5.2 The Sine Function and the Cosine Function

Student Text Pages
294 to 301

Suggested Timing
60–75 min

Tools
- grid paper
- protractor, ruler, compass
- TI-83 Plus or TI-84 Plus graphing calculator
- TI-Nspire™ CAS graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 5–3 Section 5.2 Tables for Investigate A
- BLM 5–4 Graphs of Trigonometric and Reciprocal Functions
- BLM 5–5 Section 5.2 Practice

Differentiated Instruction

- Use **what-so-what double entry** to summarize the properties of sine and cosine functions. On the left side of the T-chart, list the titles: Sketch of graph, Maximum value, Minimum value, Amplitude, Domain, Range, *x*-intercepts, *y*-intercepts, Intervals of increase, and Intervals of decrease. On the right side of the chart, fill in the properties for one cycle of $y = \sin x$. Repeat the process for one cycle of $y = \cos x$.

Teaching Suggestions

- To speed up the work in **Investigate A**, allow students to use the summary sheet they created for the trigonometric values in Chapter 4. Alternatively, if students seem to need more review on the unit circle, have them work their way around using symmetry and the "friendly" angles. You may find **BLM 5–3 Section 5.2 Tables for Investigate A** helpful.
- If time permits, consider using the pencil-and-paper method and one of the two technology methods.
- **Investigate C** (work on the cosine function) is intended as part of the homework assignment.
- After completing the summary table of properties for the sine and cosine functions, students can compare their results to the table in the **Key Concepts** and resolve any discrepancies.

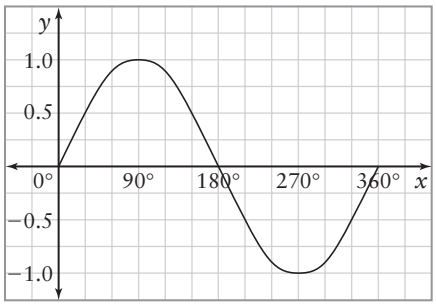
Investigate Answers (pages 294–298)

Investigate A

1.

	$\sin x$	
x	Exact Value	Rounded to One Decimal Place
0°	0	0
30°	$\frac{1}{2}$	0.5
60°	$\frac{\sqrt{3}}{2}$	0.9
90°	1	1
120°	$\frac{\sqrt{3}}{2}$	0.9
150°	$\frac{1}{2}$	0.5
180°	0	0
210°	$-\frac{1}{2}$	−0.5
240°	$-\frac{\sqrt{3}}{2}$	−0.9
270°	−1	−1
300°	$-\frac{\sqrt{3}}{2}$	−0.9
330°	$-\frac{1}{2}$	−0.5
360°	0	0

2.

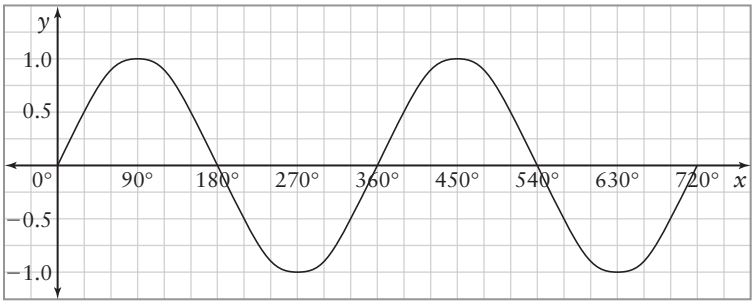


3. Answers may vary. Sample answer: The curve of the sine function looks like the profile of the cross section of a water wave.

4. Answers may vary. Sample answer: The values in the chart start to repeat in a cyclic pattern established from 0° to 360°.

5. a) Answers may vary. Sample answer: The shape beyond 360° will be the same as the characteristic shape from 0° and beyond.

b)



6.

Property	$y = \sin x$
maximum	1
minimum	−1
amplitude	1
period	360°
domain	$x \in \mathbb{R}, 0° \leq x \leq 360°$
range	$y \in \mathbb{R}, -1 \leq y \leq 1$
y-intercept	0
x-intercepts	0°, 180°, 360°
intervals of increase	$0° < x < 90°$ and $270° < x < 360°$
intervals of decrease	$90° < x < 270°$

7. Answers may vary. Sample answer: The pattern associated with the y-values is repeated every 360°.

8. Answers may vary. Sample answer: You can use the vertical line test to show that $y = \sin x$ is a function. In function notation, $f(x) = \sin x$.

Investigate B

Method 1

4. Answers may vary. Sample answer: The graph displayed on the graphing calculator is the same as that drawn in Investigate A.

5. Answers may vary. Sample answer: The table is the same as that constructed in Investigate A.

6. Answers may vary. Sample answer: The values in the chart match the predicted values. The next maximum will occur at 450°.

7. Answers may vary. Sample answer: The next maximum to the left of 0° will occur at −270°.

8. Answers may vary. Sample answer: A graph of $y = \sin x$ from −720° to 720° would have four cycles. It would have maximum values at −630°, −270°, 90°, and 450°, and minimum values at −450°, −90°, 270°, and 630°.

Method 2

2. b) Answers may vary. Sample answer: They are the same as those in the table constructed in Investigate A.

4. Answers may vary. Sample answer: The table is the same as that constructed in Investigate A.

5. Answers may vary. Sample answer: The values in the chart match the predicted values. The next maximum will occur at 450°.

6. Answers may vary. Sample answer: The next maximum to the left of 0° will occur at −270°.

7. b) Answers may vary. Sample answer: A graph of $y = \sin x$ from −720° to 720° would have four cycles. It would have maximum values at −630°, −270°, 90°, and 450°, and minimum values at −450°, −90°, 270°, and 630°.

8. b) Answers may vary. Sample answer: The graph matches the predictions from step 7.

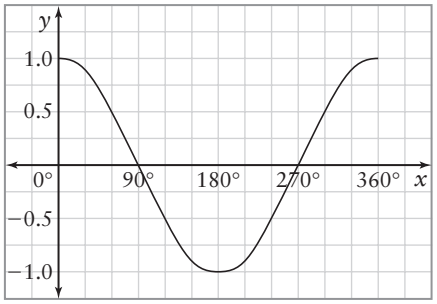
Investigate C

Repeating steps of Investigate A:

1.

	cos *x*	
x	Exact Value	Rounded to One Decimal Place
0°	1	1
30°	$\frac{\sqrt{3}}{2}$	0.9
60°	$\frac{1}{2}$	0.5
90°	0	0
120°	$-\frac{1}{2}$	−0.5
150°	$-\frac{\sqrt{3}}{2}$	−0.9
180°	−1	−1
210°	$-\frac{\sqrt{3}}{2}$	−0.9
240°	$-\frac{1}{2}$	−0.5
270°	0	0
300°	$\frac{1}{2}$	0.5
330°	$\frac{\sqrt{3}}{2}$	0.9
360°	1	1

2.

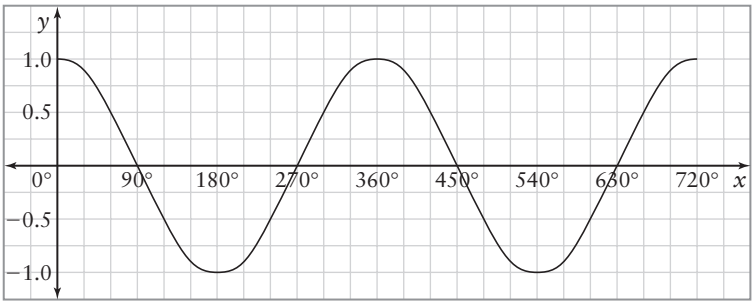


3. Answers may vary. Sample answer: The curve of the cosine function looks like the profile of the cross section of a water wave.

4. Answers may vary. Sample answer: The values in the chart start to repeat in a cyclic pattern established from 0° to 360°.

5. a) Answers may vary. Sample answer: The shape beyond 360° will be the same as the characteristic shape from 0° and beyond.

b)



6.

Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	-1	-1
amplitude	1	1
period	360°	360°
domain	$x \in \mathbb{R}, 0° \leq x \leq 360°$	$x \in \mathbb{R}, 0° \leq x \leq 360°$
range	$y \in \mathbb{R}, -1 \leq y \leq 1$	$y \in \mathbb{R}, -1 \leq y \leq 1$
y-intercept	0	1
x-intercepts	0°, 180°, 360°	90°, 270°
intervals of increase	$0° < x < 90°$ and $270° < x < 360°$	$180° < x < 360°$
intervals of decrease	$90° < x < 270°$	$0° < x < 180°$

7. Answers may vary. Sample answer: The pattern associated with the y-values is repeated every 360°.

8. Answers may vary. Sample answer: You can use the vertical line test to show that $y = \cos x$ is a function. In function notation, $f(x) = \cos x$.

Repeating steps of Method 1 in Investigate B:

4. Answers may vary. Sample answer: The graph displayed on the graphing calculator is the same as that drawn in Investigate A.

5. Answers may vary. Sample answer: The table is the same as that constructed in Investigate A.

6. Answers may vary. Sample answer: The values in the chart match the predicted values. The next maximum will occur at 720°.

7. Answers may vary. Sample answer: The next maximum to the left of 0° will occur at $-360°$.

8. Answers may vary. Sample answer: A graph of $y = \cos x$ from $-720°$ to 720° would have four cycles. It would have maximum values at $-720°$, $-360°$, 0°, 360°, and 720°, and minimum values at $-540°$, $-180°$, 180°, and 540°.

Communicate Your Understanding Responses (page 299)

C1 Answers may vary. Sample answer: The two graphs will intersect at 45° and 225°.

C2 Answers may vary. Sample answer: For the sine function, the x-intercepts occur at $180n°$, for all integer values of n. For the cosine function, the x-intercepts occur at $(90° + 180n°)$, for all integer values of n.

C3 a) the cosine function

b) the sine function

c) Answers may vary. Sample answer: As the point on the unit circle moves from the first quadrant to the second quadrant, the x-coordinate of the point increases while the y-coordinate decreases. In the second quadrant, the x-coordinate of the point becomes negative while the y-coordinate stays positive. The x-coordinate of the point (horizontal displacement) corresponds to the cosine of the angle in standard position formed by connecting the point with the origin, while the y-coordinate (vertical displacement) of the point corresponds to the sine of the same angle.

Common Errors

- Students forget to check their calculator mode and generate values and graphs based on radian measure.

R_x Have students develop a routine of checking the calculator mode before using a calculator for calculations or graphs involving trigonometry.

Practise, Connect and Apply, Extend

- Note that there are no **Practise** questions for this section. Once students are familiar with the shapes of the graphs and properties of the sine and cosine functions, proceed with the **Connect and Apply** questions.
- For **questions 1** and **2**, physical models that can be made quickly using paper, scissors, and paper clips may be helpful. Consider having students work in pairs or small groups for these questions.
- **Question 2** applies sinusoidal functions to music. Students who have studied music for some time may not know why different instruments can be distinguished, even if they are playing the same note. Technology such as *The Geometer's Sketchpad*® is a good visual aid for this question, especially if an interactive whiteboard (such as the SMART Board™) is also available. You can use action buttons to make different functions appear and disappear.
- **Question 3** enables students to select tools and use their connecting skills to represent the vertical and horizontal positions of the hour hand of a clock in two different graphs. They will use their reasoning and reflecting skills to predict the result if the minute hand is used instead of the hour hand, and will communicate the prediction they have made.
- **Question 5** gives students the opportunity to connect their present mathematical knowledge with what they have learned in the past, and to use their representing skills to sketch a graph of $y = \tan x$. Their reasoning and communicating skills will be used throughout to answer questions related to the given function.
- **Questions 4** to **8** develop graphs for the tangent function and the reciprocal trigonometric functions. Although these are not expectations of the course, it is useful for students to at least have seen these graphs before attempting Advanced Functions or Calculus. If time is a factor, consider demonstrating each function using technology, such as *The Geometer's Sketchpad*® and an interactive whiteboard, and spending a few minutes on its properties. You may find **BLM 5–4 Graphs of Trigonometric and Reciprocal Functions** useful.
- Use **BLM 5–5 Section 5.2 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	9
Reasoning and Proving	1–9
Reflecting	3, 4, 9
Selecting Tools and Computational Strategies	1–4, 6–9
Connecting	1–9
Representing	1–9
Communicating	1–5, 9

Use Technology

Student Text Pages
302 to 303

Suggested Timing
15–20 min

Tools
- graphing calculator

Dynamically Unwrap the Unit Circle

Teaching Suggestions

- This technology feature has students using their graphing calculators in parametric mode, which will be a new experience for most students. Take a few minutes to explain what parametric mode is, and why it is needed for this application.
- The factor of 60 in the calculation of $X2$ is needed to make both graphs fit on the same screen. It has no profound mathematical meaning.
- As implied in question 9, it is easy to extend the process to the other trigonometric functions.
- If time is a constraint, consider demonstrating the unwrapping process using a graphing calculator and projector, SmartView®, or an interactive whiteboard (such as the SMART Board™).

5.3

Investigate Transformations of Sine and Cosine Functions

Student Text Pages
304 to 312

Suggested Timing
60–75 min

Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 5–6 Section 5.3 Table for Investigate
- BLM 5–7 Section 5.3 Practice
- BLM 5–8 Section 5.3 Achievement Check Rubric

Teaching Suggestions

- If *The Geometer's Sketchpad*® is readily available, use it for the **Investigate**. The graphs will show up in greater clarity. In addition, students can use colour to distinguish one graph from another.
- You can use **BLM 5–6 Section 5.3 Table for Investigate** as a summary sheet for the **Investigate** results.
- An alternative method to the **Investigate** is using the **APPS** on a TI-83 Plus/TI-84 Plus graphing calculator and selecting **Transfrm**. This feature is already on the TI-84 Plus graphing calculator and can be downloaded to the TI-83 Plus. It enables students to explore on their own the transformations in the Investigate by selecting values and substituting them into the general trigonometric functions $Y1 = A\sin(X)$, $Y1 = \sin(BX)$, $Y1 = \sin(X-D)$, and $Y1 = \sin(X)+D$. To uninstall this **APPS**, press [APPS], select **Transfrm**, then select **1:Uninstall**. The windows should look like these.