2.7 Inverse of a Function

Student Text Pages
132 to 141

Suggested Timing
70 min

Tools
- grid paper
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 2–10 Section 2.7 Investigate Inverses With a Mira
- BLM 2–11 Section 2.7 Practice
- BLM 2–12 Section 2.7 Achievement Check Rubric

Teaching Suggestions

- This section deals with four expectations that all relate to inverses of a function. There is more than one way to investigate the nature of inverses. An alternative investigation using a Mira is available on **BLM 2–10 Section 2.7 Investigate Inverses With a Mira**. Inverses can be further investigated using *The Geometer's Sketchpad*® sketch 2.7 Inverses.gsp.
- **Example 1** is a nice way for students to ease into the idea of inverses since it only requires graphing of points.
- **Example 2** introduces the concept of using algebraic manipulation to find the inverse of a function. Students should be encouraged to use graphing technology to verify their answers. Note that if you are using a graphing calculator, you should use **ZSquare** from the **ZOOM** menu to make sure that the vertical and horizontal axes are the same scale. This makes the graph visually proportional. Otherwise, the graph will not look as if $y = x$ is the mirror for the function and its inverse. Remind students that if they are to graph the inverse of a quadratic function, then they must graph both the positive and negative sides for the entire function to be seen.
- For **Example 2**, instead of entering the inverse in the **Y=** editor, students can enter the function in **Y1** and enter the reflection line in **Y2** ($y = x$). Then, to graph the inverse, press 2nd [DRAW] and select **8:DrawInv**. The **DrawInv** will show up on the home screen so that students can enter **Y1** next to it and then press ENTER. Students will then see the inverse drawn on the graph. To erase the inverse line, press 2nd [DRAW] and select **1:ClrDraw**.
- **Example 3** shows that it sometimes makes more sense for the independent variable to be on the vertical axis.
- Read through the **Key Concepts** with students to ensure that they have mastered the skills developed in this section.

Differentiated Instruction

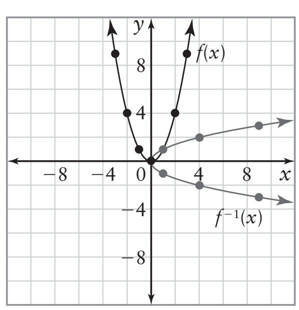
- Use **concept attainment** to define the concept of an inverse function. Organize students into groups. Provide several examples of functions and their inverses (in numerical, algebraic, and graphical forms). Then, provide a second group of non-examples. Students identify and define the concept of an inverse function, and generate their own list of examples and non-examples.
- Construct a **Frayer model** to summarize the key characteristics of an inverse function.

Investigate Answers (pages 132–133)

1. a)

Points on the Function	Points on the Inverse of the Function
(–3, 9)	(9, –3)
(–2, 4)	(4, –2)
(–1, 1)	(1, –1)
(0, 0)	(0, 0)
(1, 1)	(1, 1)
(2, 4)	(4, 2)
(3, 9)	(9, 3)

b) and **c)**



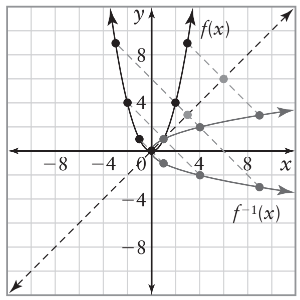
2. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

b) domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}\}$

3. a) The domain and range of $f(x)$ and the domain and range of $f^{-1}(x)$ are the reverse of each other.

b) $f^{-1}(x)$ is not a function. The x-values of 2, 4, and 9 each have two corresponding y-values.

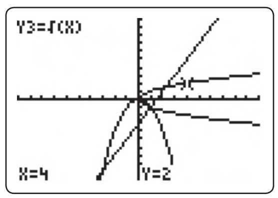
4. a) and **b)**



c) $y = x$

5. Answers may vary. Sample answer: Graph the line $y = x$ on the same set of axes as the function. Reflect each point of the function in the line $y = x$. Connect the points to draw the graph of the inverse function.

6.



7. Answers may vary. Sample answer: The graph of $k(x) = \pm\sqrt{x}$ is the same as the graph of the inverse of $f(x)$. The equations contain reverse operations. The equation of the inverse can be found by interchanging the x- and y-variables and solving the equation for y in terms of x.

Communicate Your Understanding Responses (page 137)

C1 Answers may vary. Sample answer: The equation $3x + 4 = 19$ is solved to determine a numerical value for x. In order to find the inverse of a function, the equation of the function is solved to determine an expression for y in terms of x.

C2 Answers may vary. Sample answer: The function $f(x) = 9x - 5$ is formed by multiplying by 9 and then subtracting 5. The inverse function $f^{-1}(x) = \frac{x + 5}{9}$ is formed by adding 5 and then dividing by 9. The operations to form the original function and the inverse function are the opposite operations and are in the opposite order.

C3 Answers may vary. Sample answer: In vertex form, the variable x appears only once, and thus it can be isolated (solved for).

Common Errors

- Some students may see $f^{-1}(x)$ and think that it means $\frac{1}{f(x)}$.

R_x Have students graph $\frac{1}{f(x)}$ using graphing technology to see that it is not the reflection of $f(x)$ in the line $y = x$.

- Some students may find the inverse of $f(x) = 3x^2 + 2x + 7$ by just swapping the x and y variables to get $x = 3y^2 + 2y + 7$.

R_x Have students review the various methods for finding the inverse to see that swapping the x and y only happens when there are known points of the function.

Practise, Connect and Apply, Extend

- For **question 2**, some students may use the line $y = x$ to graphically reflect the points, while others will find the coordinates of each point and reverse them.
- In **question 3**, students are required to draw the line $y = x$ and then reflect the graph of the function in that line. Encourage students to choose key points to reflect and then draw the inverse through those key reflected points.
- **Question 3** allows students to use their reasoning skills to select tools and connect with previously learned mathematical concepts to represent graphically the inverse of each given function. They will communicate whether the inverse formed is a function.
- In **questions 5** through **7**, students find the inverse of a quadratic function. In **question 5** it is fairly straightforward since all are in vertex form. In **question 6**, each equation must be put into vertex form first, and in **question 7** students have to recognize which of the two previous methods is correct.

- **Question 7** gives students the opportunity to select tools to represent each function and its inverse with a graph. They will use their connecting skills to graph each function and its inverse, and communicate whether each inverse is a function.
- For **question 8**, students may need to be reminded that only half of the function needs to be used. Part **a**) should take care of that.
- **Question 9** requires students to select tools and use connecting skills to represent the given function and its inverse graphically. They will reflect on and communicate the reasons that allow them to make predictions concerning the domain and range of the function.
- **Question 11** could also be posed as "Determine what the connection is between the function $f(x) = x + b$ and its inverse."
- For **question 12** part **c**), the answer should be where both the original and the inverse intersect the line $y = x$.
- For **question 13**, refer students back to **Example 1** to try to understand what the inverse represents in the context of the question.
- For **question 16** the answer should be that an inverse of an inverse is the original function. Note that when the inverse of the inverse is done for the quadratic, care must be taken to find the second inverse since, technically, there are restrictions on the domain and range.
- Check **questions 19** through **21** using graphing technology or software.
- For **question 22**, finding the inverse of each of the functions and seeing that they are equal to the other function should be good enough.
- Use **BLM 2–11 Section 2.7 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 141.

Achievement Check, question 17, student text page 141

This performance task is designed to assess the specific expectations covered in Section 2.7. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

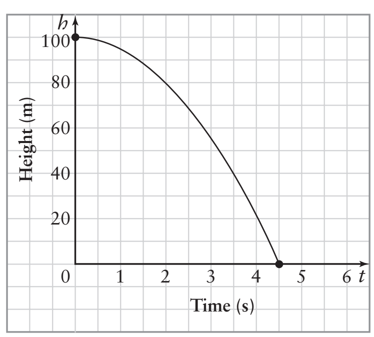
Provide students with **BLM 2–12 Section 2.7 Achievement Check Rubric** to help them understand what is expected.

a) $h(t) = 100 - 5t^2$

Solve $h(t) = 0$ to find the t-intercepts. They are $t = -\sqrt{20}$ and $t = \sqrt{20}$.

Since $t \geq 0$, only consider $t = \sqrt{20}$.

The axis of symmetry is $t = 0$. The vertex is (0, 100).



domain $\{t \in \mathbb{R}, 0 \leq t \leq \sqrt{20}\}$

range $\{h \in \mathbb{R}, 0 \leq h \leq 100\}$

b)

$$
\begin{aligned}
h &= 100 - 5t^2 \\
h - 100 &= -5t^2 \\
\frac{h - 100}{-5} &= t^2 \\
t &= \sqrt{\frac{100 - h}{5}}
\end{aligned}
$$

The inverse function is $t(h) = \sqrt{\dfrac{100 - h}{5}}$. It represents the time it takes for the rock to reach any height h.
domain $\{h \in \mathbb{R}, 0 \le h \le 100\}$
range $\{t \in \mathbb{R}, 0 \le t \le \sqrt{20}\}$

c) First, complete the square to write in vertex form.

$$\begin{aligned} h &= 100 + 10t - 5t^2 \\ &= -5t^2 + 10t + 100 \\ &= -5(t^2 - 2t) + 100 \\ &= -5(t^2 - 2t + 1 - 1) + 100 \\ &= -5(t - 1)^2 + 5 + 100 \end{aligned}$$

Now, find the inverse. Isolate t.

$$\begin{aligned} h &= -5(t - 1)^2 + 105 \\ h - 105 &= -5(t - 1)^2 \\ \frac{h - 105}{-5} &= (t - 1)^2 \\ t &= \sqrt{\frac{h - 105}{-5}} + 1 \end{aligned}$$

(Only the positive square root makes sense for this situation.)

The rock hits the ground when $h = 0$.

$$\begin{aligned} t &= \sqrt{\frac{0 - 105}{-5}} + 1 \\ &\doteq 5.6 \end{aligned}$$

Therefore, the rock hits the ground after approximately 5.6 s.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	22
Reasoning and Proving	3–6, 8–22
Reflecting	8–10, 12, 13, 16, 17, 19, 22
Selecting Tools and Computational Strategies	2, 3, 7–9, 12, 15, 17, 18, 22
Connecting	1–22
Representing	2–9, 12, 15, 17, 18
Communicating	3, 7–13, 16, 17, 19, 20