

# 6.1

## Sequences as Discrete Functions

### Student Text Pages

354 to 363

### Suggested Timing

55–75 min

### Tools

- square dot paper
- ruler
- computer with *The Geometer's Sketchpad*®
- graphing calculator
- grid paper
- computer with spreadsheet software
- isometric dot paper

### Related Resources

- G–1 Grid Paper
- T–1 Microsoft® *Excel*
- T–2 *The Geometer's Sketchpad*® 4
- BLM 6–3 Section 6.1 Investigate
- BLM 6–4 Section 6.1 Practice

### Differentiated Instruction

- Use a separate **Frayer model** to define *continuous* and *discrete* functions. Under Characteristics, include a sample sketch, a table of values, an explicit formula, and the domain and range. Under Examples and Non-examples, list real-life scenarios for each kind of function.
- Use a modified **inside/outside circle** to practise writing a sequence as an explicit formula. Provide each pair in the concentric circles with a sequence of numbers. Have students in the outside circle write the sequence using function notation. Have students in the inside circle write the sequence using an explicit formula for the  $n$ th term of the sequence. Have partners expand each other's formula to check their work. Rotate the set of sequences around the circle until each pair has tried them all.

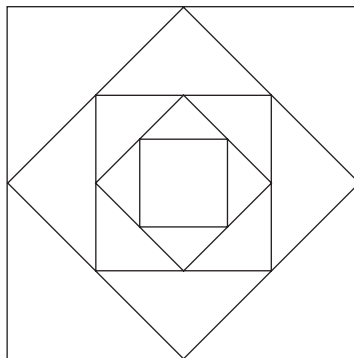
### Teaching Suggestions

- Have students complete the diagrams and tables in pairs. Students could use **BLM 6–3 Section 6.1 Investigate** to help them organize their information.
- Discuss the meaning of the “enclosed regions” mentioned in the **Investigate** so students will be able to record the correct data.
- Have students share the explicit formulas they determine with the class. Discuss any similarities and differences. Try using some of the formulas to see which are correct and which need some modification.
- Before students work through the examples, discuss the meaning of *explicit formula* and provide more examples like **Example 1** and **Example 2**, as needed. Students tend to have more difficulty in determining a formula than using one, so it may be useful to review how to use a difference table to determine the values of  $m$  and  $b$  in a linear function, and the values of  $a$ ,  $b$ , and  $c$  in a quadratic function.
- Reinforce the difference between a continuous function and a discrete one. Refer to the graph constructed in the **Investigate** to help students consolidate this concept.
- For **Investigate Method 2**, students use *The Geometer's Sketchpad*® to construct a floor tile designed with nested squares. If needed, use **T–2 The Geometer's Sketchpad® 4 to support this activity.**
- For **Example 1 Method 2**, students use a TI-83 Plus or TI-84 Plus graphing calculator to generate a sequence.
- For **question 9**, remind students 1995 represents the number zero.
- For **question 10**, students are expected to use a spreadsheet, such as Microsoft® *Excel*. If needed, use **T–1 Microsoft® Excel** to support this activity.

### Investigate Answers (pages 354–355)

#### Method 1

1. and 2.

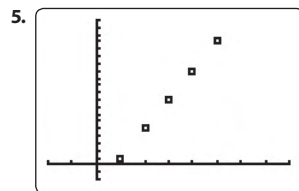


3.

Number of Squares, $n$	Number of Regions, $t$
1	1
2	5
3	9
4	13
5	17

4. a) Answers may vary. Sample answer: The first number in the Number of Regions column is 1. For each increase in the number of squares by 1, the number of regions increases by 4.

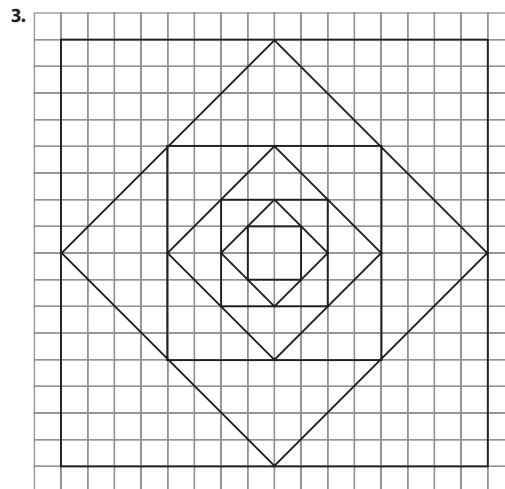
b) 1, 5, 9, 13, 17, ... , 21, 25, 29



Answers may vary. Sample answer: You should leave the points as distinct points. The numbers in the Number of Squares column are integer values and the numbers in the Number of Regions are integer values.

6.  $t_n = 1 + (n - 1)(4)$

**Method 2**

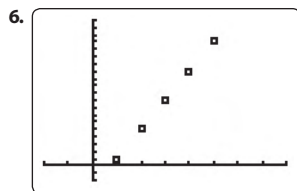


4.

Number of Squares, $n$	Number of Regions, $t$
1	1
2	5
3	9
4	13
5	17

5. a) Answers may vary. Sample answer: The first number in the Number of Regions column is 1. For each increase in the number of squares by 1, the number of regions increases by 4.

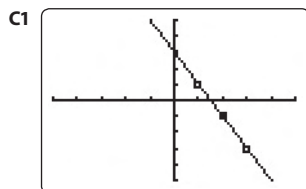
b) 1, 5, 9, 13, 17, ... , 21, 25, 29



Answers may vary. Sample answer: You should leave the points as distinct points. The numbers in the Number of Squares column are integer values and the numbers in the Number of Regions are integer values.

7.  $t_n = 1 + (n - 1)(4)$

**Communicate Your Understanding Responses (page 360)**



Answers may vary. Sample answer: Similarities: The three ordered pairs that make up the sequence are also three points on the function. Differences: The sequence is made up of three points and is a discrete function. The function defined by  $f(x) = -2x + 3$  is a continuous line. The domain of the function is the set of real numbers. The sequence can be represented by the function  $f(x) = -2x + 3$ . The domain of the sequence is the set of natural numbers.

C2 and C3 Answers may vary.

**Common Errors**

- Students have difficulty recognizing patterns in sequences.
- R<sub>x</sub> Where possible, provide extra support with manipulatives or drawings to represent the terms.
- Students have difficulty distinguishing between continuous and discrete functions.
- R<sub>x</sub> Provide graphs of both types of functions, have students describe the similarities and differences, and have them list the y-coordinates of the discrete functions.

**Practise, Connect and Apply, Extend**

- Before assigning **question 3**, have students try a few sequences with algebraic terms, such as  $2y + 1$ ,  $4y + 2$ ,  $6y + 3$ , ...
- **Questions 7 and 8** provide students with opportunities to explain their thinking in writing. Provide a few exemplars of these types of explanations to assist students with this important Communication task.
- **Question 9** requires students to use connecting skills to graph the given equation using a graphing calculator. They are asked to communicate a description of the shape of the given graph and to use reasoning and reflecting skills to determine how the graph would change if the growth rate were greater. Students need to represent the population in each year from 2007 to 2015 as a sequence.
- **Question 10** may be completed without technology as well.
- **Question 11** gives students a chance to choose an appropriate method to solve the problems. They will need to apply mathematical reasoning to recognize that the sum of  $\frac{1}{A} + \frac{1}{B}$  becomes the expression for  $\frac{1}{A}$  for the next term in the sequence.
- For **question 12**, encourage students to draw the figure in Stage 1 quite large. You may want to show them a Koch snowflake with many iterations already completed.
- **Question 15** allows students to reason through the given information and use connecting skills learned from previous mathematical knowledge to write a sequence representing the sales for the first six days that the business is in operation. They are required to select tools to write an explicit formula that is needed to determine the sales on any of the first 14 days. Students are asked to use this explicit formula to determine the sales on the 14th day and to communicate an explanation concerning the appropriateness of the answer.

- For **question 18**, encourage students to construct a diagram to help them determine the number of squares in an  $n$  by  $n$  square. Square dot paper or grid paper would be a useful aid.
- Use **BLM 6–4 Section 6.1 Practice** for remediation or extra practice.

### Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	14, 17, 18
Reasoning and Proving	3–18
Reflecting	6–10, 14, 17, 18
Selecting Tools and Computational Strategies	9, 14, 15, 18
Connecting	1–18
Representing	4, 5, 9, 10, 12–16
Communicating	3, 6–10, 15, 17

## Use Technology

### Student Text Page

364

### Suggested Timing

10–15 min

### Tools

- TI-Nspire™ CAS graphing calculator

## Use a TI-Nspire™ CAS Graphing Calculator to Write Terms in a Sequence

### Teaching Suggestions

- To enter a sequence of terms, you can use the **sequence** function:  $seq(expr, var, low, high [, step])$ .
  - **expr** is an expression for the term of a sequence.
  - **var** is the variable.
  - **low** is the starting value of the variable.
  - **high** is the finishing value of the variable.
  - **step** is the interval between values of the variable. If omitted, it is assumed to be 1.

**Example:** Enter values of  $n$  from 1 to 100 in cells A1 to A100. In the formula row for column A, enter  $seq(n, n, 1, 100)$  and press  $\left(\frac{\text{enter}}{\text{enter}}\right)$ . The integers from 1 to 100 will be entered in the cells starting at A1, and finishing at A100.