What Distinguishes Level 4

- Calculates the values in parts a) and c) with a high degree of accuracy
- Understanding of how to apply the rules for rational exponents in parts a) and c) is clearly evident
- Understanding of the relationship between frequency and mass in part b) is clearly evident
- Communicates steps of solutions with a high degree of clarity and use of proper mathematical form
- Units are indicated in final answers throughout

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 16
Reasoning and Proving	10–17
Reflecting	10, 14, 16
Selecting Tools and Computational Strategies	1–11, 15–17
Connecting	1–17
Representing	8, 9, 17
Communicating	8, 10–12, 14

3.4 Properties of Exponential Functions

Student Text Pages
178 to 187

Suggested Timing
75–150 min

Tools
- computer with *The Geometer's Sketchpad®*
- graphing calculator
- grid paper

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- T–2 *The Geometer's Sketchpad®* 4
- BLM 3–5 Section 3.4 Practice

Teaching Suggestions

- Depending on the needs of your class, this lesson may be split over two periods. Monitor student progress and understanding through the **Investigate** and **Example 1**, and break the lesson at that point if necessary. If it is preferable to cover this lesson in one period, time can be saved by presenting parts of the **Investigate** using an LCD projector or interactive whiteboard (such as a SMART Board™).
- The **Investigate** allows students to explore the effects of a and b in exponential relationships of the form $y = ab^x$. Dynamic geometry software, like *The Geometer's Sketchpad®*, is an excellent tool for exploring and visualizing the various cases, if needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- **Example 1** illustrates methods for generating the graph of an exponential function. Method 1 involves a table of values, which students should see at least once before using graphing technology, as shown in Method 2. The zoom features of graphing technology can be used to verify the asymptotic nature of exponential curves.
- In **Example 2**, students read off information from the graph of an exponential function and generate a table of values and a corresponding equation.
- In **Example 3**, students develop an exponential model to represent a real-world situation involving exponential decay. In this case, students need to restrict the domain of the function to accurately model the situation.

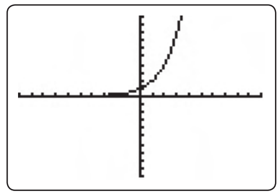
Differentiated Instruction

- Use **jigsaw** to investigate the properties of exponential functions. Create groups of four students. Assign each group member one of the following scenarios: ($a > 0$ and $b > 1$), ($a > 0$ and $0 < b < 1$), ($a < 0$ and $b > 1$), ($a < 0$ and $0 < b < 1$). Students choose a function for their scenario; construct a graph of that function; state the domain, range, equation of the asymptote, and y-intercept; and explain whether their function is increasing or decreasing. Groups use a **placemat** to record and summarize their findings.

Investigate Answers (pages 178–179)

A

1. a)



b) Answers may vary. Sample answer: The x-axis is an asymptote for this function, and the function is increasing over its domain.

2. a) domain $\{x \in \mathbb{R}\}$; range $\{y \in \mathbb{R}, y > 0\}$

b) 1

c) No.

d) i) The function is increasing over its entire domain.

ii) The function is not decreasing over any interval.

3. a) Answers may vary.

b) Answers may vary. Sample answer: Greater values of x result in a graph that is greater than the graph of $y = 2^x$ for all values greater than 0, and less than $y = 2^x$ for all values less than 0.

c) Answers may vary. Sample answer: Values of b greater than 2 have this effect because a large number raised to the same power as a smaller number will give a greater result, and a large number raised to the same negative power as a smaller number will give a lesser result.

4. Answers may vary. Sample answers:

a) The graph becomes a decreasing function.

b) The graph is decreasing over its entire domain, but all other characteristics from step 2 remain the same.

c) Each successive value is multiplied by a factor less than 1, so the values decrease as x increases.

5. Answers may vary. Sample answer: When $b = 1$, the function is the horizontal line $y = 1$. The number 1 raised to any exponent is equal to 1.

6. Answers may vary. Sample answer: When $b > 1$, the function is increasing. Greater values of b produce functions that increase more rapidly. When $0 < b < 1$, the function is decreasing, and smaller values of b produce functions that decrease more rapidly. If $b = 1$, the graph is the line $y = 1$.

B

1. Answers may vary. Sample answers:

a) When $a > 1$, the function is vertically stretched.

b) When $0 < a < 1$, the function is vertically compressed.

c) When $a < 0$, the function is reflected in the x-axis and vertically stretched or compressed.

2. Answers may vary.

Communicate Your Understanding Responses (page 184)

C1 Answers may vary. Sample answers:

a) Yes, because the value of an exponential function continues to grow or decrease as the x-values increase.

b) No, the function will continue to approach $y = 0$ for either large positive or large negative values of x, but it will never reach zero.

C2 a) i) $g(x)$ **ii)** $f(x)$

Answers may vary. Sample answer: You can tell by identifying the value of a in the equation. If $a > 0$, the graph of the function is above the x-axis. If $a < 0$, the graph of the function is below the x-axis.

b) i) neither **ii)** both

Answers may vary. Sample answer: Examine both a and b. If $a > 0$ and $0 < b < 1$, the function is decreasing. If $a < 0$ and $b > 1$, the function is decreasing. In all other cases the function is increasing.

C3 Answers may vary. Sample answer: The “asymptotic behaviour” of a function refers to the presence of, and characteristics of, the asymptote(s) of that function.

Common Errors

- When sketching graphs of exponential functions, some students may show the curve touching or intersecting the *x*-axis.

R_x Have students repeatedly zoom out along the curve and zoom in to understand and verify the nature of asymptotic behaviour.

Practise, Connect and Apply, Extend

- Students can use graphing technology to check their answers to **questions 2** to **5**.
- Connections to reciprocal functions can be made in **questions 8** and **9**. Students should note that the reciprocal functions in these questions have both horizontal and vertical asymptotes, unlike exponential functions, which have horizontal asymptotes only. Students will learn more about reciprocal functions in grade 12.
- In **question 8**, students select tools and use their representing skills to graph two functions. Students use their reasoning, reflecting, connecting, and communicating skills to determine and describe the similarities and differences between the two graphs. Students complete the question by communicating observations about the asymptotes of the functions.
- The context in **questions 11** and **13** has connections to electricity and electronics. Students interested in the physical sciences or engineering should find these questions interesting.
- **Question 12** allows students to use their reasoning and reflecting skills when interpreting the values in the given equation. Students select tools and connect prior mathematical knowledge to the real-world situation in this question in order to graph the given function and communicate answers to questions related to the graph.
- Use **BLM 3–5 Section 3.4 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13–15
Reasoning and Proving	1–15
Reflecting	2–5, 8–10, 13–15
Selecting Tools and Computational Strategies	2, 3, 7–10, 12–15
Connecting	1–15
Representing	2–5, 7–10, 12–15
Communicating	2, 3, 8–10, 12–14