b) The total profit for 16 weeks is found by calculating S_{16}.

Substitute $a = 1200$, $d = 200$, and $n = 16$ in $S_n = \frac{n}{2}[2a + (n - 1)d]$.

$$S_{16} = \frac{16}{2}[2(1200) + (16 - 1)(200)]$$
$$= 8(2400 + 3000)$$
$$= 43\ 200$$

The total profit for the season is $43 200.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	12, 13, 15–18
Reasoning and Proving	6, 7, 9–18
Reflecting	7, 10–18
Selecting Tools and Computational Strategies	1–8, 10–18
Connecting	1–18
Representing	16
Communicating	9, 15

6.7 Geometric Series

Student Text Pages
402 to 409

Suggested Timing
60–75 min

Tools
- grid paper
- counters
- computer with spreadsheet software
- graphing calculator

Related Resources
- G–1 Grid Paper
- BLM 6–10 Section 6.7 Practice
- BLM 6–11 Section 6.7 Achievement Check Rubric

Teaching Suggestions

- To introduce the **Investigate**, explain the problem to students and ask them to guess how much money there will be on the board altogether. You could give a small prize to the student with the closest guess.
- This is an excellent example to help students understand the difference between continuous and discrete functions. Both the number of nickels and the amount of money on each square are discrete values.
- When students are working on step 5, you may want to model the number of nickels as a series, since students may wish to write this as a sequence.
- Students may need support as they try to understand the derivation of the formula for a geometric series. Students could work with a partner to try to add the two expressions together. This will help them understand the process better than if they just watch it done on the board or overhead.
- Before students start **Example 1**, be sure they understand the connection between a geometric sequence and a geometric series. They need to be aware that both have the same values for the first term, the common difference, and the number of terms.
- Demonstrate the use of the formula with a supplementary example before students begin **Example 2**.
- In the **Communicate Your Understanding** section, students are asked to make important connections between arithmetic and geometric series. Describing these similarities and differences should help students to consolidate their understandings.

Differentiated Instruction

- Start the section using **blast off**. Have the class record three things they learned about geometric sequences, two numerical examples of geometric sequences, and one real-life scenario that follows a geometric sequence.
- Use **Think-Pair-Share** to complete the Investigate. Have one student read the spreadsheet instructions while the other enters the data. Discuss the results.
- Use a **what-so-what double entry** chart to compare the similarities and differences of arithmetic and geometric series.
- Use **timed retell** for question C2.
- Have one group use **think-aloud** to work through question 14 with the class.
- Use **four corners** to review arithmetic and geometric sequences and series. Assign one of the four concepts to each corner of the room. Hold up large cue cards, or reveal questions on an overhead, and have students move to the corner that represents each kind of problem.
- Have students summarize the **four corners** activity using a modified **placemat**. Divide a piece of paper into quadrants. Include the definition, the formula, and an example of each concept on the **placemat**.

Investigate Answers (pages 402–403)

Method 1

1.

Number of Squares	Number of Nickels	Total Number of Nickels
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255
9	256	511
10	512	1 023
11	1 024	2 047
12	2 048	4 095
13	4 096	8 191
14	8 192	16 383
15	16 384	32 767
16	32 768	65 535
17	65 536	131 071
18	131 072	262 143
19	262 144	524 287
20	524 288	1 048 575
21	1 048 576	2 097 151
22	2 097 152	4 194 303
23	4 194 304	8 388 607
24	8 388 608	16 777 215
25	16 777 216	33 554 431
26	33 554 432	67 108 863
27	67 108 864	134 217 727
28	134 217 728	268 435 455
29	268 435 456	536 870 911
30	536 870 912	1 073 741 823
31	1 073 741 824	2 147 483 647
32	2 147 483 648	4 294 967 295

2. 2 147 483 648 nickels; 4 294 967 295 nickels

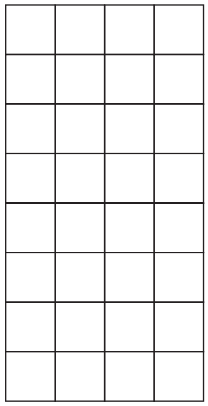
3. $S_n = 1(2^n - 1)$; 4 294 967 295 nickels

4. Answers may vary. Sample answer: The function is discrete. The numbers in the domain correspond to the number of squares on the game board. The numbers are integer values and the numbers in the range will also be integer values.

5. $1 + 2 + 4 + 8 + 16 + \dots$; The series is not arithmetic. There is a common ratio between successive terms in the series. Therefore the series is geometric.

Method 2

1.



2.

Number of Squares	Number of Nickels on the Square	Total Number of Nickels
1	1	1
2	2	3
3	4	7

3. $S_n = 1(2^n - 1)$; 4 294 967 295 nickels

4. Answers may vary. Sample answer: The function is discrete. The numbers in the domain correspond to the number of squares on the game board. The numbers are integer values and the numbers in the range will also be integer values.

5. $1 + 2 + 4 + 8 + 16 + \ldots$; The series is not arithmetic. There is a common ratio between successive terms in the series. Therefore the series is geometric.

Communicate Your Understanding Responses (page 407)

C1 Answers may vary. Sample answer: An arithmetic series is the sum of the terms in an arithmetic sequence. A geometric series is the sum of the terms in a geometric sequence. The terms in the arithmetic series have a common difference. The terms in a geometric series have a common ratio.

C2 Answers may vary.

C3 The expression is undefined when $r = 1$, since the denominator would equal zero and division by zero is not defined in the real number system.

Practise, Connect and Apply, Extend

- Remind students of the relationship between geometric series and geometric sequences before they begin to work on **question 4**.
- Make sure students are clear that they must solve for n in **question 8**.
- **Question 11** requires students to use reasoning and reflecting skills to solve the problem of writing a function that models the total amount of prize money to be given away in a lottery. Students must select appropriate tools and use connecting skills to produce a graph that represents the function, and to determine the number of prizes that can be given out based on a specific amount of prize money.
- **Question 12** enables students to reflect on how a ball bounces in a specific situation. They must select tools and use reasoning and connecting skills to determine the height reached by the ball after the sixth bounce and the total distance travelled by the ball after ten bounces.
- When students are ready to complete **question 13**, provide information about the features of a “space-filling” or Peano curve. There are many variations of this type of curve.
- Use **BLM 6–10 Section 6.7 Practice** for remediation or extra practice.

Common Errors

- Students use the formula for a geometric series incorrectly. Many students find this formula difficult to learn, and they may have the most trouble when the terms are rational numbers, as this leads to simplification of complex fractions.

R_x Have students practise simplifying some complex fractions before working with the formula on questions similar to those in Example 3. Provide supplementary practice on this skill before students work on question 6.

Ongoing Assessment

Achievement Check, question 14, on student text page 409.

Achievement Check, question 14, student text page 409

This performance task is designed to assess the specific expectations covered in Section 6.7. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing
- Communicating

Sample Solution

Provide students with **BLM 6–11 Section 6.7 Achievement Check Rubric** to help them understand what is expected.

a) $t_1 = 40$

$t_2 = 40(0.75)$
$= 30$

$t_3 = 30(0.75)$
$= 22.5$

$t_4 = 22.5(0.75)$
$= 16.88$

In the next three minutes, the balloon rises 30 m, 22.50 m, and 16.88 m.
The terms of the sequence for the first four minutes are 40, 30, 22.50, 16.88.
This is a geometric sequence. The general term is $t_n = ar^{n-1}$, where $a = 40$ and $r = 0.75$.
Thus, $t_n = 40(0.75)^{n-1}$.

b) The height of the balloon after n minutes corresponds to the sum of the geometric sequence in part a).

$S_n = \dfrac{a(r_n - 1)}{r - 1}$ Substitute $a = 40$, $r = 0.75$

$S_n = \dfrac{40(0.75n - 1)}{0.75 - 1}$
$= -160(0.75^n - 1)$

This function is continuous because decimal values make sense (or are permissible) for n, which represents time in this situation. For instance, if $n = 0.5$, this would represent half a minute. The domain is $\{n \in \mathbb{R}, n \geq 0\}$.

c) Substitute $n = 10$ to determine the height of the balloon after 10 min.

$S_{10} = -160(0.75^{10} - 1)$
$\doteq 151$

After 10 min, the height of the balloon is about 151 m.

Mathematical Processes

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 11, 13, 15–19
Reasoning and Proving	4, 5, 7–19
Reflecting	9–19
Selecting Tools and Computational Strategies	2–19
Connecting	1–19
Representing	10, 11, 13, 14
Communicating	1, 13, 14