5.4 Graphing and Modelling With $y = a\sin[k(x - d)] + c$ and $y = a\cos[k(x - d)] + c$

Student Text Pages
313 to 321

Suggested Timing
60–75 min

Tools
- grid paper
- graphing calculator
- TI-Nspire™ CAS graphing calculator (optional)
- computer with graphing software (optional)

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- BLM 5–9 Section 5.4 Practice

Differentiated Instruction
- Use **think-aloud** and have selected students graph transformations either on the board or an overhead grid for the class.
- Use **placemat** to summarize the steps in graphing transformations.
- Have students work in **cooperative task groups** to create their own sine or cosine graph and determine its equation. Use a modified **carousel** to display the constructions and have groups circulate to check the equations of their peers.

Common Errors
- Students apply transformations incorrectly to arrive at the wrong answer.

R_x Have students routinely check their answers by graphing. In the absence of technology, they can often catch errors with a quick pencil-and-paper sketch on grid paper.

Teaching Suggestions

- For Method 2 of **Example 1,** part **c)**, you can also press (menu), then select **5:Trace** and **1:Graph Trace**. Use the cursor keys to move the tracing point. Whenever you reach a maximum or minimum, the appropriate box will appear. You can also use *The Geometer's Sketchpad*®, if a computer lab is available, or an interactive whiteboard (such as the SMART Board™).
- For **Example 2**, ask students whether the order matters when transforming the graph. Could the vertical shift be applied first? Could the horizontal compression be applied first? Could the phase shift be applied first? These questions will be revisited in the **Communicate Your Understanding** feature.
- Ensure that students grasp the point of **Example 3**: either a sine curve or a cosine curve can be used to match the given values of a, k, d, and c.

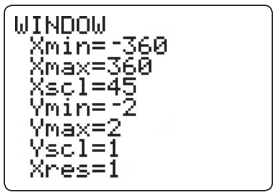
Communicate Your Understanding Responses (page 318)

C1 Answers may vary. Sample answer: To determine the phase shift, the coefficient of x must be factored out of the term with x and the angle portion. That means $3(x - 20°)$. Therefore, the phase shift is 20° to the right.

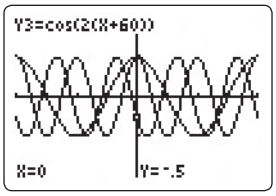
C2 **a)** to **c)** Answers may vary.

d) Answers may vary. Sample answer: Both methods give the same graph, so both are a correct procedure.

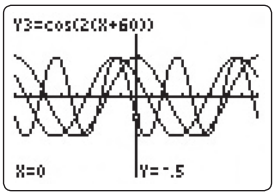
Window settings:



Graphs for part a):



Graphs for part b):



C3 Answers may vary. Sample answer: Since both the sine and cosine functions are simply phase shifts of the same curve, one function can always be expressed as a phase-shifted function of the other.

Practise, Connect and Apply, Extend

- Before assigning **questions 8** and 9, students may need to revisit the procedure for determining x- and y-intercepts.
- It is useful for students to be able to check their answers using technology. If you have arranged for home access to *The Geometer's Sketchpad*®, it can be used in place of a graphing calculator.
- **Question 14** requires students to solve the problem of representing the given graph with equations by using their reasoning and reflecting skills. In addition, to determine these equations, students will use skills that connect their present and past mathematical knowledge.
- **Question 16** gives students the opportunity to reason through and reflect on methods that will enable them to solve the problems of constructing different models for the water depth. They will select tools and use connecting skills to represent the required models, and then communicate a comparison of the models.
- Use **BLM 5–9 Section 5.4 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 321.

Achievement Check, question 17, student text page 321

This performance task is designed to assess the specific expectations covered in Section 5.4. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Connecting
- Representing
- Communicating

Sample Solution

a) Comparing $g(x) = 2\sin[4(x - 40°)] - 3$ to $y = a\sin[k(x - d)] + c$ gives $a = 2$, $k = 4$, $d = 40°$, and $c = -3$. Start with the graph of $y = \sin x$.

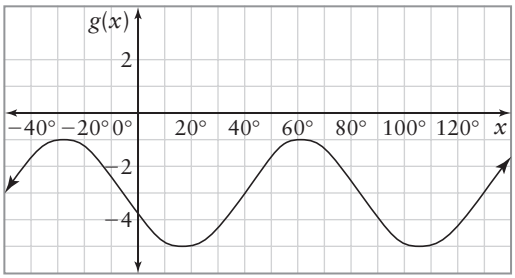
i) Since $a = 2$, apply a vertical stretch of factor 2, so the amplitude is 2.

ii) Since $c = -3$, apply a vertical shift down 3 units. Use the horizontal line $y = -3$ as a reference line so that the maximum value is -1 and the minimum value is -5.

iii) Since $k = 4$, apply a horizontal compression of 4, so the period is 90°.

iv) Since $d = 40°$, apply a phase shift of 40° to the right, so that one cycle begins at 40° and ends at 130°.

b)



c) The domain of $g(x)$ is $\{x \in \mathbb{R}\}$ because the pattern repeats itself indefinitely. The range is $\{x \in \mathbb{R}, -5 \leq y \leq -1\}$ because the amplitude is 2 and, with a vertical shift down 3 units, the maximum is -1 and the minimum is -5.

Level 3 Notes

Look for the following:

- Mostly reasonable and fully labelled sketch
- Transformations described are mostly correct
- Understanding of relationship between parameters and transformations of $y = \sin x$ is mostly evident

What Distinguishes Level 2

- Somewhat reasonable and fully labelled sketch
- Transformations described are somewhat correct
- Understanding of relationship between parameters and transformations of $y = \sin x$ is somewhat evident

What Distinguishes Level 4

- Accurate and fully labelled sketch
- Transformations described are accurate
- Understanding of relationship between parameters and transformations of $y = \sin x$ is clearly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	14–16, 18–20
Reasoning and Proving	3–9, 11–20
Reflecting	14–16, 18–20
Selecting Tools and Computational Strategies	3, 4, 10, 15–17, 19
Connecting	1–20
Representing	3–7, 11–20
Communicating	3, 15, 16, 18–20

Data Collecting and Modelling

Student Text Pages
322 to 332

Suggested Timing
60–75 min

Tools
- graphing calculator
- motion sensor
- pendulum
- computer with Internet access
- TI-Nspire™ CAS graphing calculator (optional)
- computer with graphing software (optional)

Related Resources
- BLM 5–10 Section 5.5 Practice

Differentiated Instruction
- Use **cooperative task groups** to complete the Investigate.
- Search the Internet for a Ferris wheel simulation to represent Example 2. Visit the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11* for one example.
- Add the key terms *phase shift* and *vertical shift* to the **word wall** constructed in Section 5.1.

Teaching Suggestions

- For the **Investigate**, a bowling ball makes an excellent bob for a pendulum, provided that you have a solid enough ceiling anchor to hold it. The motion sensor works best between 1 m and 3 m. Ensure that it is aimed so that the bob of the pendulum remains within its "view" during the entire cycle. Use the pendulum with the longest length that you can manage, certainly 2 m or more.
- If your graph in step 4 of the **Investigate** contains "artifacts," i.e., spikes or drop-outs, your motion sensor is not properly aimed. Adjust your set-up until you can generate a graph free of artifacts.
- You can access E-STAT directly through *www.statcan.gc.ca* and the link from Learning Resources. You may wish to go to the site to demonstrate the steps for work on **Example 1**.
- For **Example 1**, most spreadsheet applications, such as Microsoft *Excel*® and Corel *Quattro Pro*®, prefer values separated by commas. Alternatively, if you have selected an HTML table, you can often highlight, copy, and paste directly into a spreadsheet, although results are not as consistent as the CSV file download option. You can also use the highlight, copy, and paste method to insert cases into dynamic statistical software such as *Fathom*™. Ensure that the first row corresponds to the attribute names.
- In **Example 2**, some students may have trouble relating changes in the rotational speed of the Ferris wheel to changes in the period. A model using a bicycle wheel or something similar is useful in illustrating the concept.
- If you have an interactive whiteboard (such as the SMART Board™) available, you can illustrate **Example 3** using *The Geometer's Sketchpad*®. Plot a point on the sinusoidal graph, measure its coordinates, and then drag the point along the graph to illustrate the solution.