3.6 Making Connections: Tools and Strategies for Applying Exponential Models

Student Text Pages
199 to 209

Suggested Timing
75 min

Tools
- graphing calculator
- computer with *Fathom*™
- computer with Internet access
- temperature probe (optional)

Related Resources
- T–3 *Fathom*™
- BLM 3–8 Section 3.6 Practice

Teaching Suggestions

- This section focuses on using exponential functions to model contextual data. After constructing a model for each situation, various tools and strategies are used to analyse the model and use the model to make predictions. It is important in all situations for students to identify possible limitations of the model.
- In **Example 1**, an exponential model is developed to represent growth in earnings. In Method 1 the coefficient of determination, r^2 (or R^2), is a measure of goodness of fit of a regression curve to a set of data. Students need not understand how r^2 is calculated, but should understand its significance and take it into account when assessing the accuracy of their model.
- Method 2 of **Example 1** involves using sliders to find a curve of best fit. This method does not provide the option of generating the coefficient of determination, and it is worth discussing why students may come up with slightly different equations for their curves of best fit. If needed, use **T–3 *Fathom*™** to support this activity. It is useful for students to see both methods so they can discuss the strengths and weaknesses of each tool.
- While each mode in **Example 2** gives a reasonably good fit for the given data, extrapolating beyond the given points reveals flaws in the linear and quadratic models. The exponential model captures the expected behaviour of the continuing depreciation. Part **c**) gives students an opportunity to work with, and see connections between, graphical and algebraic representations of the model. Students should consider the value of r^2 when assessing the accuracy of their model.
- **Example 2** offers an opportunity for students to distinguish between the somewhat mechanical procedure of curve fitting and the reasoning process of true mathematical modelling.

Differentiated Instruction

- Use **think aloud** and have one group lead the class through Example 1 using graphing calculators or *Fathom*™. Have another group lead the class through Example 2 using graphing calculators. With students presenting, you are able to circulate and provide help as needed.

Communicate Your Understanding Responses (page 206)

C1 Answers may vary. Sample answers:
a) quadratic; the data seem to be arranged in the shape of a parabola
b) linear; the data seem to arranged in a straight line
c) none; the data do not seem to have any particular pattern
d) exponential; the data seem to have an exponential pattern

C2 Answers may vary. Sample answers:
a) *Fathom*™ provides better graphing capability, but it does not have an exponential regression function or display the value of r^2. A graphing calculator can perform an exponential regression, but has a smaller graphing screen with less resolution.
b) There may be slight variations in the exponential model because different tools may give results to a different degree of accuracy, or use a different method to produce the model.

C3 a) Answers may vary. Sample answers:
i) If the ratio of successive differences is constant (for equally spaced data values), then an exponential model may be appropriate.
ii) If the data on the scatter plot increase or decrease dramatically as the values of x increase or decrease, and the data appear to approach an asymptote, then an exponential model may be appropriate.
b) Answers may vary.

Common Errors

- Some students may assume that any curve passing close to the given data points is an effective mathematical model.

R_x It is important for students to consider whether a model will extrapolate effectively beyond the given data points. Have students use the zoom features of graphing technology to explore their models and consider the limitations of a model.

Practise, Connect and Apply, Extend

- In **questions 1** and **2**, students should see the connection between the *y*-intercept of each scatter plot and the value of *a* in the corresponding equation.
- **Question 3** requires students to reason through and reflect on the trend given in a scatter plot to estimate values and develop an exponential model. Students communicate their findings, select tools to draw a representation of the given data and use their connecting skills in answering questions.
- **Question 5** requires students to invoke reasoning skills to adjust the exponential model. For example, students may consider limits on the number of employees in the company, and may assume that the news spreads differently outside of office hours.
- **Questions 6**, **7**, **10**, and **11** involve data gathered from Statistics Canada. Statistics Canada has a vast warehouse of data that is accessible to schools. **Question 10** directs students to find another set of data that can be modelled using an exponential function, and to write a brief report of their findings.
- **Question 8** requires students to select tools and use their connecting skills to represent data using a scatter plot. Students use reasoning and reflecting skills to determine the curve of best fit using an exponential equation, and use that equation to make predictions about the size of the koala population at two different times. Students then communicate how they produced their findings and generated their predictions.
- **Question 9** has connections to economics. Interested students could explore other related indices (e.g., Gross Domestic Product, TSX Composite, NASDAQ, etc.) and investigate the purpose of each.
- **Question 12** works well using a temperature probe and a graphing calculator. Consider setting up the problem at the beginning of class and letting the technology collect data throughout the lesson.
- In **Question 13**, students use reasoning and reflecting skills to predict changes to the coffee-cooling curve when cream and sugar are added, and when a person takes sips of coffee at regular intervals. Students select tools and draw connections to prior mathematical knowledge when representing these situations graphically, and when communicating their explanation.
- Students adjust an exponential model in **Question 13**. A brief discussion of piecewise functions may be appropriate here.
- Visit the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11* for links related to **question 14**.
- Use **BLM 3–8 Section 3.6 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	4, 6, 7, 9, 13, 14
Reasoning and Proving	1, 3–14
Reflecting	3, 4, 6–11, 13, 14
Selecting Tools and Computational Strategies	2–6, 8–10, 12–14
Connecting	1–14
Representing	2–6, 8–10, 12–14
Communicating	3–5, 8–14