Use Technology

Student Text Pages
158 to 159

Suggested Timing
10–15 min

Tools
- TI-Nspire™ CAS graphing calculator

Use the Lists and Trace Features on a TI-Nspire™ CAS Graphing Calculator

Teaching Suggestions

- In **A**, the **Lists & Spreadsheet** application is used to perform repeated finite difference calculations that occur in **Example 1** of Section 3.1. In **B** and **C**, the **Lists & Spreadsheet** and **Graphs & Geometry** applications are used to represent and explore the exponential relation presented in **Example 2** of Section 3.1. These two representations can be viewed simultaneously by setting the viewing window to split-screen view. Refer to step 7 of the Use Technology activity on page 87 for instructions.
- If you know the name of the operation that you want—for example, δlist(pop) in step 3—you can type it directly into the formula cell without having to access the catalogue.
- Note that using the **Trace** function, as required in **C**, can cause the calculator to reset your window settings.

3.2 Exponential Decay: Connecting to Negative Exponents

Student Text Pages
160 to 169

Suggested Timing
75–150 min

Tools
- graphing calculator
- computer with spreadsheet software

Related Resources
- BLM 3–3 Section 3.2 Practice

Differentiated Instruction
- Complete the Investigate and answer the Reflect questions using **Think-Pair-Share**.
- Construct a **Frayer model** for exponential decay. Use the Examples and Key Concepts for definitions, characteristics, examples, and non-examples of exponential decay.
- Use **timed retell** to answer Communicate Your Understanding questions C1, C3, and C4.

Teaching Suggestions

- This lesson may be split over two periods. Monitor student progress through the **Investigate** and **Example 1**, leaving **Examples 2** to **4** and the related exercises for the next period, if necessary.
- The purpose of the **Investigate** is to illustrate the similarities and differences between exponential growth and exponential decay. Nuclear decay is one type of exponential decay encountered in the physical sciences. The method used in the **Investigate** is to perform an exponential regression analysis on data generated in a table. Students should examine the equation produced by this regression and reflect on why it makes sense.
- If you have used the paper folding activity described in the Teaching Suggestions for Section 3.1, you can use the same activity again to illustrate exponential decay. Instead of focusing on the number of layers, focus on the change in the visible area as the sheet of paper is repeatedly folded. After one fold, $\frac{1}{2}$ of the original area is showing; after two folds, $\frac{1}{4}$ of the original area is showing; and so on.
- **Example 1** requires students to develop an equation to model exponential decay. The equation is used to find an unknown appearing in the exponent. Three methods of solution are presented: using a table or spreadsheet, carrying out a systematic trial, and using graphing technology. Many students will benefit from seeing this variety of approaches. Students will learn an algebraic method to solve this type of problem when they study logarithms in grade 12.
- **Examples 2** to **4** highlight the relationship between powers with negative exponents and corresponding powers with positive exponents.

Investigate Answers (pages 160–161)

1.

Time (years)	Number of Half-Life Periods (2 years)	Amount of U-239 Remaining (mg)
0	0	1000
2	1	500
4	2	250
6	3	125
8	4	62.5

2. Answers may vary. Sample answers:

a) The amount decreases to one half of the value before it.

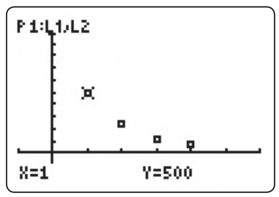
b) Exponential, since the terms are decreasing by a constant factor

3. a) **i)** 13.2 years

ii) 20 years

b) Answers may vary. Sample answer: No, the mass will never reach zero. The initial value is positive, and multiplying that value by a positive number will always give a non-zero number.

4. a)



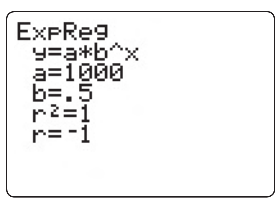
b) Answers may vary. Sample answer: They are alike in the general shape of the curve, but the curve in Section 3.1 increases as x increases, while this curve decreases as x increases.

5. a)

Time (years)	Number of Half-Life Periods (2 years)	Amount of U-239 Remaining (mg)	First Differences	Second Differences
0	0	1000		
			−500	
2	1	500		250
			−250	
4	2	250		125
			−125	
6	3	125		62.5
			−62.5	
8	4	62.5		

b) Answers may vary. Sample answer: Yes, this relationship is exponential, since the ratio of successive finite differences is constant.

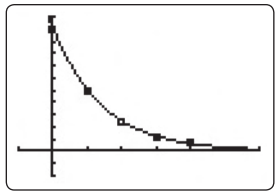
6. a)



b) $y = 1000\left(\frac{1}{2}\right)^x$

c) Answers may vary. Sample answer: The value of a represents the initial amount. The value of b represents the factor by which terms are being multiplied to give the next term in the pattern.

7. a)



b) Answers may vary. Sample answer: Yes, because the curve passes through all points from the table.

8. Answers may vary. Sample answer: The curve continues to get closer and closer to zero as the value of x increases, but never reaches zero.

Communicate Your Understanding Responses (page 166)

C1 Answers may vary. Sample answer: The two are alike in that both have a horizontal asymptote. However, exponential growth is an increasing function and exponential decay is a decreasing function.

C2 a) Answers may vary. Sample answer: The functions are equivalent, since $\frac{1}{2} = 2^{-1}$, and the power of a power rule can be used to show $\left(\frac{1}{2}\right)^n = 2^{-n}$.

b) Answers may vary. Sample answer: Values of x can be substituted into each expression to show that they generate the same y-values. Also, I can use technology to graph the functions and check that the graphs overlap.

C3 Answers may vary. Sample answer: An expression with a negative exponent, like 2^{-3}, can be written using a fraction with a positive exponent in the denominator: $2^{-3} = \frac{1}{2^3}$.

C4 Answers may vary. Sample answer: A term involving a fraction with a positive exponent can be written as a fraction with a negative exponent in the denominator by taking the reciprocal: $\left(\frac{2}{3}\right)^4 = \left(\frac{3}{2}\right)^{-4}$.

Common Errors

- Some students may confuse raising a power to a negative exponent with multiplying by a negative number.

R_x Have students work through the conceptual development of the meaning of a negative exponent, as presented in the Investigate. Another approach is to use patterning: refer to Section 3.1 Example 2 on student text page 153. The pattern developed in that example can be continued to include negative exponents.

Practise, Connect and Apply, Extend

- **Question 8** requires students to reason and reflect when creating a table of values for the given situation, and then to use an equation to represent that information. Students select tools and use their connecting skills to represent the equation by a graph, and use their communicating skills to answer questions related to that graph. Students use their problem solving, reasoning, and reflecting skills to produce a second function that models the same situation. Finally, students communicate why the two functions are equivalent.
- **Question 12** introduces depreciation, another context often modelled using exponential decay, and frequently used in business and accounting applications.
- **Question 13** allows students to reason through and reflect on how the exponential model in **Example 1** can be used to extrapolate the decay occurring prior to the beginning of the study. Students communicate their findings and use their connecting skills to answer questions related to the mass of this sample.
- The financial applications of exponential growth and decay that feature in **questions 16** and **17** are explored in greater depth in Chapter 7.
- There are connections to physics in **questions 18** to **20**.
- Use **BLM 3–3 Section 3.2 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 16, on student text page 168.

Achievement Check, question 16, student text page 168

This performance task is designed to assess the specific expectations covered in Sections 3.1 and 3.2. The following mathematical process expectations can be assessed.

- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) i) Substitute $A = 660$, $n = 1$, and $i = 0.03$.

$$\begin{aligned} P &= A(1 + i)^{-n} \\ &= 660(1 + 0.03)^{-1} \\ &= 660(1.03^{-1}) \\ &\doteq 640.78 \end{aligned}$$

One year ago there was $640.78 in the account.

ii) Substitute $A = 660$, $n = 5$, and $i = 0.03$.

$$\begin{aligned} P &= A(1 + i)^{-n} \\ &= 660(1 + 0.03)^{-5} \\ &= 660(1.03^{-5}) \\ &\doteq 569.32 \end{aligned}$$

Five years ago there was $569.32 in the account.

b) Answers may vary. Sample answer: The formula represents exponential decay. The sign of the exponent in the formula is negative, and the corresponding value with a positive exponent has a base that is less than 1. So, as n increases, $(1 + i)^{-n}$ decreases, as does the value of P.

c) $P = A(1 + i)^{-n}$ Express using a positive exponent.

$P = \dfrac{A}{(1 + \mathrm{i})^n}$ Multiply each side by $(1 + i)^n$.

$A = P(1 + i)^n$

This formula represents exponential growth. The exponent n is positive and the base is greater than 1, so the value of A will increase.

Level 3 Notes

Look for the following:

- Calculations in part a) are accurate
- Understanding of how to distinguish between exponential decay and exponential growth in parts b) and c) is mostly evident
- Rewrites the formula in part c) with considerable accuracy
- Justifications and explanations of solutions, where required, are mostly valid

What Distinguishes Level 2

- Calculations in part a) are somewhat accurate
- Understanding of how to distinguish between exponential decay and exponential growth in parts b) and c) is somewhat evident
- Rewrites the formula in part c) with some accuracy
- Justifications and explanations of solutions, where required, are somewhat valid

What Distinguishes Level 4

- Calculations in part a) are highly accurate
- Understanding of how to distinguish between exponential decay and exponential growth in parts b) and c) is clearly evident
- Rewrites the formula in part c) with a high degree of accuracy
- Justifications and explanations of solutions, where required, are clearly valid

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	8, 13–15, 18–20
Reasoning and Proving	8–11, 13–15, 18–20
Reflecting	8–11, 13–15, 18–20
Selecting Tools and Computational Strategies	4–9, 12, 14–20
Connecting	1–20
Representing	8–10, 12
Communicating	8, 9, 12, 13, 19

3.3 Rational Exponents

Student Text Pages
170 to 177

Suggested Timing
75 min

Tools
- graphing calculator

Related Resources
- BLM 3–4 Section 3.3 Practice

Differentiated Instruction
- Use a **gallery walk**. Create a set of examples similar to Communicate Your Understanding question C3. Have groups add a written explanation for one step of each example. Rotate until all the steps of each example are explained. Groups can check the explanations when an example is completed.

Teaching Suggestions

- You may wish to pause after covering **Example 2**, discuss **C1** and **C2**, assign **questions 1** to **4**, and then return to **Examples 3** and **4**. Assess student engagement and readiness through the first two examples before deciding whether a break is needed.
- The purpose of the **Investigate** is to provide students with an opportunity to discover the meaning of a rational exponent. As the activity develops, algebraic reasoning is used to connect simple powers involving rational exponents to their corresponding radicals.
- Strong students could extend the methods of the **Investigate** to look at other rational exponents before moving on to **Example 1**.
- **Example 1** illustrates how to evaluate powers with unit fraction as exponents (fractions of the form $\frac{1}{n}$). Part **a**) and the Connections feature on student book page 172 highlight the more general version of the result encountered in the **Investigate**. In the remaining parts of this example, special attention should be paid to the presence and placement of parentheses, which define the base of each power.
- **Example 2** extends the concept of rational exponents to those having any integral numerator. A solid understanding of the power of a power rule of exponents is critical for students to be able to understand the impact of the m and n on the value of an expression in the form $a^{\frac{m}{n}}$. Students will also need to apply rules for evaluating negative exponents in part **c**).
- **Example 3** draws on all of the exponent rules to simplify algebraic expressions involving a variety of rational exponents.
- **Example 4** has students apply rational exponents in a real-world situation. In such cases it is often preferable to use a calculator to obtain an approximate value for the final result. This is in contrast to the preceding examples, for which exact answers are expected.