6.4 Arithmetic Sequences

Student Text Pages
380 to 387

Suggested Timing
60–75 min

Tools
- grid paper
- computer with spreadsheet software
- TI-Nspire™ CAS graphing calculator (optional)

Related Resources
- G–1 Grid Paper
- T–1 Microsoft® *Excel*
- BLM 6–7 Section 6.4 Practice

Teaching Suggestions

- There are many interesting mathematical and historical facts to be discovered about the building of the Great Pyramid. You may wish to spend time helping students to explore this structure, which is the last of the "Seven Wonders of the Ancient World" still in existence.
- After completing the **Investigate** and examples, you might encourage students to discover arithmetic sequences involved with the Great Pyramid.
- Students may want to draw or build a model of a section of the wall they are exploring in the **Investigate**. Encourage them to work with a partner and to share the formulas they develop with their classmates.
- As an alternative for **Method 2** of the **Investigate**, students could use Microsoft® *Excel*. Enter the given formulas in **A3**, **B3**, and **C3**. Highlight the entered row by holding down the left-click of the mouse and placing the cursor over the bottom right corner of box C3 until a bolded plus sign appears. Then drag downward. If needed, use **T–1 Microsoft® *Excel*** to support this activity.
- Challenge students to develop a definition of an arithmetic sequence based on their observations of the two sequences created in the **Investigate**.
- Students should have a clear understanding of the properties of an arithmetic sequence before they start work on the examples. Use the general form of the terms in an arithmetic sequence, $a, a + d, a + 2d, a + 3d, \ldots$, to help students discover the general formula: $t_n = a + (n - 1)d$.
- In **Example 1** part **c**), make sure students realize that they need to use the formula to determine the first few terms before they can find the common difference.
- Before completing **Example 4**, you may need to review how to solve a linear system with students.

Differentiated Instruction

- Use **concept attainment** to define the concept of an arithmetic sequence. Provide examples and non-examples of arithmetic sequences in graphical, table, equation, and written form. Use a **journal** to summarize the key concepts.
- Construct a **word wall** and include the terms *arithmetic sequence*, *common difference*, *geometric sequence*, *constant ratio*, *arithmetic series*, and *geometric series* as they arise in the chapter. Also include the formula for the general term of an arithmetic or geometric sequence, and the formula for arithmetic and geometric series on the **word wall**.

Investigate Answers (pages 380–382)

Method 1

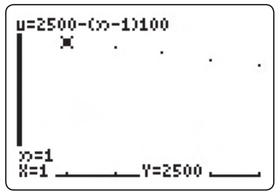
1. a)

Row Number	Number of Blocks in the Row	Row Length (cm)
1	2500	100 000
2	2400	96 000
3	2300	92 000
4	2200	88 000
5	2100	84 000

b) 18 rows; Answers may vary. Sample answer: The height of the wall at the centre is 3.6 m, which is 3600 cm. Each cinder block is 20 cm in height. Divide the height of the wall at the centre by the height of each cinder block to find that the number of blocks in the top row of the wall is 18.

2. a) 2500, 2400, 2300, 2200, 2100, …

b)

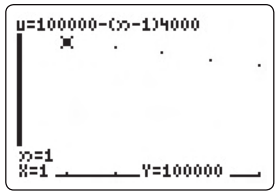


c) $NB(n) = 2500 - (n - 1)(100)$, where NB is the number of blocks in the row.

d) $n = 18$; $f(18) = 800$

3. a) 100 000, 96 000, 92 000, 88 000, 84 000, …

b)



c) $RL(n) = 100\ 000 - (n - 1)(4000)$, where RL is the row length in centimetres.

d) length of row = 32 000 cm

4. a) Answers may vary. Sample answer: Both graphs are decreasing linear functions. The sequences in both cases are arithmetic sequences and are therefore discrete functions.

b) Answers may vary. Sample answer: The first term of each sequence is the starting number in both formulas. The constant difference for each sequence is a negative number. Both formulas calculate the values by starting with a number and subtracting the common difference from each term to find each subsequent term.

Method 2

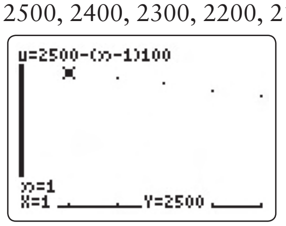
1. a)

Row	Number of Blocks in the Row	Row Length (cm)
1	2500	100 000
2	2400	96 000
3	2300	92 000
4	2200	88 000
5	2100	84 000

b) 18 rows; Answers may vary. Sample answer: The height of the wall at the centre is 3.6 m which is 3600 cm. Each cinder block is 20 cm in height. Divide the height of the wall at the centre by the height of each cinder block to find that the number of blocks in the top row of the wall is 18.

2. a) 2500, 2400, 2300, 2200, 2100, …

b)

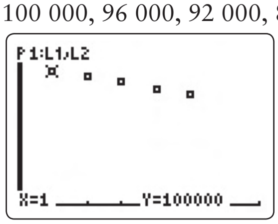


c) $NB(n) = 2500 - (n - 1)(100)$, where NB is the number of blocks in the row.

d) $n = 18$; $f(18) = 800$

3. a) 100 000, 96 000, 92 000, 88 000, 84 000, …

b)



c) $RL(n) = 100\ 000 - (n - 1)(4000)$, where RL is the row length in centimetres.

d) length of row = 32 000 cm

4. a) Answers may vary. Sample answer: Both graphs are decreasing linear functions. The sequences in both cases are arithmetic sequences and are therefore discrete functions.

b) Answers may vary. Sample answer: The first term of each sequence is the starting number in both formulas. The constant difference for each sequence is a negative number. Both formulas calculate the values by starting with a number and subtracting the common difference from each term to find each subsequent term.

Communicate Your Understanding Responses (page 385)

C1 Answers may vary. Sample answer: The first sequence is an arithmetic sequence. The first term is $a = 1$, and the common difference is $d = 2$. The second sequence is not an arithmetic sequence. The first term is $a = 2$, but there is no common difference between the consecutive terms.

C2 Answers may vary. Sample answer: The first term in an arithmetic sequence is a and the common difference is d. Each subsequent term in the arithmetic sequence can be found by adding the common difference multiplied by the term number less 1 to the first term using the formula $t_n = a + (n - 1)d$. For example, the fifth term in an arithmetic sequence would be $t_5 = a + 4d$.

Common Errors

- Students have difficulty recognizing t_n as the last term in a finite sequence.

R_x Before attempting question 7, have students write out a sequence with the term numbers to show that t_n is the final term.

Practise, Connect and Apply, Extend

- For **question 8**, remind students that *verify* means to show that something is always true. Although this will not be a formal proof, students should be prepared to give a thoughtful and organized solution including a conclusion.
- **Question 8** gives students the opportunity to reason out and reflect on how to determine whether the sequence formed by a given recursion formula is arithmetic. They need to select tools and use connecting skills to verify their findings.
- In **question 13**, students may decide to keep subtracting $500 from the $10 000 initial prize. Since the numbers in this sequence are easy to work with, this is a natural and suitable way to do this problem. If students choose this method, have them check their solution using the formula.
- **Question 18** requires students to use reasoning and reflecting skills to connect material learned in this chapter with previously learned concepts in order to establish whether the given recursion formulas produce arithmetic sequences. They must communicate the method that they used to identify an arithmetic sequence from a given recursion formula.
- Use **BLM 6–7 Section 6.4 Practice** for remediation or extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	22
Reasoning and Proving	6–8, 10–22
Reflecting	8, 11, 16–19, 21, 22
Selecting Tools and Computational Strategies	5–15, 17, 21, 22
Connecting	1–22
Representing	3, 5, 10–12, 21
Communicating	2, 13, 18