# 6.5 Geometric Sequences

**Student Text Pages**
388 to 394

**Suggested Timing**
60–75 min

**Tools**
- grid paper
- large grid paper sheets and markers

**Related Resources**
- G–1 Grid Paper
- BLM 6–8 Section 6.5 Practice

## Teaching Suggestions

- In a class discussion, help students understand the meaning of the becquerel. Although some students who have studied physics may know this unit, most students will not.
- You might search the Internet for a suitable video of a heart procedure like the one discussed in the introduction to this lesson.
- Step 3 of the **Investigate** gives students an opportunity to use knowledge from Section 6.4 and helps them make connections between the topics.
- Ask students to explain what the pattern in the first differences can tell them about the function that represents this sequence.
- As you observe students working through the steps of the **Investigate**, guide them to multiply the terms in the sequence by the common difference.
- Before having students work through the examples, be sure they know the definitions of arithmetic and geometric sequences, and that they realize that these are two special kinds of sequences. In general, most sequences are neither arithmetic nor geometric.
- Before introducing **Example 2**, be sure students can identify the first term and the common difference in a geometric sequence.
- The **Communicate Your Understanding** questions give students the opportunity to make important connections between the various representations of geometric sequences.

**Differentiated Instruction**

- Use **Think-Pair-Share** to complete the Investigate. Have one student perform the calculations, and the other record the data. Discuss the results.
- Use **concept attainment** to define the concept of a geometric sequence. Provide examples and non-examples of geometric sequences in graphical, table, equation, and written form. Use a **journal** to summarize the key concepts.
- Use **think-aloud** and have pairs or groups of students work through questions C1 to C3 with the class.
- Complete question 15 using **cooperative task groups**. Have one student perform the calculations and another record the data for each option. Graph each option using technology and compare the rate of change on each graph.

**Investigate Answers (pages 388–389)**

**1.**

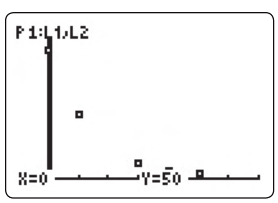
| Time (73-h periods) | Amount of Thallium-201 (MBq) | First Differences |
|---|---|---|
| 0 | 50 | |
| 1 | 25 | −25 |
| 2 | 12.5 | −12.5 |
| 3 | 6.25 | −6.25 |
| 4 | 3.125 | −3.125 |
| 5 | 1.5625 | −1.5625 |

**2.** 50, 25, 12.5, 6.25, 3.125, 1.5625, …

**3.** Answers may vary. Sample answer: No, this is not an arithmetic sequence. The first term is 50, but there is not a common difference between each term in the sequence.

**4.** Answers may vary. Sample answer: The first first difference, calculated by subtracting the amount of thallium-201 for term 4 from the amount of thallium-201 for term 5 is −1.5625. Each of the consecutive first differences calculated in the table is twice the previous first difference.

**5.**



**6.** Answers may vary. Sample answer: When each term after the first term is divided by the previous term, the answer is always a common ratio, $\frac{1}{2}$.

**7.** $t_1 = 50\left(\frac{1}{2}\right)^1$; $t_2 = 50\left(\frac{1}{2}\right)^2$; $t_3 = 50\left(\frac{1}{2}\right)^3$; $t_4 = 50\left(\frac{1}{2}\right)^4$; $t_5 = 50\left(\frac{1}{2}\right)^5$; $t_n = 50\left(\frac{1}{2}\right)^n$

**8.** 12.3 years

**Communicate Your Understanding Responses (page 392)**

**C1** Answers may vary. Sample answer: An arithmetic sequence is formed by adding a common difference to each previous term; for example, 3, 5, 7, … is an arithmetic sequence. The first term is 3 and the common difference is 2. A geometric sequence is formed by multiplying the previous term by a common ratio; for example, 2, 6, 18, … is a geometric sequence. The first term is 2 and the common ratio is 3. A sequence is neither arithmetic nor geometric if it is not formed by adding a common difference or by multiplying each term by a common ratio. For example, 1, 5, 18, … is neither an arithmetic nor a geometric sequence.

**C2** Answers may vary. Sample answer: The first term in the geometric sequence is $a = 5$. The common ratio is $d = -2$. Substitute these values in the general term for a geometric sequence, $t_n = ar^{n-1}$ to determine the formula for the general term, $t_n = 5(-2)r^{n-1}$.

**C3** Answers may vary. Sample answer: The sequence represented by the first graph is an arithmetic sequence. The sequence is 1, 3, 5, 7, 9, … . The first term is $a = 1$ and the common difference is $d = 2$. The sequence represented by the second graph is a geometric sequence. The sequence is 1, 2, 4, 8, 16, … . The first term is $a = 1$ and the common ratio is $d = 2$.

## Practise, Connect and Apply, Extend

- Remind students of what a formula for a general term represents before they attempt **question 3**.
- Generate interest in **question 9** by discussing the causes and effects of food poisoning with students. Visit the McGraw-Hill Ryerson Web site *www.mcgrawhill.ca/books/functions11* for links to some useful information.
- **Question 10** allows students to select tools and use connecting skills to determine the amount of cesium-137 (Cs-137) that exists per square kilometre if specific information about the release of this radioactive substance is given. They must examine the half-life of Cs-137 and solve the problem of writing an explicit formula that represents the level of Cs-137 left after *n* years. They are required to use reasoning and reflecting skills to solve this problem. These skills are also needed to search the Internet to research the dangerous contamination that the explosion of a Chernobyl nuclear reactor caused in 1986. Students must use communicating skills to demonstrate the research they have done.
- Extend **question 14** by having students determine how many computers could be infected by a virus in a chain e-mail message if you assume people check their e-mail four times per day.
- For **question 12**, provide chart-sized grid paper for students to complete their Sierpinski carpet drawings. Display these drawings in the classroom. Students may require additional scaffolding in the form of a chart to record their observations as they draw five stages of this fractal.
- **Question 13** requires students to select tools and use reasoning and connecting skills to develop an equation that represents a model for the number of voters at an election in a specific country and to graph the related function. They must also use reasoning skills to establish whether this function is continuous or discontinuous, and use communicating skills to explain their findings.

**Common Errors**

- Students have difficulty with operations with fractions.

**$R_x$** Have students practise calculations involving rational bases or rational coefficients.

- Students have difficulty with the concept of a rational value of the common ratio.

**$R_x$** If the common ratio is $\frac{3}{4}$, encourage students to change this to the decimal equivalent and then perform the multiplication. They must, however, revert back to the rational values for the sequence. The process can be repeated with the rational value as the multiplier. Have students reflect on this connection.

- Students may benefit from another example of a geometric mean before attempting **question 18**. They should reflect on the relationship between numbers, their geometric means, and a geometric sequence.
- **Question 20** allows students to extend their thinking about geometric sequences to a more abstract algebraic example.
- Use **BLM 6–8 Section 6.5 Practice** for remediation or extra practice.

**Ongoing Assessment**

Achievement Check, question 15, on student text page 394.

**Achievement Check, question 15, student text page 394**

This performance task is designed to assess the specific expectations covered in Section 6.5. The following mathematical process expectations can be assessed.

- Reflecting
- Connecting
- Representing
- Communicating

**Sample Solution**

Write the first terms of each option and calculate the differences between consecutive terms.

**Option 1**

| Term | Amount ($) | Difference |
|---|---|---|
| 1 | 25 | |
| | | 25 − 25 = 0 |
| 2 | 25 | |
| | | 25 − 25 = 0 |
| 3 | 25 | |
| | | 25 − 25 = 0 |
| 4 | 25 | |

**Option 2**

| Term | Amount ($) | Difference |
|---|---|---|
| 1 | 0.25 | |
| | | 0.50 − 0.25 = 0.25 |
| 2 | 0.50 | |
| | | 1.00 − 0.50 = 0.50 |
| 3 | 1.00 | |
| | | 2.00 − 1.00 = 1.00 |
| 4 | 2.00 | |

Option 1 forms an arithmetic sequence since the differences between consecutive terms are constant (0).

Option 2 does not form an arithmetic sequence since the differences between consecutive terms are not constant. An arithmetic sequence can be written as $a$, $a + d$, $a + 2d$, $a + 3d$, … , where $a$ is the first term and $d$ is the common difference.

For option 1, $a = \$25.00$ and $d = 0$.

The general term for this sequence is

$$\begin{aligned} t_n &= a + (n - 1)d \\ &= 25.00 + (n - 1)0 \\ &= 25.00 \end{aligned}$$

**b)** Write the first terms of each option and determine the ratio between each pair of consecutive terms.

**Option 1**

| Term | Amount ($) | Ratio |
|---|---|---|
| 1 | 25 | |
| | | $\frac{25}{25} = 1$ |
| 2 | 25 | |
| | | $\frac{25}{25} = 1$ |
| 3 | 25 | |
| | | $\frac{25}{25} = 1$ |
| 4 | 25 | |

**Option 2**

| Term | Amount ($) | Ratio |
|---|---|---|
| 1 | 0.25 | |
| | | $\frac{0.50}{0.25} = 2$ |
| 2 | 0.50 | |
| | | $\frac{1.00}{0.50} = 2$ |
| 3 | 1.00 | |
| | | $\frac{2.00}{1.00} = 2$ |
| 4 | 2.00 | |

Option 1 forms a geometric sequence since the ratio between each pair of consecutive terms is the same.

Option 2 also forms a geometric sequence for the same reason. A geometric sequence can be written as $a$, $ar$, $ar^2$, $ar^3$, … , where $a$ is the first term and $r$ is the common ratio.

For option 1, $a = 25.00$ and $r = 1$.
The general term for this sequence is

$$\begin{aligned} t_n &= ar^{n-1} \\ &= 25(1^{n-1}) \\ &= 25 \end{aligned}$$

For option 2, $a = 0.25$ and $r = 2$.
The general term for this sequence is

$$\begin{aligned} t_n &= ar^{n-1} \\ &= 0.25(2^{n-1}) \end{aligned}$$

**c)** With option 1, Aika would receive \$25 each week for 52 weeks, or \$1300.
With option 2, Aika would receive less for the first seven weeks (\$0.25, \$0.50, \$1.00, \$2.00, \$4.00, \$8.00, \$16.00), but from there onward the amount she received each week would be more than \$25, and would grow rapidly (\$32.00, \$64.00, \$128.00, …) up to an impossible sum of money. Aika's dad should choose option 1, as this would involve much less money.

## Level 3 Notes

Look for the following:

- Demonstrates understanding and justification of why one sequence is arithmetic and one is geometric
- Determines the general term of each sequence with considerable accuracy
- Communicates steps of solutions with considerable clarity and uses proper mathematical form most of the time

## What Distinguishes Level 2

- Demonstrates little understanding and justification of why one sequence is arithmetic and one is geometric
- Determines the general term of each sequence with some accuracy
- Communicates steps of solutions with some clarity and uses proper mathematical form some of the time

## What Distinguishes Level 4

- Demonstrates clear understanding and justification of why one sequence is arithmetic and one is geometric
- Determines the general term of each sequence with a high degree of accuracy
- Communicates steps of solutions with a high degree of clarity and uses proper mathematical form consistently

### Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectation | Selected Questions |
|---|---|
| Problem Solving | 10–12, 15–20 |
| Reasoning and Proving | 1, 5–20 |
| Reflecting | 9–12, 15–20 |
| Selecting Tools and Computational Strategies | 1, 3, 5–20 |
| Connecting | 1–20 |
| Representing | 3, 10, 12, 13, 20 |
| Communicating | 10, 12, 13, 15 |