6.3 Pascal's Triangle and Expanding Binomial Powers

Student Text Pages
373 to 379

Suggested Timing
60–75 min

Tools
- graphing calculator with computer algebra system (CAS) (optional)

Related Resources
- BLM 6–6 Section 6.3 Practice

Teaching Suggestions

- Have students work individually on step 1 of the **Investigate**. They may need support to complete parts c) and d). Working through steps 1, 2, and 3 as a whole class activity may be beneficial.
- Have students work on step 4 of the **Investigate** with a partner. They can put their solutions on the board to discuss. This may lead to a discussion of common errors.
- Encourage students to work slowly when expanding binomials with complex or fractional terms. Carefully completing each step will reduce the number of errors in the final expansion.
- Before presenting **Example 4**, have students complete the expansions in the chart (exposition) so they will understand how to arrive at each individual term. To access the **expand** function on a TI-Nspire™ CAS graphing calculator, press (menu), and then select **3:Algebra**. Select **3:Expand.** Type the binomial you want to expand, and press (≈ enter). The CAS engine will display the terms of the expansion.
- Students will have an opportunity to consolidate their understanding of Pascal's triangle in the **Communicate Your Understanding** section.

Differentiated Instruction

- Use **Think-Pair-Share** to complete the Investigate. Have partners compare and correct each other's expansions in question 1 before completing the investigation.
- After reading Example 4, have students used **timed retell** to answer question C4.
- Complete question 17 using **cooperative task groups**, a checkerboard, and checker for each group. Or**,** complete the question as a class by constructing a large checkerboard on the floor with masking tape and have a student act as the checker. Assign a recorder to keep track of the total number of pathways as the student (checker) moves down the grid.

Investigate Answers (page 373)

1. a) $a + b$

b) $a^2 + 2ab + b^2$

c) $a^3 + 3a^2b + 3ab^2 + b^3$

d) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^3$

2. Answers may vary. Sample answer: The coefficients of the terms of the binomial expansions in question 1 are the same as the numbers in Pascal's triangle as follows: part a) row 1 of Pascal's triangle, part b) row 2 of Pascal's triangle, part c) row 3 of Pascal's triangle, and part d) row 4 of Pascal's triangle.

3. Answers may vary. Sample answer: The degree of each term in the expansion for each row is the same as the power of the binomial.
For row 1, representing the binomial expansion of $(a + b)^1$, the degree of each term is 1.
For row 2, representing the binomial expansion of $(a + b)^2$, the degree of each term is 2.
For row 3, representing the binomial expansion of $(a + b)^3$, the degree of each term is 3.
For row 4, representing the binomial expansion of $(a + b)^4$, the degree of each term is 4.

4. Answers may vary. Sample answer: $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$.

Communicate Your Understanding Responses (page 377)

C1 Answers may vary. Sample answer: The term $t_{6,\,3}$ is the sum of the terms immediately above it, $t_{5,\,2} + t_{5,\,3}$. The term $t_{3,\,6}$ is not a term in Pascal's triangle and therefore will not have a value.

C2 Answers may vary. Sample answer: Starting at the top, find the correct row number and locate the designated term in the diagonal row number position.

C3 Answers may vary.

C4 Answers may vary. Sample answer: Since the exponent is 8, the coefficients occur in row 8 of Pascal's triangle. The powers of a will decrease and the powers of b will increase.

$(a + b)^8$
$= 1a^8b^0 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + 1a^0b^8$
$= a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$

Common Errors

- Students have trouble simplifying terms in complex binomial expansions.

R_x Have students practise expanding a single term, such as $35(2a)^4\left(\frac{1}{a^2}\right)^3$.

- Students cannot locate some of the sequences they need to find in Pascal's triangle.

R_x Seeing this right triangle version of Pascal's triangle may help students to see patterns more clearly.

$t_{0,0}$
$t_{1,0}$ $t_{1,1}$
$t_{2,0}$ $t_{2,1}$ $t_{2,1}$
$t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$
$t_{4,0}$ $t_{4,1}$ $t_{4,2}$ $t_{4,3}$ $t_{4,4}$
…

Practise, Connect and Apply, Extend

- Before having students complete **question 1**, it may help to demonstrate an example for a hockey stick on an overhead model of Pascal's triangle.
- **Question 7** encourages students to reason through and reflect on patterns to determine the value of k in each given term of $(x + y)^{12}$. When they recognize the patterns, they need to select tools and use connecting skills with past mathematical knowledge to determine these values of k.
- In **question 10**, the hexagonal diagrams represent sections taken from Pascal's triangle. Encourage students to use their copy of the triangle to help them locate these sections.
- For **question 11**, students should be expected to make a 10–16 row version of Pascal's triangle.
- Make sure students know the terms of the Fibonacci sequence before completing **question 12**.
- **Question 13** enables students to reflect on, reason through, and connect knowledge learned in this chapter with previously learned material to solve the problem of determining the sum of the squares of the terms in the horizontal rows of Pascal's triangle. They must select appropriate tools and use connecting skills to represent the sequence by a recursion formula.
- Students may benefit from working with a partner on **question 14**. This will enable them to discuss possible strategies for solving the problem.
- Provide checkerboards to help students visualize the context of **question 17**.
- Use **BLM 6–6 Section 6.3 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13–16
Reasoning and Proving	3–16
Reflecting	7, 10–16
Selecting Tools and Computational Strategies	7, 13–16
Connecting	5, 7, 8, 10–16
Representing	4, 9, 13, 15
Communicating	11, 12, 15, 16