Use Technology

Student Text Pages
302 to 303

Suggested Timing
15−20 min

Tools
- graphing calculator

Dynamically Unwrap the Unit Circle

Teaching Suggestions

- This technology feature has students using their graphing calculators in parametric mode, which will be a new experience for most students. Take a few minutes to explain what parametric mode is, and why it is needed for this application.
- The factor of 60 in the calculation of X2 is needed to make both graphs fit on the same screen. It has no profound mathematical meaning.
- As implied in question 9, it is easy to extend the process to the other trigonometric functions.
- If time is a constraint, consider demonstrating the unwrapping process using a graphing calculator and projector, SmartView®, or an interactive whiteboard (such as the SMART Board™).

5.3 Investigate Transformations of Sine and Cosine Functions

Student Text Pages
304 to 312

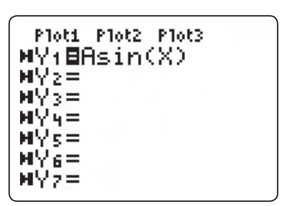
Suggested Timing
60–75 min

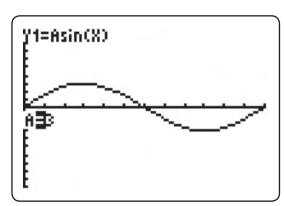
Tools
- grid paper
- graphing calculator
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- BLM 5–6 Section 5.3 Table for Investigate
- BLM 5–7 Section 5.3 Practice
- BLM 5–8 Section 5.3 Achievement Check Rubric

Teaching Suggestions

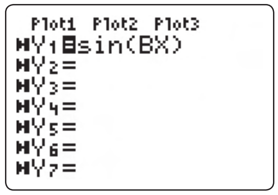
- If *The Geometer's Sketchpad*® is readily available, use it for the **Investigate**. The graphs will show up in greater clarity. In addition, students can use colour to distinguish one graph from another.
- You can use **BLM 5–6 Section 5.3 Table for Investigate** as a summary sheet for the **Investigate** results.
- An alternative method to the **Investigate** is using the **APPS** on a TI-83 Plus/TI-84 Plus graphing calculator and selecting **Transfrm**. This feature is already on the TI-84 Plus graphing calculator and can be downloaded to the TI-83 Plus. It enables students to explore on their own the transformations in the Investigate by selecting values and substituting them into the general trigonometric functions Y1 = Asin(X), Y1 = sin(BX), Y1 = sin(X−D), and Y1 = sin(X)+D. To uninstall this **APPS**, press APPS, select **Transfrm**, then select **1:Uninstall**. The windows should look like these.

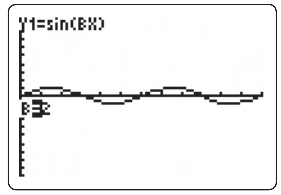


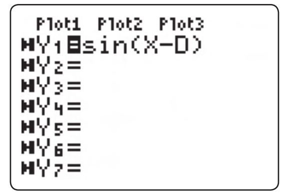


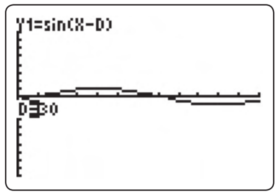
Differentiated Instruction

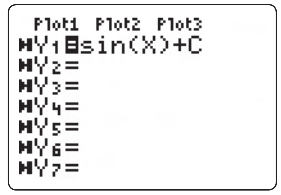
- Use **jigsaw** to complete the Investigate. Divide the class into four expert groups. Assign the graphs of $y = a \sin x$ to Group 1, $y = \sin kx$ to Group 2, $y = \sin (x - d)$ to Group 3, and $y = \sin x + c$ to Group 4. Each group completes the part of the Investigate that corresponds to their assigned equation.
- Create home groups consisting of one member from each expert group. In their home groups, students take turns teaching the effect their factor has on the sine function. Students summarize their results in a **journal** or **what-so-what double entry.**

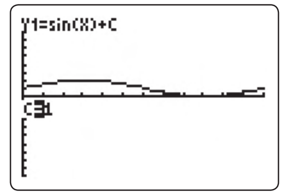










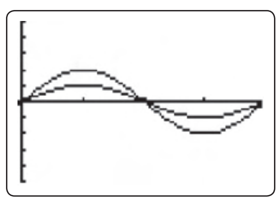


- Graphs for the functions in the **Examples**, and many of the **Practise** questions, are expected to be quick sketches, not point by point graphs. Axes need only have enough detail to determine the properties of a graph.
- For **Example 1**, part **b**), you can treat finding the period as solving the equation $p = \frac{360°}{k}$. This comes in handy later when trying to fit a function to a given period.
- In **Example 3**, point out that the brackets need to be inserted to identify the horizontal shift correctly. Visit the McGraw-Hill Ryerson Web site, *www.mcgrawhill.ca/books/functions11*, to obtain a *The Geometer's Sketchpad®* interactive sine curve that can be used to demonstrate the effects of the values of a, k, d, and c on the sine curve.

Investigate Answers (pages 304–308)

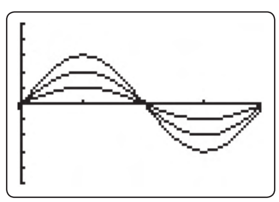
Part A

1. a)



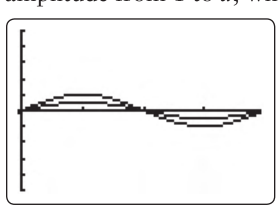
b) Answers may vary. Sample answer: They have the same period and zeros, but differ in amplitude.

2. a)



b) The effect of multiplying the sine function by a constant factor is a change in amplitude from 1 to a, where a represents the multiplying factor.

3. a)

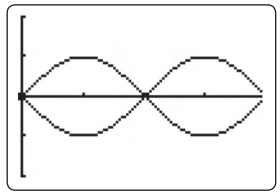


Answers may vary. Sample answer: The amplitude of the graph will be 0.5, but all other properties will remain the same.

b) Answers may vary. Sample answer: The graph matches the prediction.

4. a) Answers may vary. Sample answer: The graph will be a reflection of $y = \sin x$ in the x-axis.

b)



Answers may vary. Sample answer: The prediction was correct.

c) Answers may vary. Sample answer: When compared to the graph of $y = \sin x$, the graph will be a reflection in the x-axis, with the amplitude increasing from 1 to 2.

5. a) Answers may vary. Sample answer: For $a > 0$, the amplitude of the function increases by a factor of a. The amplitude will be larger than the amplitude of $y = \sin x$.

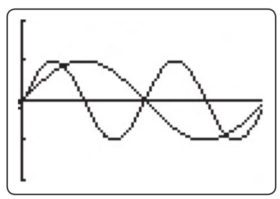
b) Answers may vary. Sample answer: For $0 < a < 1$, the amplitude of the function will change by a factor of a. The amplitude will be smaller than the amplitude of $y = \sin x$.

c) Answers may vary. Sample answer: For $a < -1$, the function will be reflected in the x-axis. The amplitude of the function increases by a factor of a. The amplitude will be larger than the amplitude of $y = -\sin x$.

d) Answers may vary. Sample answer: For $-1 < a < 0$, the function will be reflected in the x-axis. The amplitude of the function will change by a factor of a. The amplitude will be smaller than the amplitude of $y = -\sin x$.

Part B

1. a)

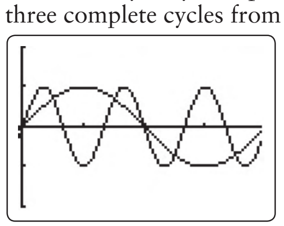


b) Answers may vary. Sample answer: The two graphs have the same amplitude, and they start and end at the same position. For $y = \sin x$, there is one complete cycle; for $y = \sin 2x$, there are two complete cycles from 0° to 360°.

c) Answers may vary. Sample answer: The period of $y = \sin x$ is 360° and the period for $y = \sin 2x$ is 180°.

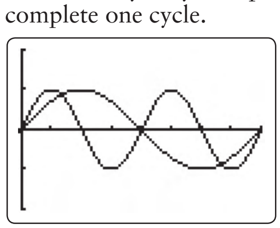
2. a) Answers may vary. Sample answer: The graph of $y = \sin 3x$ should go through three complete cycles from 0° to 360°.

b)



3. a) Answers may vary. Sample answer: The graph should require 0° to 720° to complete one cycle.

b)



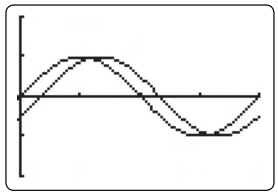
4. a) Answers may vary. Sample answer: The period for $y = \sin kx$ is given by $\frac{360°}{k}$.

b) Answers may vary. Sample answer: When $k > 1$, the period of the function decreases, and when $0 < k < 1$, the period of the function increases.

c) When $k < -1$, the period decreases and the function reflects in the y-axis. When $-1 < k < 0$, the period increases and the function reflects in the y-axis.

Part C

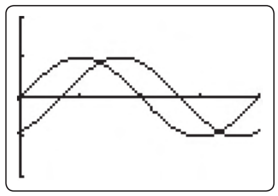
1. a)



b) Answers may vary. Sample answer: The two graphs have the same period and amplitude, but one is shifted horizontally relative to the other.

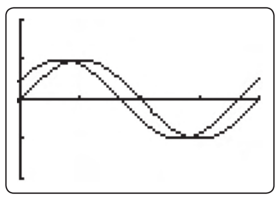
2. a) Answers may vary. Sample answer: The graph of $y = \sin x$ will shift 60° to the right.

b)



3. a) Answers may vary. Sample answer: The graph of $y = \sin x$ will shift 30° to the left.

b)

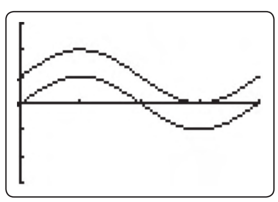


4. a) Answers may vary. Sample answer: When $d > 0$, the graph shifts d units to the left.

b) Answers may vary. Sample answer: When $d < 0$, the graph shifts d units to the right.

Part D

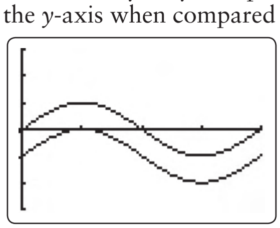
1. a)



b) Answers may vary. Sample answer: The two graphs have the same shape, period, and amplitude, but they differ in their vertical position.

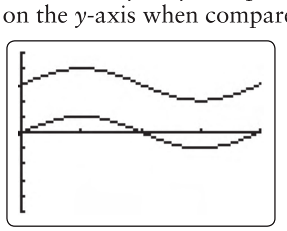
2. a) Answers may vary. Sample answer: The graph will be located one unit lower on the y-axis when compared to $y = \sin x$.

b)



3. a) Answers may vary. Sample answer: The graph will be located three units higher on the y-axis when compared to $y = \sin x$.

b)



4. a) Answers may vary. Sample answer: When $c > 0$, the function moves up c units.
b) Answers may vary. Sample answer: When $c < 0$, the function moves down c units.

5.

Factor	Value	Effect
a	$a > 1$	amplitude is greater than 1
	$0 < a < 1$	amplitude is less than 1
	$-1 < a < 0$	amplitude is less than 1 and the function reflects in the x-axis
	$a < -1$	amplitude is greater than 1 and the function reflects in the x-axis
k	$k > 1$	period decreases
	$0 < k < 1$	period increases
	$-1 < k < 0$	period increases and the function reflects in the y-axis
	$k < -1$	period decreases and the function reflects in the y-axis
d	$d > 0$	function moves d units to the right
	$d < 0$	function moves d units to the left
c	$c > 0$	function moves up c units
	$c < 0$	function moves down c units

Communicate Your Understanding Responses (page 309)

C1 Answers may vary. Sample answer: The value of c must be greater than 5. This is due to the fact that the amplitude is 5, which results in a minimum value of -5 if $c = 0$. Translating the function slightly more than 5 units up will result in the minimum value of y being positive.

C2 Answers may vary. Sample answer: The two graphs have the same y-intercepts. This point will not change after a reflection in the y-axis. As well, points on the curve to the left of $x = 0$ in graph a) are in the same relative position to $x = 0$ as those points on the right of $x = 0$ on graph b). This is also true for the points to the right of $x = 0$ on graph a) when compared to those points on the left of $x = 0$ on graph b).

C3 Answers may vary. Sample answer: Graphs such as $y = \cos(x - 90°)$ and $y = \cos(x + 270°)$ are translated functions that would be identical to the function $y = \sin x$.

Common Errors

- Students use the wrong direction for a horizontal translation.

R_x Have students use a mnemonic to remember "add left, subtract right," such as All Lions Sit and Roar.

Practise, Connect and Apply, Extend

- Ensure that students understand the point of **questions 4** and **5**: either a sine function or a cosine function may be used to model a graph.
- **Question 10** requires students to reason through and reflect upon the maximum and minimum vertical positions that the point on the rim of the bicycle will reach. They will use the above skills along with connecting skills from previously learned concepts and communicating skills to determine and explain how the given equation will change if the period of rotation of the wheel is tripled.
- For **question 10**, part **d**), have students discuss this question: If the period increases, is the wheel speeding up or slowing down?
- For **question 11**, part **b**), if the TI-83 Plus/TI-84 Plus graphing calculator is used, remind students to set the calculator to **Degree** mode and use a suitable window to display the graph.
- In **question 12**, there is little evidence that biorhythms really work. Consider asking students to do a quick study using the Internet, and share their results.
- **Question 13** is an example of an inverse square law. Other such laws include the gravitational force between heavenly bodies, and the electrical force between charged particles.
- **Question 14** allows students to use their reasoning, reflecting, and connecting skills to solve the problem of writing the equation of the transformed function in part **a**) and of determining what phase shift is necessary to produce a

specific y-intercept in part **b**). They will select tools necessary to represent the graph in part **b**) and the diagram(s) required in part **c**). They will then use communicating skills to explain whether the y-intercept in part **b**) can be achieved by altering the period rather than the phase shift.

- Use **BLM 5–7 Section 5.3 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 312.

Achievement Check, question 17, student text page 312

This performance task is designed to assess the specific expectations covered in Section 5.3. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Connecting
- Selecting Tools and Computational Strategies
- Representing
- Communicating

Sample Solution

Provide students with **BLM 5–8 Section 5.3 Achievement Check Rubric** to help them understand what is expected.

a) Comparing $y = 6\cos[5(x + 45°)] - 3$ to $y = a\cos[k(x - d)] + c$ gives $a = 6$, $k = 5$, $d = -45°$, and $c = -3$.

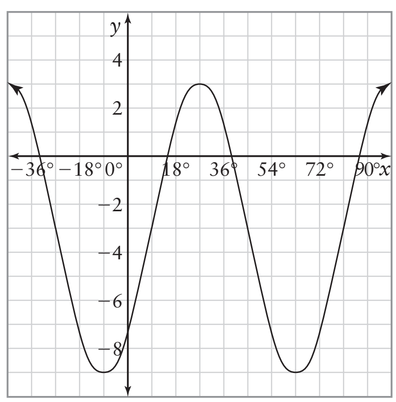
i) $a = 6$, so the amplitude is 6.

ii) $k = 5$, so the period is $\frac{360°}{5}$ or 72°.

iii) $d = -45°$, so the phase shift is 45° to the left.

iv) $c = -3$, so the vertical shift is 3 units down.

b) Use a basic cosine graph. Stretch it by factor of 6. Change the period to 72°, so one cycle begins at 0° and ends at 72°. Shift the graph three units down so that the maximum value is 3 and the minimum value is -9. Then, shift 45° left, so that one cycle begins at $-45°$ and ends at 27°.



Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13–16, 18, 19
Reasoning and Proving	1–5, 10, 12–19
Reflecting	10, 13–16, 18, 19
Selecting Tools and Computational Strategies	1, 2, 8, 9, 11, 12, 14–18
Connecting	1–19
Representing	1, 2, 4, 5, 8, 9, 11–18
Communicating	10–12, 14–16, 18, 19