# 5.1 Modelling Periodic Behaviour

**Student Text Pages**
284 to 293

**Suggested Timing**
60–75 min

**Tools**
- grid paper
- protractor, ruler, compass
- graphing calculator
- computer with Internet access
- computer with *The Geometer's Sketchpad®*

**Related Resources**
- G–1 Grid Paper
- BLM 5–2 Section 5.1 Practice

## Teaching Suggestions

- At the end of the **Investigate**, consider asking students to model the situation using *The Geometer's Sketchpad®*, and then animate the point that represents Suzanne moving around the circle. If you have arranged for home access, this makes an illuminating and short technology exercise. **Question 19** in the **Practise** asks students to perform the animation. Alternatively, you can make this part of **Investigate**.
- If graphing calculators are available, students can use the **Trace** function to analyse the graph of the function in **Investigate**.
- In **Example 2**, extend the graph to $x = 15$ or so to assist visual learners in verifying the answers to part c).
- You can also use the graph in **Example 3** to investigate long-term trends. Is the month of January becoming colder, warmer, or neither? Have student plot the natural gas consumption for January only, and observe any apparent trend.
- Be sure to review the **Key Concepts** with the class to ensure that students understand all the concepts.

**Differentiated Instruction**

- Build a **model** of a carousel for the Investigate. Cut out a cardboard circle. Put a hole in the centre of the circle. Push a brass fastener (a split pin) through the hole and attach the circle to a piece of paper. Mark John's seat on the circle, and Suzanne's position on the piece of paper. Use a protractor to rotate the circle at 30° intervals. Measure the distance between John and Suzanne using a ruler. Record the results for one complete revolution.
- Use **concept attainment** and **cooperative task groups** to determine whether various functions are periodic. Provide each group with a set of graphs demonstrating examples and non-examples of periodic functions. Students compare the two sets of graphs to determine the key attributes of periodic functions, and then generate their own examples and/or non-examples of periodic functions.
- Construct a **word wall** of the key terms *cycle, period, periodic function,* and *amplitude* to keep these definitions accessible to the class throughout the chapter.

**Investigate Answers (pages 284–285)**

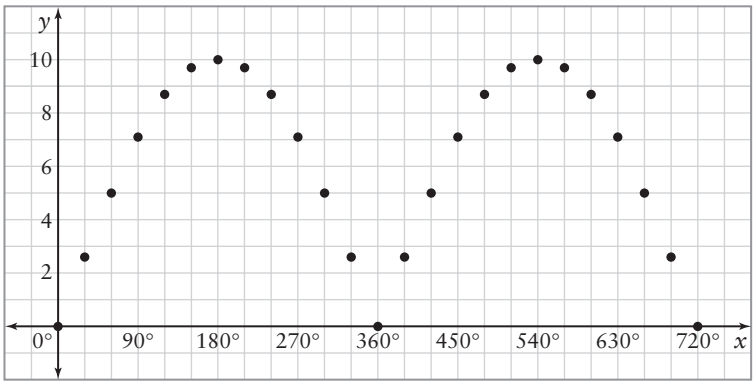
**1.** Provided in student text.

**2.**

| Angle | Distance (m) |
|---|---|
| 0° | 0 |
| 30° | 2.6 |
| 60° | 5.0 |
| 90° | 7.1 |
| 120° | 8.7 |
| 150° | 9.7 |
| 180° | 10.0 |
| 210° | 9.7 |
| 240° | 8.7 |
| 270° | 7.1 |
| 300° | 5.0 |
| 330° | 2.6 |
| 360° | 0 |

**3.** Answers may vary. Sample answer: As the carousel continues to rotate through 360° to 720°, Suzanne's distance from John will repeat the pattern of distances in the chart for 0° to 360°.

**4.**




**5.** Answers may vary.

**6.** **a)** 1800° **b)** 4320°

**7.** **a)** 106° and 254° **b)** 826° and 974°

**8.** 2 cycles

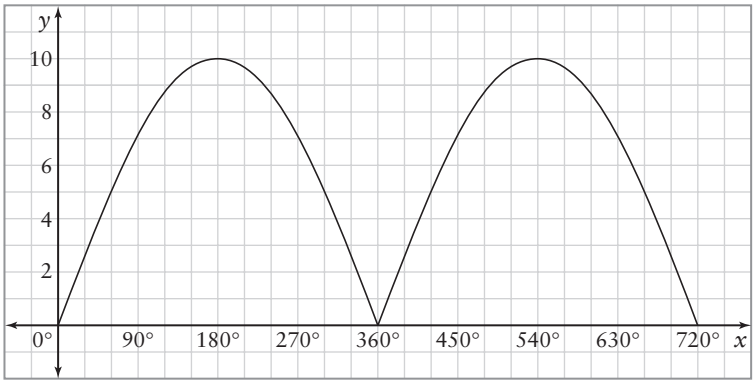
**9.** 360°

**10.** Answers may vary. Sample answer: The *y*-values of a periodic function repeat over and over again in a period fashion.

**11.** minimum 0 m; maximum 10 m

**12.** 5 m

**13.** Answers may vary. Sample answer: The curve would be shifted up 2 m, as the closest distance occurs when Suzanne is 3 m away from John. The amplitude of the function would change. Instead of 5 m, the amplitude would be 2 m with a minimum of 3 m and a maximum of 7 m.

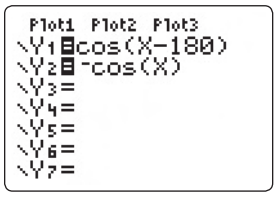



### Communicate Your Understanding Responses (page 289)

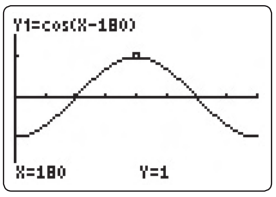
**C1** Answers may vary. Sample answer: It would depend on the pattern associated with the increase and decrease in the population. For the function to be periodic, the population values would need to be in a repeating pattern over equal time intervals.

**C2** **a)** Answers may vary. Sample answer:

Functions:



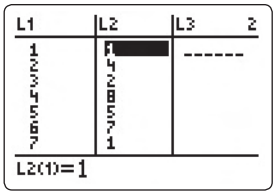

Graph:




**b)** Answers may vary. Sample answer: Yes. A pattern of *y*-values repeats at regular intervals.

**C3** Answers may vary. Sample answer: When the digits of the decimal expansion of $\frac{1}{7}$ are graphed against their decimal places, the result is a periodic pattern of these digits: 1, 4, 2, 8, 5, 7.

Table of values:

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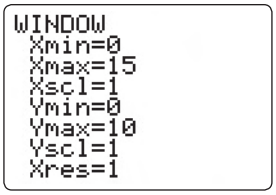
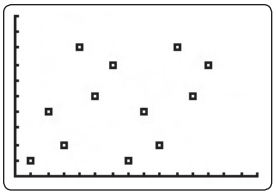



Graph:



**Common Errors**

- Students develop the belief that periodic functions are always vertically symmetric about the horizontal axis.

**R$_x$** When working through a problem such as Example 3 or Practise question 1, take time to discuss whether this is true, and why. When determining the amplitude of a periodic function, it is important to consider both the maximum and minimum values.

- Students have difficulty in applying a known period to known values of a function in order to predict other values.

**R$_x$** Encourage students to make and extend graphs to visually check their answers until they become comfortable using algebra only.

## Practise, Connect and Apply, Extend

- **Question 6** encourages students to use reasoning and reflecting skills to make predictions for the values of a function that has a specific period for different values of $x$. Students will select tools and use connecting skills from previous mathematical knowledge to make predictions. If a prediction is not possible, students will communicate the reason.
- Advise those students who have difficulty with **question 6** to sketch a graph of a periodic function that matches the given period and points, and extend the graph accordingly.
- **Question 8** requires students to include a sketch of a function that has a period equal to the difference between $p$ and $q$.
- **Question 9** illustrates how a boater will know which navigation light is being observed. Marine navigation charts include flash patterns for each light.
- **Question 11** allows students to reflect upon and reason through several given statements to determine which ones involve a periodic pattern. Students will communicate a justification for each answer.
- For **question 12**, visit the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11* to find a graph of the sunspot activity.
- **Question 15** requires students to solve the problem of selecting an appropriate trigonometric tool and using connecting skills to generate a table of values for $d$ as a function of $t$. Students will use their reasoning and reflecting skills to determine why a graph of $d$ versus $t$ will show a periodic pattern and why the tool selected is the most appropriate. Students will then use their communicating skills to explain their choice of the trigonometric tool and to describe how to use the tool to represent the table of values.
- For **question 19**, using *The Geometer's Sketchpad*®, students can follow these steps:
  - From the **Graph** menu, choose **Show Grid**.
  - Choose the **Circle** tool and create a circle that has a diameter of 10 cm.
  - From the **Graph** menu, choose **Hide Grid**.
  - From the **Display** menu, choose **Hide Axes**.
  - Select the circle, and from the **Construct** menu, choose **Point on Circle**.
  - Select the centre of the circle and the point on the circle, and from the **Construct** menu, choose **Line**.
  - With the line selected, from the **Construct** menu, choose **Point on Line**.
  - Select the point on the circle, and from the **Transform** menu, choose **Rotate**. Enter **30°** for the rotation and press **Rotate**.
  - Select the three points, and from the **Construct** menu, choose **Segment**.
  - Select the point on the circle and the rotated one, and from the **Measure** menu, choose **Coordinate Distance**.
  - Repeat this for one complete revolution.
- Data such as those in **question 20** can be obtained from sources such as the U.S. Naval Observatory.

- You can demonstrate the situation in **question 22** using a laser pointer held by a rotating student. Note: Ensure that the pointer is never directly aimed at anyone in the class.
- Use **BLM 5–2 Section 5.1 Practice** for remediation or extra practice.

**Ongoing Assessment**

Achievement Check, question 21, on student text page 292.

**Achievement Check, question 21, student text page 292**

This performance task is designed to assess the specific expectations covered in Section 5.1. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Connecting
- Communicating

**Sample Solution**

The blood pressure pattern is periodic because the high and low values follow a pattern that repeats itself at regular intervals.

The period is 15 s or 18 beats.

The highest value is 120, and the lowest value is 80. Therefore, the amplitude is $(120 - 80) \div 2 = 20$.

## Level 3 Notes

Look for the following:

- Understanding of how to identify a periodic pattern is mostly evident
- Period and amplitude are mostly correct
- Justification and explanation of why blood pressure is periodic is mostly valid

## What Distinguishes Level 2

- Understanding of how to identify a periodic pattern is somewhat evident
- Period and amplitude are somewhat correct
- Justification and explanation of why blood pressure is periodic is somewhat valid

## What Distinguishes Level 4

- Understanding of how to identify a periodic pattern is clearly evident
- Period and amplitude are correct
- Justification and explanation of why blood pressure is periodic is clearly valid

### Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectation | Selected Questions |
|---|---|
| Problem Solving | 12, 16, 17 |
| Reasoning and Proving | 1–23 |
| Reflecting | 6, 11, 12, 14–20, 22, 23 |
| Selecting Tools and Computational Strategies | 3, 4, 6–10, 16, 17, 19, 22, 23 |
| Connecting | 3, 4, 6–10, 12, 14–16, 19–23 |
| Representing | 3, 4, 7–10, 13, 17, 19, 22, 23 |
| Communicating | 5, 6, 8, 9, 11–16, 18, 20–23 |