Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	8, 13–15, 18–20
Reasoning and Proving	8–11, 13–15, 18–20
Reflecting	8–11, 13–15, 18–20
Selecting Tools and Computational Strategies	4–9, 12, 14–20
Connecting	1–20
Representing	8–10, 12
Communicating	8, 9, 12, 13, 19

3.3 Rational Exponents

Student Text Pages
170 to 177

Suggested Timing
75 min

Tools
- graphing calculator

Related Resources
- BLM 3–4 Section 3.3 Practice

Differentiated Instruction
- Use a **gallery walk**. Create a set of examples similar to Communicate Your Understanding question C3. Have groups add a written explanation for one step of each example. Rotate until all the steps of each example are explained. Groups can check the explanations when an example is completed.

Teaching Suggestions

- You may wish to pause after covering **Example 2**, discuss **C1** and **C2**, assign **questions 1** to **4**, and then return to **Examples 3** and **4**. Assess student engagement and readiness through the first two examples before deciding whether a break is needed.
- The purpose of the **Investigate** is to provide students with an opportunity to discover the meaning of a rational exponent. As the activity develops, algebraic reasoning is used to connect simple powers involving rational exponents to their corresponding radicals.
- Strong students could extend the methods of the **Investigate** to look at other rational exponents before moving on to **Example 1**.
- **Example 1** illustrates how to evaluate powers with unit fraction as exponents (fractions of the form $\frac{1}{n}$). Part **a**) and the Connections feature on student book page 172 highlight the more general version of the result encountered in the **Investigate**. In the remaining parts of this example, special attention should be paid to the presence and placement of parentheses, which define the base of each power.
- **Example 2** extends the concept of rational exponents to those having any integral numerator. A solid understanding of the power of a power rule of exponents is critical for students to be able to understand the impact of the m and n on the value of an expression in the form $a^{\frac{m}{n}}$. Students will also need to apply rules for evaluating negative exponents in part **c**).
- **Example 3** draws on all of the exponent rules to simplify algebraic expressions involving a variety of rational exponents.
- **Example 4** has students apply rational exponents in a real-world situation. In such cases it is often preferable to use a calculator to obtain an approximate value for the final result. This is in contrast to the preceding examples, for which exact answers are expected.

Investigate Answers (pages 170–171)

1. Answers may vary. Sample answer: P represents the number of planets captured and t represents the time, in decades.

2. a) Answers may vary. Sample answer: The coordinates of the point at the P-intercept are (0, 100), which represents the number of planets (100) now under the control of the Empire (at $t = 0$).

b) i) 400

ii) 1600

The graph can be used to find the number of planets under the control of the Empire after 1 and 2 decades by finding the coordinates with x-values 1 and 2, respectively.

3. Answers will vary. Sample answers:

a) The graph can be used to find the number of planets under the control of the Empire after 5 years by finding the coordinates of the point on the curve with an x-value of $\frac{1}{2}$. The Empire will control 200 planets after 5 years.

b) $P\left(\frac{1}{2}\right) = 200$; Yes, this agrees with my answer in part a). Both the equation and the graph represent the same function.

4. a) 200 **b)** same value **c)** The two are equal.

5. b) Yes

6. a) Yes **b)** Answers may vary. **c)** They are the same.

Communicate Your Understanding Responses (page 175)

C1 Answers may vary. Sample answers:

a) The cube root of a number n is the value a such that a multiplied by itself three times equals n. For example, the cube root of 27 is 3, as 3 multiplied by itself three times equals 27.

b) A fourth root of a number n is a value a such that a multiplied by itself four times equals n. For example, a fourth root of 81 is 3, as 3 multiplied by itself four times equals 81.

C2 Answers may vary. Sample answers:

a) i) For a number n, $\sqrt[5]{n}$ represents its fifth root as a radical.

ii) For a number n, $n^{\frac{1}{5}}$ represents its fifth root as a power.

b) $\sqrt[5]{32} = 32^{\frac{1}{5}}$, which is equal to 2.

C3 step 1: Apply the power of a power rule.
step 2: Apply the product rule.
step 3: Find a common denominator.
step 4: Simplify.
step 5: Apply the definition of a negative exponent.
step 6: Express the final answer as a radical.

Common Errors

- Some students may make computational errors when working with rational exponents.

R_x Have students review operations with fractions, integers, and exponent rules, as needed. A CAS can be used to check algebraic results.

Practise, Connect and Apply, Extend

- A scientific or graphing calculator can be used to verify answers in **questions 1** to **4**. A computer algebra system (CAS) can be used to verify answers in **questions 5** and **6**.
- It may be useful to provide linking cubes for **questions 8** to **11**. The square-cube law has applications in biology and biophysics, as explained in the Connections feature on student text page 176.
- In **question 10**, students need to apply exponent rules in order to rearrange a given formula to express it in terms of a different variable. It is important that students recognize that powers and roots can be treated as inverse operations in some contexts.

- **Question 10** gives students the opportunity to connect previously acquired mathematical knowledge and to select tools in order to determine the formula that correctly relates the surface area and volume of a cube. Students determine whether more than one formula correctly describes this relationship, and use their reasoning and reflecting skills to produce their answers. Student use their communicating skills to explain why the relationship is called the square-cube law.
- **Question 14** provides a connection to health and medical science.
- In **question 16**, students select tools to find the formula for the surface area of a cylinder, and to express the surface area of a cylinder in terms of its volume. Students use their reasoning, reflecting, and connecting skills in finding these formulas and applying them in parts **c**) and **d**).
- **Question 17** provides a connection to chemistry.
- Use **BLM 3–4 Section 3.3 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 14 , on student text page 177.

Achievement Check, question 14, student text page 177

This performance task is designed to assess the specific expectations covered in Section 3.3. The following mathematical process expectations can be assessed.

- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

a) **i)** Substitute $m = 6400$ into each equation.

$$\begin{aligned} h &= 241m^{-\frac{1}{4}} \\ &= 241(6400^{-\frac{1}{4}}) \\ &= \frac{241}{6400^{\frac{1}{4}}} \\ &\doteq 27 \end{aligned}$$

The heartbeat frequency for the killer whale is approximately 27 beats per minute.

$$\begin{aligned} b &= \frac{107}{2}m^{-\frac{1}{4}} \\ &= \frac{107}{2}(6400^{-\frac{1}{4}}) \\ &= \frac{107}{2(6400^{\frac{1}{4}})} \\ &\doteq 12 \end{aligned}$$

The respiratory frequency for the killer whale is approximately 12 breaths per minute.

ii) Substitute $m = 6.4$ into each equation.

$$\begin{aligned} h &= 241m^{-\frac{1}{4}} \\ &= 241(6.4^{-\frac{1}{4}}) \\ &= \frac{241}{6.4^{\frac{1}{4}}} \\ &\doteq 152 \end{aligned}$$

The heartbeat frequency for the dog is approximately 152 beats per minute.

$$\begin{aligned} b &= \frac{107}{2}m^{-\frac{1}{4}} \\ &= \frac{107}{2}(6.4^{-\frac{1}{4}}) \\ &= \frac{107}{2(6.4^{\frac{1}{4}})} \\ &\doteq 34 \end{aligned}$$

The respiratory frequency for the dog is approximately 34 breaths per minute.

iii) Substitute $m = 0.064$ into each equation.

$$\begin{aligned} h &= 241m^{-\frac{1}{4}} \\ &= 241(0.064^{-\frac{1}{4}}) \\ &= \frac{241}{0.064^{\frac{1}{4}}} \\ &\doteq 479 \end{aligned}$$

The heartbeat frequency for the mouse is approximately 479 beats per minute.

$$\begin{aligned} b &= \frac{107}{2}m^{-\frac{1}{4}} \\ &= \frac{107}{2}(0.064^{-\frac{1}{4}}) \\ &= \frac{107}{2(0.064^{\frac{1}{4}})} \\ &\doteq 106 \end{aligned}$$

The respiratory frequency for the mouse is approximately 106 breaths per minute.

b) Both the respiratory and heartbeat frequencies increase as the mass decreases.

c) **i)** Substitute $m = 6400$ into the equation.

$$B = \frac{1}{100}m^{\frac{2}{3}}$$
$$= \frac{1}{100}(6400^{\frac{2}{3}})$$
$$= \frac{(\sqrt[3]{6400})^2}{100}$$
$$\doteq 3.4$$

The brain mass for the killer whale is approximately 3.4 kg.

ii) Substitute $m = 6.4$ into the equation.

$$B = \frac{1}{100}m^{\frac{2}{3}}$$
$$= \frac{1}{100}(6.4^{\frac{2}{3}})$$
$$= \frac{(\sqrt[3]{6.4})^2}{100}$$
$$\doteq 0.034$$

The brain mass for the dog is approximately 0.034 kg.

iii) Substitute $m = 0.064$ into the equation.

$$B = \frac{1}{100}m^{\frac{2}{3}}$$
$$= \frac{1}{100}(0.064^{\frac{2}{3}})$$
$$= \frac{(\sqrt[3]{0.064})^2}{100}$$
$$= 0.0016$$

The brain mass for the mouse is approximately 0.0016 kg.

Level 3 Notes

Look for the following:

- Calculates the values in parts a) and c) with considerable accuracy
- Understanding of how to apply the rules for rational exponents in parts a) and c) is mostly evident
- Understanding of the relationship between frequency and mass in part b) is mostly evident
- Communicates steps of solutions with considerable clarity and use of proper mathematical form
- Units are mostly indicated in final answers

What Distinguishes Level 2

- Calculates the values in parts a) and c) with some accuracy
- Understanding of how to apply the rules for rational exponents in parts a) and c) is somewhat evident
- Understanding of the relationship between frequency and mass in part b) is somewhat evident
- Communicates steps of solutions with some clarity and use of proper mathematical form
- Units are sometimes indicated in final answers

What Distinguishes Level 4

- Calculates the values in parts a) and c) with a high degree of accuracy
- Understanding of how to apply the rules for rational exponents in parts a) and c) is clearly evident
- Understanding of the relationship between frequency and mass in part b) is clearly evident
- Communicates steps of solutions with a high degree of clarity and use of proper mathematical form
- Units are indicated in final answers throughout

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 16
Reasoning and Proving	10–17
Reflecting	10, 14, 16
Selecting Tools and Computational Strategies	1–11, 15–17
Connecting	1–17
Representing	8, 9, 17
Communicating	8, 10–12, 14

3.4 Properties of Exponential Functions

Student Text Pages
178 to 187

Suggested Timing
75–150 min

Tools
- computer with *The Geometer's Sketchpad®*
- graphing calculator
- grid paper

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- T–2 *The Geometer's Sketchpad®* 4
- BLM 3–5 Section 3.4 Practice

Teaching Suggestions

- Depending on the needs of your class, this lesson may be split over two periods. Monitor student progress and understanding through the **Investigate** and **Example 1**, and break the lesson at that point if necessary. If it is preferable to cover this lesson in one period, time can be saved by presenting parts of the **Investigate** using an LCD projector or interactive whiteboard (such as a SMART Board™).
- The **Investigate** allows students to explore the effects of a and b in exponential relationships of the form $y = ab^x$. Dynamic geometry software, like *The Geometer's Sketchpad®*, is an excellent tool for exploring and visualizing the various cases, if needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- **Example 1** illustrates methods for generating the graph of an exponential function. Method 1 involves a table of values, which students should see at least once before using graphing technology, as shown in Method 2. The zoom features of graphing technology can be used to verify the asymptotic nature of exponential curves.
- In **Example 2**, students read off information from the graph of an exponential function and generate a table of values and a corresponding equation.
- In **Example 3**, students develop an exponential model to represent a real-world situation involving exponential decay. In this case, students need to restrict the domain of the function to accurately model the situation.