6.6 Arithmetic Series

Student Text Pages
395 to 401

Suggested Timing
60–75 min

Tools
- graphing calculator

Related Resources
- BLM 6–9 Section 6.6 Practice

Differentiated Instruction
- Start the section using **blast off**. Have the class record three things they learned about arithmetic sequences, two numerical examples of arithmetic sequences, and one real-life scenario that follows an arithmetic sequence.
- Use **Think-Pair-Share** to complete the Investigate. Have one student perform the calculations, and the other record the data. Discuss the results.

Teaching Suggestions

- Students may be interested in some of the other stunts performed by Dar Robinson. There are many sites dedicated to this great stuntman.
- As students start the **Investigate**, you may need to clarify the term *mean*, as they may be more familiar with the term *average*.
- For **Investigate** step 5, help students understand what it means to add the two series together. They are adding the columns together to come up with another sum.
- Students may find it difficult to follow the process of adding the general form of the series to itself. They may benefit from having you work through this on the board as they assist.
- Have students work with a partner through the **Communicate Your Understanding** section. It would be valuable for students to share their ideas for C2 and C3 with the whole class.
- For **Example 1, Method 2,** a TI-83 Plus or TI-84 Plus graphing calculator is used to generate an arithmetic sequence.

Investigate Answers (page 395)

1.

Number of Terms	Indicated Sum	Sum	Mean of All Terms	Mean of First and Last Terms
1	1	1	1	1
2	2 + 4	6	3	3
3	2 + 4 + 6	12	4	4
4	2 + 4 + 6 + 8	20	5	5
5	2 + 4 + 6+ 8 + 10	30	6	6

2. Answers may vary. Sample answer: The sum of the terms is equal to the number of terms multiplied by the mean of all the terms. The mean of the first and last terms is the same as the mean of all the terms.

3. a) $S_n = \frac{n}{2}(t_1 + t_n)$

b) $S_{100} = \frac{100}{2}(2 + 200) = 10\,100$

4. $S_6 = \frac{6}{2}(1 + 11) = 36$

5. $\frac{14 + 14 + 14 + 14 + 14 + 14}{2} = 42$

6. Answers may vary. Sample answer: Using the formula to calculate the sum of the terms in Q5, the result is $S_6 = \frac{6}{2}(2 + 12) = 42$; The results are the same in both cases.

Communicate Your Understanding Responses (page 399)

C1 Answers may vary. Sample answer: The first term and the consecutive terms in an arithmetic sequence and an arithmetic series are the same. The terms in an arithmetic sequence have commas between them. The terms in an arithmetic series have addition signs between them.

C2 Answers may vary. Sample answer: It is better to use the formula $S_n = \frac{n}{2}(a + t_n)$ when you are required to determine the sum of an arithmetic series and are given the first term, the last term, and the number of terms in the series. It is better to use the formula $S_n = \frac{n}{2}[a + (n - 1)d]$ when you are required to determine the sum of an arithmetic series and are given the first term, the common difference, and the number of terms in the series.

C3 Answers may vary. Sample answer: A real-life situation that could be defined by an arithmetic series is a situation in which you are offered two jobs with different hourly starting wages and different increases in hourly wages that would have the same amount of total hours. You could use the arithmetic series formula to calculate the total wages you would earn if you worked either job and this could be one of the factors that would help you make your decision about which job to accept.

C4 Answers may vary.

Common Errors

- Students have trouble determining the common difference for a series in which the terms are algebraic expressions.

R_x Have students practise this application with some simpler examples before attempting question 14.

- Students confuse the terms *sequence* and *series*.

R_x Give students an opportunity to clarify these terms by writing the definition in their notebooks along with a few examples.

Practise, Connect and Apply, Extend

- Remind students they need to determine the value of n before completing **questions 4** and **5**.
- For **question 8**, remind students how to add and subtract expressions with radicals.
- **Question 7** gives information about a specific series. Students are required to use that information when selecting tools, and to use reasoning, reflecting, and connecting skills to determine the sum of the first 50 terms of this series.
- Encourage students to draw a diagram to assist them with **question 10**, or have foam cups on hand so they can build a model of the pyramid.
- **Question 14** gives students the opportunity to reason through and reflect on the first three terms of a given arithmetic sequence. By selecting tools and connecting concepts learned in this chapter with previously learned mathematical material, they are required to determine the value of x as it is seen in those terms and to establish the sum of the first ten terms of the sequence.
- It is important for students to explain their thinking about mathematics in words. **Question 15** provides a good opportunity for students to practise this skill. It can be helpful for them to see how others explain things, so you may consider having students volunteer to write their explanations on the board or read them aloud.
- It is important to continue to reinforce the relationships among the different representations of a series. **Question 16** has students construct a graph after they determine the terms in the series.
- Use **BLM 6–9 Section 6.6 Practice** for remediation or extra practice.

Ongoing Assessment

Achievement Check, question 15, on student text page 401.

Achievement Check, question 15, student text page 401

This performance task is designed to assess the specific expectations covered in Section 6.6. The following mathematical process expectations can be assessed.

- Reflecting
- Connecting
- Representing
- Communicating

Sample Solution

a) The terms of the sequence are 1200, 1400, 1600, …

This represents an arithmetic sequence with first term $a = 1200$ and the common difference $d = 200$. The corresponding general term is

$$\begin{aligned} t_n &= 1200 + (n - 1)(200) \\ &= 1200 + 200n - 200 \\ &= 1000 + 200n \end{aligned}$$

Since the terms form an arithmetic sequence, the total profit corresponds to the sum of the sequence, which represents an arithmetic series.

b) The total profit for 16 weeks is found by calculating S_{16}.

Substitute $a = 1200$, $d = 200$, and $n = 16$ in $S_n = \frac{n}{2}[2a + (n - 1)d]$.

$$\begin{aligned} S_{16} &= \frac{16}{2}[2(1200) + (16 - 1)(200)] \\ &= 8(2400 + 3000) \\ &= 43\ 200 \end{aligned}$$

The total profit for the season is $43 200.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	12, 13, 15–18
Reasoning and Proving	6, 7, 9–18
Reflecting	7, 10–18
Selecting Tools and Computational Strategies	1–8, 10–18
Connecting	1–18
Representing	16
Communicating	9, 15

6.7 Geometric Series

Student Text Pages
402 to 409

Suggested Timing
60–75 min

Tools
- grid paper
- counters
- computer with spreadsheet software
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 6–10 Section 6.7 Practice
- BLM 6–11 Section 6.7 Achievement Check Rubric

Teaching Suggestions

- To introduce the **Investigate**, explain the problem to students and ask them to guess how much money there will be on the board altogether. You could give a small prize to the student with the closest guess.
- This is an excellent example to help students understand the difference between continuous and discrete functions. Both the number of nickels and the amount of money on each square are discrete values.
- When students are working on step 5, you may want to model the number of nickels as a series, since students may wish to write this as a sequence.
- Students may need support as they try to understand the derivation of the formula for a geometric series. Students could work with a partner to try to add the two expressions together. This will help them understand the process better than if they just watch it done on the board or overhead.
- Before students start **Example 1**, be sure they understand the connection between a geometric sequence and a geometric series. They need to be aware that both have the same values for the first term, the common difference, and the number of terms.
- Demonstrate the use of the formula with a supplementary example before students begin **Example 2**.
- In the **Communicate Your Understanding** section, students are asked to make important connections between arithmetic and geometric series. Describing these similarities and differences should help students to consolidate their understandings.