3.5 Transformations of Exponential Functions

Student Text Pages
188 to 198

Suggested Timing
75 min

Tools
- computer with *The Geometer's Sketchpad®*
- graphing calculator
- grid paper
- TI-Nspire™ CAS graphing calculator (optional)
- toothpicks or drinking straws (optional)

Related Resources
- G–1 Grid Paper
- T–2 *The Geometer's Sketchpad®* 4
- BLM 3–6 Section 3.5 Practice
- BLM 3–7 Section 3.5 Achievement Check Rubric

Teaching Suggestions

- In the **Investigate**, students discover the transformations that result from changing the parameters of an exponential function. Students discover that these are the same effects that they have observed previously with other functions, such as quadratic functions. Accordingly, the general concepts of function transformations should be review for most students, although students should take care to observe the effects on characteristics unique to exponential functions (e.g., asymptotes). If needed, use **T–2 *The Geometer's Sketchpad®* 4** to support this activity.
- Throughout this lesson students should make connections between the parameters of the base function and the parameters of the transformed function when expressed in function notation. For example, when $f(x) = b^x$, then $y = af(x)$ corresponds to $y = ab^x$ and $y = f(kx)$ corresponds to $y = b^{kx}$, etc.
- The examples that follow the **Investigate** consolidate the concepts that students should recognize in the **Investigate**. Graphing technology is strongly recommended to provide a quick visual representation of the transformations. *The Geometer's Sketchpad®* and the TI-Nspire™ CAS graphing calculator are excellent tools to use, as both have click-and-drag capabilities. If these tools are unavailable, the TI-83 Plus/TI-84 Plus family of graphing calculators could be used.
- In **Example 1**, students explore the effects of horizontal and vertical translations. In **Example 2** they examine the effects of vertical and horizontal stretches, compressions, and reflections. In **Example 3** students examine the effects of combined transformations.
- For **Examples 1** to **3**, remind students to use a different line style for each curve they graph on the same set of axes.
- In **Example 4** students explore the effects of transformations in a real-world context. In this example, it is necessary to restrict the domain of the function used to model the physical situation. This example shows methods involving a graphing calculator and *The Geometer's Sketchpad®*, and illustrates techniques for analysing the transformed functions.

Differentiated Instruction

- Use an **anticipation guide** to review the types of transformations previously learned and to predict whether these transformations apply to exponential functions.
- Use **Think-Pair-Share** to complete the Investigate.
- Construct a **what-so-what double entry chart** with the headings Type of Transformation, Domain, Range, and Asymptote. List types of transformations, including a sample equation and graph, in the first column. Describe the effect of each transformation on the domain, range, and asymptote in the respective columns.

Investigate Answers (pages 188–189)

1. Answers may vary. Sample answers:
a) The function will shift up 3 units.
b) The function shifted up 3 units.
c) **i)** The function will shift down 4 units.
ii) Answers may vary.
d) When $c > 0$, the function shifts c units up; when $c < 0$, the function shifts down.

2. Answers may vary. Sample answers:
a) The function will shift 3 units to the left, rather than 3 units down.
b) Answers may vary.
c) When $d > 0$, the function moves d units to the right; when $d < 0$, the function moves left.

3. Answers may vary. Sample answers:
a) The function will stretch vertically by a factor of 2.
b) When $a > 1$, the function is stretched vertically by a factor of a, when $0 < a < 1$, the function is compressed vertically by a factor of a. If $a < 0$, the function is reflected in the x-axis, and then stretched or compressed.

4. to **6.** Answers may vary.

7. Answers may vary. Sample answer:

a) Yes, the function $y = 4^x$ is equivalent, because $4 = 2^2$.

Communicate Your Understanding Responses (page 195)

C1 **a)** no match **b)** A **c)** B **d)** G **e)** no match **f)** no match **g)** C

C2 **a)** $y = 10^{\frac{x}{3}}$ **e)** $y = 3(10^x)$ **f)** $y = 10^{x-3}$

C3 Answers may vary. Sample answers:

a) Horizontally compress $y = 5^x$ by a factor of $\frac{1}{2}$.

b) Using the power of a power rule, $y = 5^{2x}$ is equivalent to $y = 25^x$, so sketching $y = 25^x$ also produces the graph of $y = 5^{2x}$.

C4 The value of a gives a vertical stretch or compression, k gives a horizontal stretch or compression, d gives a horizontal translation, and c gives a vertical translation. If $a < 0$, then the vertical stretch or compression is combined with a reflection in the x-axis. If $k < 0$ the horizontal stretch or compression is combined with a reflection in the y-axis.

C5 Answers may vary. Sample answers:

a) You can express the function in the form $y = ab^{k(x-d)} + c$, then apply transformations to the base function $y = ab^x$ to determine if there is a horizontal asymptote. If the function is increasing, and lies below that asymptote, it will increase indefinitely as it approaches the asymptote.

b) The function is increasing and has a horizontal asymptote at $I = 0.9$, and so it does increase indefinitely, getting closer and closer to 0.9.

Common Errors

- Some students may mix up the direction of a translation, particularly horizontal translations.

R_x Have students perform a simple horizontal translation using a table of values on a familiar function (such as a line or parabola), and then check their answers using graphing technology.

Practise, Connect and Apply, Extend

- Students need to be careful of the case-sensitive variables appearing in the equation in **question 9**.
- The ability to express powers using different bases, which figures in **questions 11** and **12**, is important for future work in grade 12, in which students will learn various techniques for solving exponential equations.
- **Question 11** gives students the opportunity to select tools to represent a given exponential function by a graph. Students use their reasoning and connecting skills to rewrite the given function in two other ways, and use these equivalent functions to explore transformations that yield the original function. Finally, students communicate their findings concerning the equivalency of the three functions.
- Graphing technology, particularly *The Geometer's Sketchpad*® or the TI-Nspire™ CAS graphing calculator, is particularly useful when exploring **questions 13** to **15**.
- **Question 15** requires students to use their reasoning and reflecting skills to create an exponential function by applying at least one of three types of transformations. Students will connect mathematical concepts learned in the past to knowledge acquired in this chapter when creating and analysing their chosen function. Students will use their reasoning skills to predict whether the order in which the transformations are applied affects the graph that results. Students will select tools and use connecting skills to represent the function by a graph, applying the transformations in the order that they choose.
- Toothpicks or drinking straws may be useful to students when answering **question 17**. Coming up with the function that gives the number of squares (in **question 16**) is relatively straightforward, but students may struggle to come up with the function that gives the number of toothpicks. Have students try to visualize the different parts of each part of the pattern (i.e., fixed versus growing), and use different colours (if available) to distinguish these.
- **Questions 18** and **19** provide opportunities for students to use counterexamples as a reasoning and proving tool.
- **Question 22** has connections to architecture.
- Use **BLM 3–6 Section 3.5 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 20, on student text page 198.

Achievement Check, question 20, student text page 198

This performance task is designed to assess the specific expectations covered in Sections 3.4 and 3.5. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Connecting
- Representing
- Communicating

Sample Solution

Provide students with **BLM 3-7 Section 3.5 Achievement Check Rubric** to help them understand what is expected.

a) Rewrite the equation in the form $y = a3^{k(x-d)} + c$.

$$y = -\left(\frac{1}{3}\right)^{12-3x} + 2$$
$$= -(3^{-1})^{-3(x-4)} + 2$$
$$= -\ 3^{3(x-4)} + 2$$

Compare the equation $y = -3^{3(x-4)} + 2$ to $y = a3^{k(x-d)} + c$.

The parameters and corresponding transformations are:

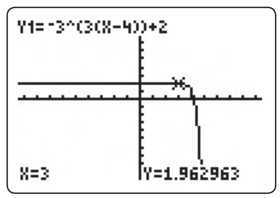
$a = -1$, reflection in the x-axis

$k = 3$, horizontal compression by a factor of $\frac{1}{3}$

$d = 4$, shift right 4 units

$c = 2$, shift up 2 units

b)



c) **i)** $\{x \in \mathbb{R}\}$

ii) $\{y \in \mathbb{R}, y < 2\}$

iii) $y = 2$

iv) x-intercept at approximately 4.2, y-intercept at approximately 2

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13–19, 21
Reasoning and Proving	9–22
Reflecting	13–19, 21, 22
Selecting Tools and Computational Strategies	2, 5, 7–22
Connecting	1–22
Representing	2, 3, 5–22
Communicating	1, 4, 9, 11–22