4.4 Problems in Two Dimensions

Student Text Pages
249–258

Suggested Timing
60–70 min

Tools
- geometry set
- grid paper
- computer with *The Geometer's Sketchpad®* (optional)

Related Resources
- G–1 Grid Paper
- BLM 4–8 Section 4.4 Practice
- BLM 4–9 Section 4.4 Achievement Check Rubric
- BLM 4–10 Solving Triangles Decision Tree

Differentiated Instruction

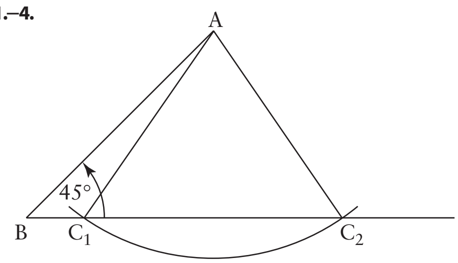
- Construct a **decision tree** for determining when to solve trigonometric problems using primary trigonometric ratios, the sine law, or the cosine law. A sample decision tree is given in **BLM 4–10 Solving Triangles Decision Tree**, but students could be encouraged to construct their own. The **decision tree** can be posted in the classroom to help students solve problems throughout the chapter.

Teaching Suggestions

- The ambiguous case is often confusing to students. Consider making models of the triangles in the **Investigate** using pieces of wood or cardboard marked and cut to appropriate lengths. Show how one of the possible triangles can "morph" into the other while maintaining the lengths of the two sides and the given angle. Ask students to describe what changes to the model would result in a) a single possible triangle, and b) no possible triangles.
- If you have arranged for home access to *The Geometer's Sketchpad®*, consider having students reinforce their understanding of the ambiguous case, and the conditions under which it occurs, by drawing sketches for **Examples 2** and **3**.

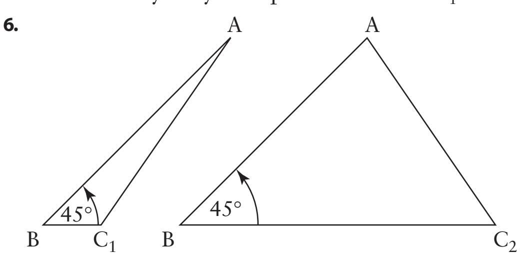
Investigate Answers (pages 249–250)

1.–4.



5. Answers may vary. Sample answer: $\angle AC_1B$ and $\angle AC_2B$ sum to 180°.

6.



The acute triangle is $\triangle ABC_2$ and the obtuse triangle is $\triangle ABC_1$.

7. Answers may vary. Sample answers:

a) When b, the side length of AC, is equal to $c \sin B$, a right triangle is created with the right angle at vertex C. This is the only triangle satisfying the given conditions that can be constructed.

b) No triangle can be constructed when b, the side length of AC, is smaller than $c \sin B$, because the length of AC would be too short to create a triangle that satisfies the given conditions.

Communicate Your Understanding Responses (page 254)

C1 Answers may vary. Sample answer: There is only one possible triangle that can be drawn.

C2 Answers may vary. Sample answer: When the cosine law is appropriate to use, it is not possible to have an ambiguous case. There is only one triangle that can be drawn with three given sides or with two sides and their contained angle (the only two scenarios that require the use of the cosine law).

C3 Answers may vary. Sample answer: It is possible to have a triangle where an unknown side can be found using either the sine or cosine law.

Common Errors

- The most common source of error when solving trigonometric problems is an incorrect diagram.

R_x It is important that diagrams "look" right. If an angle is given as 35°, ensure that it looks like 35°. Carefully label vertices and sides, and label known lengths and angles. Carefully identify the side or angle that must be determined.

Ongoing Assessment

Achievement Check, question 21, on student text page 257.

Practise, Connect and Apply, Extend

- **Questions 6**, **7**, and **11** are good candidates for solutions using *The Geometer's Sketchpad®*.
- **Question 11** requires students to reason through the boat's route to represent that route with a diagram and find Charles's distance from the marina. In order to solve this problem algebraically, students select appropriate tools and connect this situation to prior mathematical knowledge.
- For **question 13**, advise students to check whether a given sine law calculator takes the ambiguous case into account. Some possible applets can be found on the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11*.
- **Question 19** allows students to use reasoning and reflecting skills to find the distance by which an awning should extend over a window. Students represent this situation with a diagram, select tools, and use connecting skills to solve the problem before communicating their findings.
- Some students may need a hint for **question 26**, such as "Draw an altitude from point P."
- Use **BLM 4–8 Section 4.4 Practice** for remediation or extra practice.

Achievement Check, question 21, student text page 257

This performance task is designed to assess the specific expectations covered in Section 4.4. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

Provide students with **BLM 4–9 Section 4.4 Achievement Check Rubric** to help them understand what is expected.

a) There are two possible locations for Enrico's second stop because we are given the measures of two sides and one opposite angle that is not the contained angle. This represents the ambiguous case of the sine law.
Let T represent the tower, let E_1 represent Enrico's first stop, and let E_2 represent Enrico's second stop.

Diagram 1

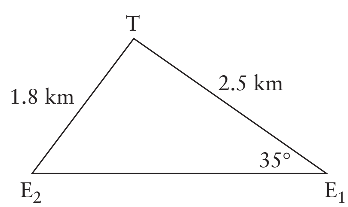
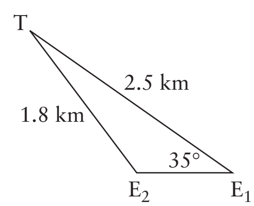


Diagram 2



b) $$\frac{\sin 35°}{1.8} = \frac{\sin E_2}{2.5}$$
$$\sin E_2 = \frac{2.5 \sin 35°}{1.8}$$
$$\doteq 0.7966$$

Therefore, $\angle E_2 \doteq 53°$ or $\angle E_2 = 180° - 53°$
$= 127°$

c) $\angle E_2 = 127°$ results in the shorter distance because it forms an obtuse triangle.

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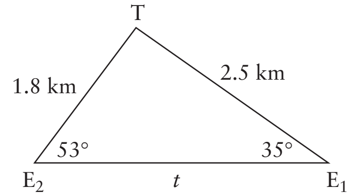
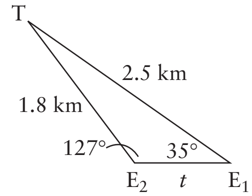
d) **Diagram 1: $\angle E_2 = 53°$**



$$\angle T = 180° - 53° - 35°$$
$$= 92°$$
$$\frac{t}{\sin 92°} = \frac{1.8}{\sin 35°}$$
$$t = \frac{1.8 \sin 92°}{\sin 35°}$$
$$\doteq 3.14$$

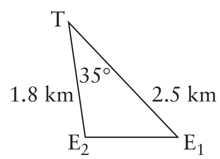
Diagram 2: $\angle E_2 = 127°$



$$\angle T = 180° - 127° - 35°$$
$$= 18°$$
$$\frac{t}{\sin 18°} = \frac{1.8}{\sin 35°}$$
$$t = \frac{1.8 \sin 18°}{\sin 35°}$$
$$\doteq 0.97$$

Therefore, the second stopping point is either approximately 3.14 km from the first, or 0.97 km from the first.

e) If the problem were altered as suggested it would have exactly one solution, since this situation would no longer involve the ambiguous case.
To solve the new problem, use the cosine law, since we know two sides and the contained angle.



$$t^2 = 1.8^2 + 2.5^2 - 2(1.8)(2.5)\cos 35°$$
$$t \doteq 1.46$$

Therefore, the distance between Enrico's first and second stops would be approximately 1.46 km.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	3–5, 7–12, 14, 16–26
Reasoning and Proving	1, 3–12, 14–26
Reflecting	3–5, 7–10, 12, 14, 16, 18–26
Selecting Tools and Computational Strategies	1, 3–12, 14, 16–25
Connecting	1–26
Representing	5, 6, 11, 14, 17, 19–21, 24
Communicating	1, 4, 6, 14–16, 19, 21, 22, 24, 25

Use Technology

Student Text Pages
259–260

Suggested Timing
10–15 min

Tools
- computer with *The Geometer's Sketchpad®*

Use Geometry Software to Test for the Ambiguous Case

Teaching Suggestions

- If you have provided home access for *The Geometer's Sketchpad®*, consider having students work through this **Use Technology** feature before attempting homework for Section 4.4. They can then use *The Geometer's Sketchpad®* to test for the ambiguous case as needed.

4.5 Problems in Three Dimensions

Student Text Pages
261–269

Suggested Timing
60–70 min

Related Resources
- BLM 4–11 Section 4.5 Practice

Teaching Suggestions

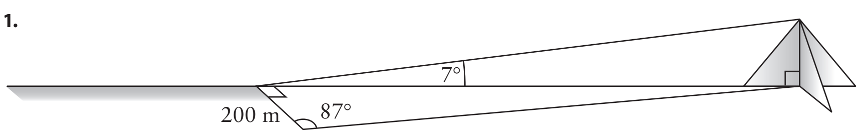
- Many students have trouble visualizing three-dimensional problems. Before beginning the **Investigate**, consider modelling the problem using concrete materials, such as drinking straws, string, tape, and modelling clay. Write labels on small pieces of paper, and attach them to the model. Clearly label the side or angle to be determined. Leave the model clearly visible to students as they work through the **Investigate**.
- Ensure that angles and lengths in a model look "right." A side of 12 cm should not appear shorter than a side of 10 cm.
- When presenting the **Examples**, take time to develop each diagram in a stepwise manner from the data given in the question. Consider having students take turns adding lines and labels until the diagram is complete. If students have difficulty, consider using a concrete model for one of the **Examples**.
- The problem in **Example 3** draws on both reasoning and reflecting skills. Students select tools, draw a diagram to represent the situation, and draw connections when finding the required angle of elevation.

Differentiated Instruction

- Use **think-aloud** and have groups of two to four students present solutions to different problems using the chalkboard, overheads, or large poster paper. Remember to provide markers, overhead markers, chart paper, or overhead transparencies as needed.

Investigate Answers (page 261)

1.



2. Using the tangent ratio, the base of the hill is approximately 3816.2 m from the departure end of the runway.

3. The hill is approximately 470 m high.

4. A rate of climb of approximately 123 m/km would be required for an aircraft to just clear the hill.

5. A rate of climb of approximately 210 m/km would be required for an aircraft to clear the hill in accordance with the safety specifications.