4.3 Reciprocal Trigonometric Ratios

Student Text Pages
243–248

Suggested Timing
60–70 min

Tools
- geometry set
- grid paper
- computer with *The Geometer's Sketchpad®*
- computer algebra system (CAS) (optional)

Related Resources
- G–1 Grid Paper
- BLM 4–6 Use a Computer Algebra System to Find Reciprocal Trigonometric Ratios
- BLM 4–7 Section 4.3 Practice

Differentiated Instruction
- Use a **cooperative task group** to complete the Investigate and Communicate Your Understanding questions. Write **journal** responses to Investigate step 5 and to questions C1, C2, and C3.
- Add the term *reciprocal* and the formulas for the cosecant ratio, secant ratio, and cotangent ratio to the **word wall** constructed in the previous sections.

Teaching Suggestions

- After students finish the **Investigate**, use **BLM 4–6 Use a Computer Algebra System to Find Reciprocal Trigonometric Ratios** to demonstrate how a computer algebra system (CAS) can directly calculate exact and approximate values for the reciprocal trigonometric ratios.
- **Example 2** can be checked directly using a TI-Nspire™ CAS graphing calculator, by choosing the appropriate function from the catalogue.
- **Example 3** can be checked using a TI-Nspire™ CAS graphing calculator to solve the equation. Refer to the method in the **Use Technology** feature on student text pages 241–242.

Investigate Answers (page 243–244)

1. a) Answers may vary. Sample answer: The exact value of $\sin 30°$ is $\frac{1}{2}$. I predict $\csc 30° = 2$.
b) Yes, the result confirms my prediction.

2. a) Answers may vary. Sample answer: The exact value of $\cos 60°$ is $\frac{1}{2}$. I predict $\sec 60° = 2$.
b) Yes, the result confirms my prediction

3. a) Answers may vary. Sample answer: The exact value of $\tan 30°$ is $\frac{1}{\sqrt{3}}$. I predict $\cot 30° = \sqrt{3}$.
b) Yes, the result confirms my prediction.

4.

θ	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
0°	0	undefined	1	1	0	undefined
30°	$\frac{1}{2}$	2	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{2}$	2	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
90°	1	1	0	undefined	undefined	0

5. Answers may vary. Sample answer: As the measure of angle θ increases, the value of $\sin\theta$ increases while the value of $\csc\theta$ decreases. As the measure of angle θ increases, the value of $\cos\theta$ decreases while the value of $\sec\theta$ increases. As the measure of angle θ increases, the value of $\tan\theta$ increases while the value of $\cot\theta$ decreases.

Communicate Your Understanding Responses (page 246)

C1 Answers may vary. Sample answer: In the first quadrant, the minimum value for the sine ratio occurs at 0°, where $\sin 0° = 0$, and the maximum value for the sine ratio occurs at 90°, where $\sin 90° = 1$. The function $\csc\theta$ will have a minimum value of 1 at 90° in the first quadrant. There is no maximum value for $\csc\theta$ in this quadrant, as the closer the angle is to 0°, the larger the value of the function. As a result, as the angle approaches 0° the value of $\csc\theta$ approaches infinity.

C2 $\csc\theta = \frac{r}{y}$, $\sec\theta = \frac{r}{x}$, $\cot\theta = \frac{x}{y}$

C3 Answers may vary. Sample answer: The cosine of 90° is equal to zero. So, $\sec 90°$ will be undefined, since division by zero is undefined.

Common Errors

- Some students may use inappropriate calculator sequences when determining reciprocal trigonometric ratios.

R_x Ensure that students are familiar with the keystrokes required on their calculators. Checking answers with peers is good insurance that proper keystrokes have been used.

Ongoing Assessment

Achievement Check, question 16, on student text page 247.

Practise, Connect and Apply, Extend

- The answers to **questions 2**, **4**, **6**, and **7** can be checked using a TI-Nspire™ CAS graphing calculator.
- **Question 5** gives students the opportunity to use reasoning and connecting skills to find the measure of an angle in the first quadrant that satisfies each given ratio. Students will also use communicating skills to explain why in some cases no such angle can be found.
- If you have arranged for home access to *The Geometer's Sketchpad*®, you can assign **question 14** for homework. Otherwise, you can save it and others in this chapter for a session in the computer lab.
- **Question 17** requires students to use reasoning and proving skills to prove the given identity. Students will establish connections to other mathematical concepts they have learned when performing the algebraic manipulations needed for this solution.
- Students may find the following hint useful when answering **questions 18** and **19**: Use the given ratio to determine expressions for x, y, and r.
- Use **BLM 4–7 Section 4.3 Practice** for remediation or extra practice.

Achievement Check, question 16, student text page 247

This performance task is designed to assess the specific expectations covered in Section 4.3. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Connecting
- Reflecting
- Communicating
- Selecting Tools and Computational Strategies

Sample Solution

a) $$-1 = \csc\theta\cos\theta = \frac{1}{\sin\theta}\cos\theta = \frac{\cos\theta}{\sin\theta}$$

b) The ratio $\frac{\cos\theta}{\sin\theta} = -1$, so $\sin\theta = -\cos\theta$, which means that either $\sin\theta$ is positive and $\cos\theta$ is negative, or $\sin\theta$ is negative and $\cos\theta$ is positive. This is true for angles in the second or fourth quadrant.

c) In the second quadrant, $\theta = 135°$. In the fourth quadrant, $\theta = 315°$.

d) A unique value of θ can be determined if the quadrant of the angle is known, or if the sign of either the sine or cosine ratio is known.

e) If $\sec\theta$ is negative, then so is $\cos\theta$. Thus, $\sin\theta$ and $\csc\theta$ are positive and the angle is in the second quadrant. Therefore, $\theta = 135°$.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15, 17–21
Reasoning and Proving	5–21
Reflecting	17–21
Selecting Tools and Computational Strategies	8–13, 15, 20, 21
Connecting	1–21
Representing	2, 6, 7, 14–16, 21
Communicating	5, 14, 16, 20, 21