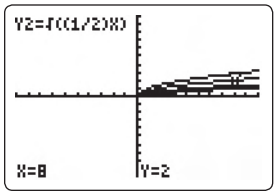
1.



2. Answers may vary. Sample answer: The equation of $g(x)$ is the same as the equation of $f(x)$ with the x-variable multiplied by $\frac{1}{2}$ before the function is squared. The equation of $h(x)$ is the same as the equation of $f(x)$ with the x-variable multiplied by $\frac{1}{5}$ before the function is squared. The graph of $g(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 5.

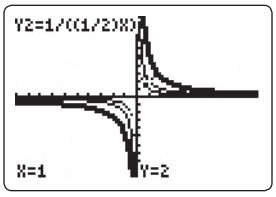
3. Answers may vary. Sample answer:

Set 2:



The equation of $g(x)$ is the same as the equation of $f(x)$ with the x-variable in the square root function multiplied by $\frac{1}{2}$. The equation of $h(x)$ is the same as the equation of $f(x)$ with the x-variable in the square root function multiplied by $\frac{1}{5}$. The graph of $g(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 5.

Set 3:



The equation of $g(x)$ is the same as the equation of $f(x)$ with the x-variable in the denominator of the rational function multiplied by $\frac{1}{2}$. The equation of $h(x)$ is the same as the equation of $f(x)$ with the x-variable in the denominator of the rational function multiplied by $\frac{1}{5}$. The graph of $g(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 2. The graph of $h(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 5.

4. a) Answers may vary. Sample answer: The value of k in $g(x) = f(kx)$ stretches the graph horizontally by a factor of $\frac{1}{k}$ if $k > 0$.

b) The point (0, 0) is an invariant point.

c) The domain and range for the function do not change when the graph of the function is stretched horizontally by a factor of $\frac{1}{k}$, $k > 0$.

5. Answers may vary. Sample answer: The transformations in part A of the Investigate stretch the graph of a function vertically. The transformations in part B of the Investigate stretch the graph of a function horizontally.

Communicate Your Understanding Responses (page 119)

C1 Answers may vary. Sample answer: The graph of $f(x)$ is a vertical stretch of the graph of $f(x)$ and not a translation, since the two functions intersect at the points $(-3, 0)$ and $(1, 0)$.

C2 Answers may vary. Sample answer: The graph of $g(x) = af(x)$, $a > 0$, stretches the graph of $f(x)$ vertically, since the y-values are each multiplied by a factor of a.

C3 The graph of the base function is stretched vertically by a factor of 3.

Common Errors

- Some students may confuse the factor used when speaking of a compression.

R_x Have students review proper terminology. You may want to post examples of each type of transformation and their descriptions on your classroom walls.

Practise, Connect and Apply, Extend

- For **question 5**, students may be confused by the fact that there are two variables in the equation. Remind them to use the table values for the value of g to create three different functions.
- **Question 5** allows students to select tools and use connecting skills to represent a graph of the period versus the length of pendulums related to the theoretical values of acceleration due to gravity for different planets. Students will reflect on and communicate their reasoning concerning the representation of a similar graph of a comet whose gravity is one tenth that of Earth.
- Especially for part **b)** of **question 6**, students may state their answers as either horizontal or vertical stretches.
- **Question 7** requires students to select tools to allow them to represent different transformations related to the graph of $f(x) = x^2$. The students will connect mathematical concepts that they have learned in the past with transformation knowledge learned in this chapter to reason, reflect on, and solve the problem of using a single stretch on the base function that will have the same effect as two different stretches. Students will communicate their results.
- For **question 7**, students should be encouraged to verify their answers using *The Geometer's Sketchpad*® function notation and graphing capabilities. In particular, part **f)** can be done quickly by changing the base function in the sketch.
- **Question 8** is an example of how mathematics can help in fields that may not at first seem mathematical.
- For **question 11**, you may have to point out that the -4.9 at the front of the equation is just $-9.8 \div 2$.
- **Question 12** revisits the idea of computer animation using *The Geometer's Sketchpad*®. If needed, use **T–2 *The Geometer's Sketchpad*® 4** to support this activity.
- **Question 13** is a nice preview of grade 12 Advanced Functions.
- Use **BLM 2–7 Section 2.5 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	7, 12
Reasoning and Proving	1–7, 9–13
Reflecting	1, 5, 7, 9, 10, 12, 13
Selecting Tools and Computational Strategies	1, 2, 4, 5, 7–9, 13
Connecting	1–13
Representing	1, 2, 4, 5, 7–9, 11, 13
Communicating	1, 3–7, 10, 12, 13

Use Technology

Student Text Pages
123 to 124

Suggested Timing
60 min

Tools
- computer with *The Geometer's Sketchpad®*
- Translations1.gsp
- Translations2.gsp
- Stretches1.gsp
- Stretches2.gsp

Related Resources
- T–2 *The Geometer's Sketchpad®* 4

Use *The Geometer's Sketchpad®* to Explore Transformations

Teaching Suggestions

- These sketches don't require much prior knowledge of *The Geometer's Sketchpad®* to complete. A reminder of how the **Selection Arrow Tool** works may be helpful. At the end of each translation and stretch sketch, students are asked to measure the coordinates of each point. Remind them that they should click anywhere on the white space to deselect everything before selecting the points to be measured. If needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- The last sketch in each of the stretch files is very useful in showing the effects of stretches and which types of points are invariant under various stretches.
- The provided sketches deal with the following topics:
  - Translations1.gsp (vertical translations)
  - Translations2.gsp (horizontal translations)
  - Stretches1.gsp (vertical stretches)
  - Stretches2.gsp (horizontal stretches)
- The intent is that these be done as needed as you work through Sections 2.3 to 2.5, rather than being done all at once.

2.6 Combinations of Transformations

Student Text Pages
125 to 131

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 2–8 Section 2.6 Summary Table
- BLM 2–9 Section 2.6 Practice

Teaching Suggestions

- In this section, it may be advantageous to use a graphing calculator projection unit or show graphing software using a data projector to quickly show the effect of transformations.
- The **Investigate** can be completed using pencil and paper, a graphing calculator, or graphing software, such as *The Geometer's Sketchpad®*.
- Much like the order of operations, the order of transformations is important when stretches and translations are combined.
- For **Example 1**, stress the importance of factoring the inside of the function to show the individual stretches and translations in the form $y = af[k(x - d)] + c$. Also, use a projector with a computer to show the transformations via *The Geometer's Sketchpad®*.
- For **Example 2** part **b)**, an approximate answer is determined using the graph from part **a)**. A more exact answer can be found when students determine the inverse of each function in Section 2.7, question 14.
- **BLM 2–8 Section 2.6 Summary Table** summarizes transformations and their effects. Give it to students to help them consolidate what they know.
- Read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.
- The intent with **question C2** is to remind students that they must factor any stretches out of the inside of a function to accurately describe both the horizontal compression and translation.