Use Technology

Student Text Pages
259–260

Suggested Timing
10–15 min

Tools
- computer with *The Geometer's Sketchpad®*

Use Geometry Software to Test for the Ambiguous Case

Teaching Suggestions

- If you have provided home access for *The Geometer's Sketchpad®*, consider having students work through this **Use Technology** feature before attempting homework for Section 4.4. They can then use *The Geometer's Sketchpad®* to test for the ambiguous case as needed.

4.5

Problems in Three Dimensions

Student Text Pages
261–269

Suggested Timing
60–70 min

Related Resources
- BLM 4–11 Section 4.5 Practice

Teaching Suggestions

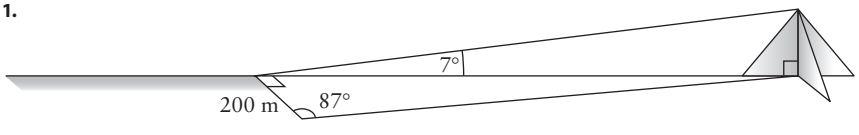
- Many students have trouble visualizing three-dimensional problems. Before beginning the **Investigate**, consider modelling the problem using concrete materials, such as drinking straws, string, tape, and modelling clay. Write labels on small pieces of paper, and attach them to the model. Clearly label the side or angle to be determined. Leave the model clearly visible to students as they work through the **Investigate**.
- Ensure that angles and lengths in a model look "right." A side of 12 cm should not appear shorter than a side of 10 cm.
- When presenting the **Examples**, take time to develop each diagram in a stepwise manner from the data given in the question. Consider having students take turns adding lines and labels until the diagram is complete. If students have difficulty, consider using a concrete model for one of the **Examples**.
- The problem in **Example 3** draws on both reasoning and reflecting skills. Students select tools, draw a diagram to represent the situation, and draw connections when finding the required angle of elevation.

Differentiated Instruction

- Use **think-aloud** and have groups of two to four students present solutions to different problems using the chalkboard, overheads, or large poster paper. Remember to provide markers, overhead markers, chart paper, or overhead transparencies as needed.

Investigate Answers (page 261)

1.



2. Using the tangent ratio, the base of the hill is approximately 3816.2 m from the departure end of the runway.

3. The hill is approximately 470 m high.

4. A rate of climb of approximately 123 m/km would be required for an aircraft to just clear the hill.

5. A rate of climb of approximately 210 m/km would be required for an aircraft to clear the hill in accordance with the safety specifications.

6. Answers may vary. Sample answer: If you need to find the height of a distant skyscraper that you cannot measure directly, you could use trigonometry and nearby measurements to calculate its height.

Communicate Your Understanding Responses (page 265)

Answers may vary. Sample answers:

C1 Three-dimensional problems are more difficult to solve, since a solution usually requires solving more than one two-dimensional problem. It may also be more difficult to draw three-dimensional diagrams to represent the given situations.

C2 He could measure the angle of elevation to the top of the Taj Mahal from where he is standing and then move to a second location that would create a right triangle with his first location and the Taj Mahal. In addition to the angle of elevation at his first location, he would need to measure the distance between his first and second locations, and the angle between his first location and the Taj Mahal when viewed from his second location. This would require the minimum number of three measurements to find the height of the Taj Mahal.

Common Errors

- The actual trigonometry used in this section is no different from what has been used in previous sections. The main difficulties, and sources of error, are in visualizing the problem and sketching a correct diagram.

R_x Allow students to use concrete models until they feel comfortable creating a diagram directly from the information given in the problem. Should they subsequently encounter a problem that they find confusing, encourage them to return to a concrete model until the confusion is resolved.

Practise, Connect and Apply, Extend

- As students work through the homework problems, encourage them to use concrete models whenever they find a scenario confusing. The main source of error in dealing with three-dimensional problems is a diagram that is incorrectly constructed or labelled.
- The construction of the diagram in **question 4** draws on students' reasoning and reflecting skills. Students select the most appropriate tools and draw connections to prior mathematical concepts to produce a solution. Students also use communicating skills to explain how they can find the height of the tallest tree and determine whether Dave can perform a safe takeoff.
- **Question 8** gives students the opportunity to reflect and reason to determine the optimal route through the race and represent the situation with a diagram. In determining the optimal route, students select tools for calculation, and draw on their background mathematical knowledge.
- The intent of **question 9** is to get students to use models and measurements rather than calculations. One method students could use to construct the molecule is to first construct the tetrahedron using full-length drinking straws. Students could place the tetrahedron on a piece of paper and then use measurement to determine the centre of the base. After marking the centre of the base, a piece of a straw can be used to form an altitude of the tetrahedron, with other pieces of straw taped to hold that straw in place. Rotate the tetrahedron to find another altitude. The carbon molecule should be located at the intersection of the two altitudes.
- For **question 14**, students may need to review the shape of a regular tetrahedron.
- Students often have difficulty working with symbols rather than numbers, as in **questions 14** to **17**. Advise students to substitute numbers for the variables to first work out a numerical solution using the substituted values. Then, work out a parallel solution replacing the numbers with symbols.
- Use **BLM 4–11 Section 4.5 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	1–19
Reasoning and Proving	1–19
Reflecting	4–10, 12–19
Selecting Tools and Computational Strategies	2–13, 15–17
Connecting	1–19
Representing	4, 8, 10, 11
Communicating	4, 6, 9, 13, 18, 19

Trigonometric Identities

Student Text Pages
270–275

Suggested Timing
60–70 min

Tools
- graphing calculator

Related Resources
- BLM 4–2 Trigonometric Ratios of Special Angles
- BLM 4–12 Section 4.6 Practice

Teaching Suggestions

- Some teachers prefer to teach identities after covering the graphs of trigonometric functions. Therefore, you may prefer to move this section to the end of Chapter 5.
- The formula in the introductory text for this section is only true for the given speed. The general form of the trajectory equation is beyond the scope of this chapter.
- Before beginning the **Investigate**, establish the difference between equations that are not identities and those that are. An equation such as $\sin\theta = \cos\theta$ may be true for certain values of θ, such as $\theta = 45°$, but not true for other values, such as $\theta = 60°$. An identity, such as $\tan\theta = \frac{\sin\theta}{\cos\theta}$, is true for any value of θ. Have students try different values of θ for each of these.
- After completing the **Investigate**, emphasize what does and what does not constitute a proof.
- Before presenting **Example 3**, it may be helpful to review the concept of difference of squares.
- After completing the **Examples**, encourage students to look at both sides of an equation before beginning a proof.
- Consider having students add the Pythagorean, quotient, and reciprocal identities to their summary sheet of exact trigonometric ratios (**BLM 4–2 Trigonometric Ratios of Special Angles**), for use on homework assignments, quizzes, and tests.
- Students will likely have little experience with formal proofs. Take time to establish why the L.S./R.S. method must be used.
- Be sure students understand that only one counterexample is needed to show that an equation is not an identity.