Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	14–16, 18–20
Reasoning and Proving	3–9, 11–20
Reflecting	14–16, 18–20
Selecting Tools and Computational Strategies	3, 4, 10, 15–17, 19
Connecting	1–20
Representing	3–7, 11–20
Communicating	3, 15, 16, 18–20

Data Collecting and Modelling

Student Text Pages
322 to 332

Suggested Timing
60–75 min

Tools
- graphing calculator
- motion sensor
- pendulum
- computer with Internet access
- TI-Nspire™ CAS graphing calculator (optional)
- computer with graphing software (optional)

Related Resources
- BLM 5–10 Section 5.5 Practice

Differentiated Instruction
- Use **cooperative task groups** to complete the Investigate.
- Search the Internet for a Ferris wheel simulation to represent Example 2. Visit the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11* for one example.
- Add the key terms *phase shift* and *vertical shift* to the **word wall** constructed in Section 5.1.

Teaching Suggestions

- For the **Investigate**, a bowling ball makes an excellent bob for a pendulum, provided that you have a solid enough ceiling anchor to hold it. The motion sensor works best between 1 m and 3 m. Ensure that it is aimed so that the bob of the pendulum remains within its "view" during the entire cycle. Use the pendulum with the longest length that you can manage, certainly 2 m or more.
- If your graph in step 4 of the **Investigate** contains "artifacts," i.e., spikes or drop-outs, your motion sensor is not properly aimed. Adjust your set-up until you can generate a graph free of artifacts.
- You can access E-STAT directly through *www.statcan.gc.ca* and the link from Learning Resources. You may wish to go to the site to demonstrate the steps for work on **Example 1**.
- For **Example 1**, most spreadsheet applications, such as Microsoft *Excel*® and Corel *Quattro Pro*®, prefer values separated by commas. Alternatively, if you have selected an HTML table, you can often highlight, copy, and paste directly into a spreadsheet, although results are not as consistent as the CSV file download option. You can also use the highlight, copy, and paste method to insert cases into dynamic statistical software such as *Fathom*™. Ensure that the first row corresponds to the attribute names.
- In **Example 2**, some students may have trouble relating changes in the rotational speed of the Ferris wheel to changes in the period. A model using a bicycle wheel or something similar is useful in illustrating the concept.
- If you have an interactive whiteboard (such as the SMART Board™) available, you can illustrate **Example 3** using *The Geometer's Sketchpad*®. Plot a point on the sinusoidal graph, measure its coordinates, and then drag the point along the graph to illustrate the solution.

Investigate Answers (pages 322–323)

4. Answers may vary. Sample answer: The graph appears to be sinusoidal.

5. to **9.** Answers will be based on experimental set-up.

Communicate Your Understanding Responses (page 331)

C1 Answers may vary. Sample answer: The amplitude, period, and vertical shift are the same. The only difference is the phase shift. For the motion to start at a position of 0 (which a sine curve allows), the cosine curve will require an interpretation so that the pendulum starts at a maximum position.

C2 Answers may vary. Sample answer: From 1976 to 1980, it appears that the population of Canadians aged 20 to 24 years can be modelled with an increasing linear function. From 1981 to 1988, it appears that the population of Canadians aged 20 to 24 years can be modelled with a decreasing quadratic or sinusoidal function.

C3 Answers may vary. Sample answer: The vertical shift will change from +12 to +13, and the phase shift will change from 30° to 20°, which corresponds to a new location of the loading platform.

Common Errors

- Students have difficulty visualizing a situation from a written description.

R_x Have students construct and use physical models to simulate a situation before attempting to model it with a trigonometric function.

Practise, Connect and Apply, Extend

- **Question 2** enables students to use their reasoning and connecting skills to read the required information and make specific estimates from the given graph. They will select appropriate tools to represent a sketch of a horizontal reference line and graph the model required. Representing skills will also be needed to construct a model for the motion.
- **Question 9** gives students the opportunity to use reasoning, reflecting, and connecting skills to solve several problems related to the motion of a pendulum. Representing skills will be needed in parts **c**) and **e**) to modify the equation that models the motion of the pendulum. Students will be required to communicate their predictions in part **d**).
- Consider using **question 10** as a class investigation. Small groups can each assemble a double pendulum with assigned lengths. Have the groups demonstrate their pendulums in turn.
- **Question 11** requires students to perform an experiment and use their reasoning skills to determine how closely deep breathing can be modelled using a sinusoidal function. They will represent volume exhaled versus time by producing a table of values and then sketching a graph of the data. A representation to model the data as a sinusoidal function and a sketch of its graph will follow. Communicating skills to describe their findings will be necessary.
- A sharp student may suggest that **question 5** or **11** could generate a function that is periodic, but not necessarily sinusoidal. You can use *The Geometer's Sketchpad*® to illustrate how a sum of sinusoidal functions can be used to approximate any periodic function, similar to the method used to build up sound waves of varying timbre in **question 14**. This is another example of Fourier analysis.
- Before assigning **question 18**, consider using a flashlight and model to illustrate the changes in the shadow at different times of the day.
- Use **BLM 5–10 Section 5.5 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 332.

Achievement Check, question 17, student text page 332

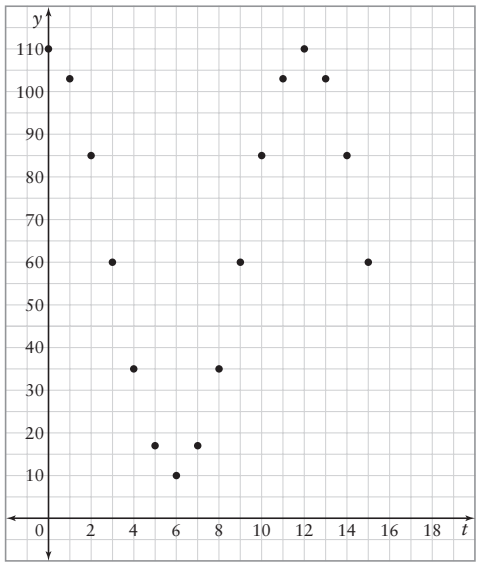
This performance task is designed to assess the specific expectations covered in Section 5.5. The following mathematical process expectations can be assessed.

- Problem Solving
- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

Answers may vary. Sample answer: Follow these steps.

i) Graph the data.



ii) Use the graph to determine the values of a, k, d, and c in $y = a \sin[k(x - d)] + c$.

The amplitude is $(110 - 10) \div 2 = 50$, and therefore $a = 50$.

The vertical shift is $10 + 50 = 60$, and therefore $c = 60$.

The pattern repeats itself after 12 s, so the period is 12 s.

$\frac{360}{k} = 12$, and therefore $k = 30$.

The sine curve begins at 60; this occurs at 3 s and 9 s.

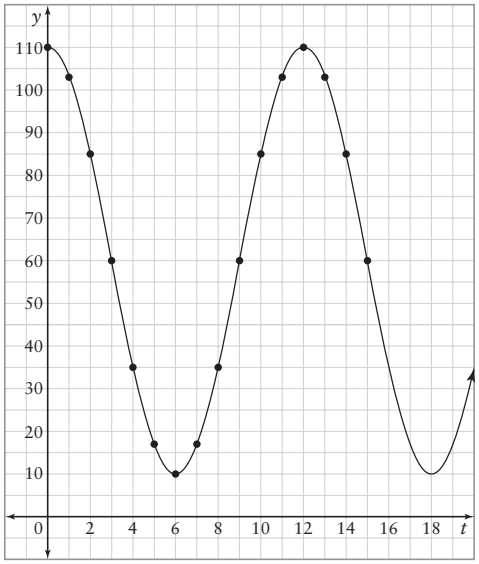
iii) Use the values of a, k, d, and c to determine the equation.

If the sine curve begins at 3 s, then $d = 3$ and $a = -50$ (the y-values decrease then increase, and therefore the sine curve is reflected in the horizontal reference line $y = 60$).

In this case, the function that models the data is $y = -50 \sin[30(t - 3)] + 60$. This equation fits the data, as shown in this graph.

If the sine curve begins at 9 s, then $d = 9$ and $a = 50$ (since the y-values increase, and then decrease).

In this case, the function that models the data is $y = 50 \sin[30(t - 9)] + 60$. This equation fits the data, as shown in the same graph.



Level 3 Notes

Look for the following:

- Mostly reasonable and fully labelled scatter plot and graph
- Values of parameters are mostly correct
- Understanding of steps required to determine an algebraic model is mostly evident
- Understanding of relationship between parameters and algebraic model is mostly evident

What Distinguishes Level 2

- Somewhat reasonable and somewhat labelled scatter plot and graph
- Values of parameters are somewhat correct
- Understanding of steps required to determine an algebraic model is somewhat evident
- Understanding of relationship between parameters and algebraic model is somewhat evident

What Distinguishes Level 4

- Accurate and clearly labelled scatter plot and graph
- Values of parameters are correct
- Understanding of steps required to determine an algebraic model is clearly evident
- Understanding of relationship between parameters and algebraic model is clearly evident

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	3, 4, 6, 7, 9, 10, 14–18
Reasoning and Proving	1–18
Reflecting	3, 4, 6, 7, 9, 10, 12, 14–18
Selecting Tools and Computational Strategies	1, 2, 14, 17, 18
Connecting	1–18
Representing	1, 2, 5–7, 9–11, 14, 15, 17, 18
Communicating	6, 7, 9–12, 14–17