Use Technology

Student Text Pages
123 to 124

Suggested Timing
60 min

Tools
- computer with *The Geometer's Sketchpad®*
- Translations1.gsp
- Translations2.gsp
- Stretches1.gsp
- Stretches2.gsp

Related Resources
- T–2 *The Geometer's Sketchpad®* 4

Use *The Geometer's Sketchpad®* to Explore Transformations

Teaching Suggestions

- These sketches don't require much prior knowledge of *The Geometer's Sketchpad®* to complete. A reminder of how the **Selection Arrow Tool** works may be helpful. At the end of each translation and stretch sketch, students are asked to measure the coordinates of each point. Remind them that they should click anywhere on the white space to deselect everything before selecting the points to be measured. If needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- The last sketch in each of the stretch files is very useful in showing the effects of stretches and which types of points are invariant under various stretches.
- The provided sketches deal with the following topics:
  - Translations1.gsp (vertical translations)
  - Translations2.gsp (horizontal translations)
  - Stretches1.gsp (vertical stretches)
  - Stretches2.gsp (horizontal stretches)
- The intent is that these be done as needed as you work through Sections 2.3 to 2.5, rather than being done all at once.

2.6 Combinations of Transformations

Student Text Pages
125 to 131

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 2–8 Section 2.6 Summary Table
- BLM 2–9 Section 2.6 Practice

Teaching Suggestions

- In this section, it may be advantageous to use a graphing calculator projection unit or show graphing software using a data projector to quickly show the effect of transformations.
- The **Investigate** can be completed using pencil and paper, a graphing calculator, or graphing software, such as *The Geometer's Sketchpad®*.
- Much like the order of operations, the order of transformations is important when stretches and translations are combined.
- For **Example 1**, stress the importance of factoring the inside of the function to show the individual stretches and translations in the form $y = af[k(x - d)] + c$. Also, use a projector with a computer to show the transformations via *The Geometer's Sketchpad®*.
- For **Example 2** part **b)**, an approximate answer is determined using the graph from part **a)**. A more exact answer can be found when students determine the inverse of each function in Section 2.7, question 14.
- **BLM 2–8 Section 2.6 Summary Table** summarizes transformations and their effects. Give it to students to help them consolidate what they know.
- Read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.
- The intent with **question C2** is to remind students that they must factor any stretches out of the inside of a function to accurately describe both the horizontal compression and translation.

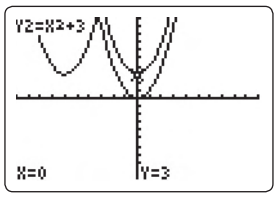
Differentiated Instruction

- Start this section with **blast off**. Ask students to record three important things they learned about transformations, two questions they have about transformations, and one reflection about transformations (possibly a tip for their peers on graphing transformations).
- Complete the Investigate using **Think-Pair-Share**.
- Use **think-aloud** and have two groups follow a different order of transformations when graphing a given function. Check the functions with graphing calculators to determine which result is correct. Emphasize that order matters when combining transformations to graph functions.
- Play **function aerobics** to review functions and transformations. Create large cue cards representing a variety of functions, all written in the form $y = af[k(x - d)] + c$. Have students imagine they are standing at the origin on a Cartesian grid. Hold cue cards up for the class one at a time. Students use their arms to represent the shape of the function, and then hop in the direction of each translation. Students who demonstrate incorrect arm shape or hop in the wrong direction are asked to sit down. Continue the game until one student is left standing and a winner is declared. Add music to enhance the appeal of the game.

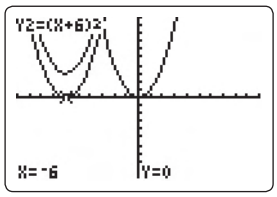
Investigate Answers (pages 125–126)

A

1. a)



b)



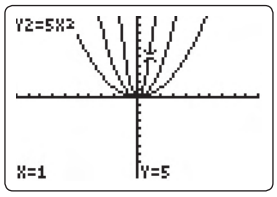
2. Answers may vary. Sample answer: The graph of $g(x)$ is the graph of $f(x)$ translated up 3 units. The graph of $h(x)$ is the graph of $g(x)$ translated left 6 units. The graph of $m(x)$ is the graph of $f(x)$ translated left 6 units. The graph of $r(x)$ is the graph of $m(x)$ translated up 3 units.

3. $h(x) = (x + 6)^2 + 3$; $r(x) = (x + 6)^2 + 3$

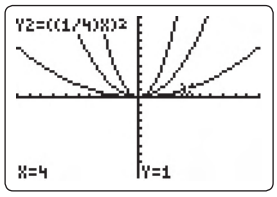
4. Answers may vary. Sample answer: The graphs of $h(x)$ and $r(x)$ are the same, and the equations of $h(x)$ and $r(x)$ are the same. The order of the translations does not matter. The final graph is still the same if the translations are done in either order.

B

1. a)



b)



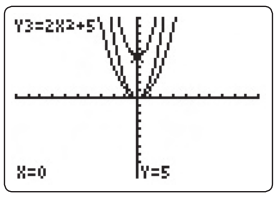
2. Answers may vary. Sample answer: The graph of $b(x)$ is the graph of $f(x)$ stretched vertically by a factor of 5. The graph of $p(x)$ is the graph of $b(x)$ stretched horizontally by a factor of 4. The graph of $n(x)$ is the graph of $f(x)$ stretched horizontally by a factor of 4. The graph of $s(x)$ is the graph of $n(x)$ stretched vertically by a factor of 5.

3. $p(x) = 5\left(\frac{1}{4}x\right)^2$; $s(x) = 5\left(\frac{1}{4}x\right)^2$

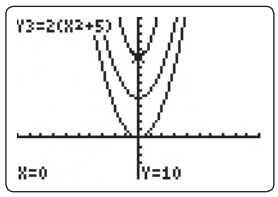
4. Answers may vary. Sample answer: The graphs of $p(x)$ and $s(x)$ are the same, and the equations of $p(x)$ and $s(x)$ are the same. The order of the stretches does not matter. The final graph is still the same if the stretches are done in either order.

C

1. a)



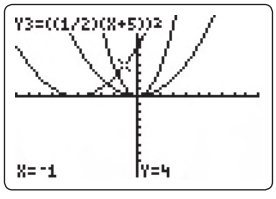
b)



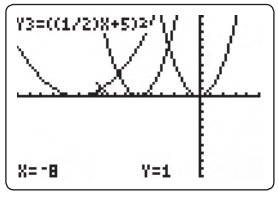
2. The graph of $j(x)$ is the graph of $f(x)$ stretched vertically by a factor of 2. The graph of $s(x)$ is the graph of $j(x)$ translated up 5 units. The graph of $q(x)$ is the graph of $f(x)$ translated up 5 units. The graph of $t(x)$ is the graph of $q(x)$ stretched vertically by a factor of 2.

3. $s(x) = 2x^2 + 5$; $t(x) = 2x^2 + 10$

4. a)



b)



5. The graph of $w(x)$ is the graph of $f(x)$ stretch horizontally by a factor of 2. The graph of $u(x)$ is the graph of $w(x)$ translated left 5 units. The graph of $v(x)$ is the graph of $f(x)$ translated left 5 units. The graph of $z(x)$ is the graph of $v(x)$ stretched horizontally by a factor of 2.

6. $u(x) = \left(\left(\frac{1}{2}\right)(x+5)\right)^2$; $z(x) = \left(\frac{1}{2}x + 5\right)^2$

7. Answers may vary. Sample answer: When combining transformations, order matters. To accurately sketch the graph of a function, apply transformations represented by the parameters a and k before transformations represented by the parameters d and c. That is, stretches, compressions, and reflections occur before translations. This is similar to the order of operations, where multiplication and division occur before addition and subtraction.

Communicate Your Understanding Responses (page 129)

C1 Answers will vary. Sample answer: The parameters representing these transformations are all multiplied with the function and multiplication is commutative.

C2 Answers may vary. Sample answer: $g(x)$ can be expressed as $g(x) = f[3(x + 4)]$. This will be a horizontal compression by a factor of $\frac{1}{3}$ followed by a horizontal translation of 4 units left of the base function $f(x)$.

Common Errors

- Some students may look at a function, such as $g(x) = f(3x + 6)$, and say that the horizontal translation is 6 to the left.

R_x Have students confirm their ideas using graphing software.

Practise, Connect and Apply, Extend

- **Questions 1** and **2** are essentially the same question. However, **question 1** has at most two transformations, while **question 2** has more complex transformations. **Questions 3** and **4** follow a similar pattern. Students should be encouraged to check their answers using technology. As you take up the homework, use a data projector with a computer with *The Geometer's Sketchpad®* to speed up the process.
- In **question 5**, the questions get progressively more complex and so should be assigned with that in mind.
- **Question 6** assumes negligible wind resistance. Inquiring students may have concerns about this. This could lead to a discussion on how the functions might change if wind resistance came into play.
- **Question 6** requires students to select tools and use their connecting skills to represent graphically the motion of two skydivers. They will use their reasoning and reflecting skills, and communicate the results found when examining the two graphs.
- With **question 7**, students should be encouraged to check their answers with graphing software or a graphing calculator.
- **Question 9** enables students to solve a general algebraic problem involving transformations by using their reasoning and reflecting skills. They will connect previous mathematical knowledge with material learned in this chapter to solve the problem and communicate an explanation.
- Although **question 12** could be seen as a preview of Advanced Functions, students should be encouraged to deal with these transformations in the same way as they would for **question** (**i.e.,** key points and sketch).
- **Question 13** should be a bit of a review of grade 10 work.
- In **question 14**, the animation sketch of previous sections is revisited, now using the full complement of transformations.
- Use **BLM 2–9 Section 2.6 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 11, on student text page 131.

Achievement Check, question 11, student text page 131

This performance task is designed to assess the specific expectations covered in Sections 2.5 and 2.6. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Communicating

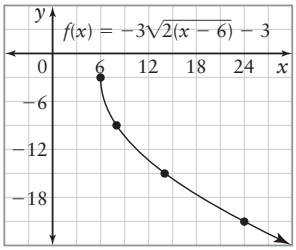
Sample Solution

a) The parameters are $a = -3$, $k = 2$, $d = 6$, and $c = -3$, so the transformed equation is $g(x) = -3\sqrt{2(x - 6)} - 3$.

b) Answers may vary. Sample answer:

x	$f(x) = \sqrt{x}$	$g(x) = -3\sqrt{2(x-6)} - 3$
6	$\sqrt{6}$	-3
8	$\sqrt{8}$	-9
14	$\sqrt{14}$	-15
24	-18	-21

c)



d) domain $\{x \in \mathbb{R}, x \geq 6\}$; range $\{y \in \mathbb{R}, y \leq -3\}$

Level 3 Notes

Look for the following:

- Image points are mostly correct
- Mostly accurate and fully labelled graph
- Equation of transformed function is mostly correct
- Domain and range for transformed function are correct

What Distinguishes Level 2

- Image points are somewhat correct
- Somewhat accurate and partially labelled graph
- Equation of transformed function is somewhat correct
- Domain and range of transformed function are somewhat correct

What Distinguishes Level 4

- Image points are correct
- Accurate and fully labelled graph
- Equation of transformed function may contain a minor error
- Domain and range for transformed function are correct

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	9, 14
Reasoning and Proving	1–6, 8–14
Reflecting	6, 8, 9, 11, 14
Selecting Tools and Computational Strategies	3–7, 10–13
Connecting	1–14
Representing	3–7, 10–13
Communicating	1–4, 6, 9, 14