Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	13, 14
Reasoning and Proving	2, 3, 5–14
Reflecting	11, 13, 14
Selecting Tools and Computational Strategies	1, 2, 4, 7–14
Connecting	1–14
Representing	2, 4–7, 12
Communicating	4, 12

7.2 Compound Interest

Student Text Pages
426 to 435

Suggested Timing
75 min

Tools
- graphing calculator
- computer with graphing software
- TI-Nspire™ CAS graphing calculator (optional)

Related Resources
- BLM 7–3 Section 7.2 Practice

Differentiated Instruction
- Add the terms *compound interest* and *compounding period* to the **word wall**.
- Use **Think-Pair-Share** to complete the Investigate. One partner reads the instructions while the other calculates and enters data.
- Use **jigsaw** to compare various compounding periods. Assign the same problem to each group member. Assign each a different compounding period (annual, semi-annual, quarterly, and monthly) for the problem. Have group members compare and summarize their results in a **journal**.

Teaching Suggestions

- In the **Investigate**, students discover the nature of compound interest. Students should recognize that the amount in a compound interest account grows exponentially. This is in contrast to the linear nature of simple interest growth. Have students work in pairs or groups of three or four. Allow 15 min for the activity. For step 6a) students should consider the first differences.
- The discourse following the investigation provides a general algebraic development of the compound interest formula. This derivation is dependent on an understanding of geometric sequences.
- In **Example 1**, students should begin to understand that the difference between compound interest and the corresponding simple interest becomes more significant as the interest rate or time period increases. Point out to students that the simple interest formula leads to an *interest* calculation from which the amount can then be determined, while the compound interest formula leads to an *amount* calculation from which interest can be determined.
- For **Example 2**, it is important for students to remember to multiply the number of years by the number of compounding periods per year, and to divide the annual interest rate by the number of compounding periods per year prior to substituting into the formula.
- In **Example 3**, two methods for determining annual interest rate are shown. The first uses graphing technology. This method can be used to solve a number of equations that may be difficult to handle algebraically. Refer to the Technology appendix for additional support for the TI-83 Plus/TI-84 Plus graphing calculators. If using the TI-Nspire™ CAS graphing calculator, refer to the Technology Tip. The second method applies algebraic reasoning and requires the fourth root of both sides of the equation to be taken to "undo" the power of 4 that appears. If possible, students should see both methods and make connections between the graphical and algebraic representations.

Investigate Answers (pages 426–427)

1.

Year	Balance at Start of Year ($)	Interest Earned During Year ($)	Balance at End of Year ($)
1	1000.00	50.00	1050.00
2	1050.00	52.50	1102.50
3	1102.50	55.13	1157.63
4	1157.63	57.88	1215.51
5	1215.51	60.78	1276.29

2. a) Answers may vary. Sample answer: As time increases, the amount of interest paid increases year to year.

b) Answers may vary. Sample answer: Compound interest causes the amount of the investment to increase more rapidly over time than simple interest with the same annual rate.

3. a)

Year	Balance at Start of Year ($)	Interest Earned During Year ($)	Balance at End of Year ($)	First Differences
1	1000.00	50.00	1050.00	
2	1050.00	52.50	1102.50	52.50
3	1102.50	55.13	1157.63	55.13
4	1157.63	57.88	1215.51	57.88
5	1215.51	60.78	1276.29	60.78

b) Answers may vary. Sample answer: The first differences represent the increasing values of the interest being paid.

c) Answers may vary. Sample answer: The relationship is not linear, since consecutive terms do not differ by a constant amount.

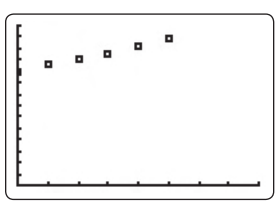
4. a)

Year	Balance at Start of Year ($)	Interest Earned During Year ($)	Balance at End of Year ($)	First Differences	Common Ratios
1	1000.00	50.00	1050.00		
2	1050.00	52.50	1102.50	52.50	1.05
3	1102.50	55.13	1157.63	55.13	1.05
4	1157.63	57.88	1215.51	57.88	1.05
5	1215.51	60.78	1276.29	60.78	1.05

b) Answers may vary. Sample answer: The common ratios are constant.

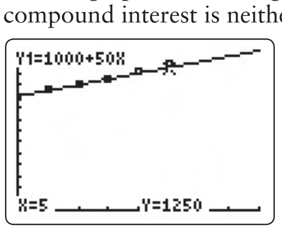
c) Answers may vary. Sample answer: This represents a geometric function, since consecutive terms differ by a common ratio.

5. a)



b) Yes, the graph is increasing. The function representing the amount with compound interest is neither linear nor quadratic; it is exponential.

c)



Both graphs have vertical intercepts of 1000. Both have the same domain and range. The graph of the amount with simple interest is linear; the graph of the amount with compound interest is exponential.

d) The amount with compound interest will be farther and farther above the amount with simple interest. This happens because more interest is earned with compound interest.

6. a) Answers may vary. Sample answer: With simple interest, the amount of interest earned is constant at the end of each year, but with compound interest, the amount of interest increases as time increases. This is because interest is paid on the amount invested and on the interest already paid in compound interest. With simple interest, the amount of interest paid is based only on the amount of the original investment.

b) \$77.46

Communicate Your Understanding Responses (page 433)

C1 Answers may vary. Sample answer: The advantage of compound interest is that interest is paid on interest earned in all previous interest payment periods. Simple interest always adds the same amount of interest each payment period, and is based on the initial investment.

Example:

Option A: \$400 is deposited into an account that earns 5% simple interest annually.

Time (years)	Amount (\$)
0	400
1	420
2	440
3	460
4	480

Option B: \$400 is deposited into an account that earns 5% compound interest, compounded annually.

Time (years)	Amount (\$)
0	400.00
1	420.00
2	441.00
3	464.05
4	487.25

C2 a) Answers may vary. Sample answer: This is a geometric sequence, since all consecutive terms in the sequence are multiplied by the same factor.

b) Annual compound interest rate = 4%; multiplying each term by 1.04 gives the next term in the sequence $(1 + i) = 1.04$, so $i = 0.04$ or 4%.

C3 a) Answers may vary. Sample answer: The graph is a positive exponential curve. The amount grows exponentially over time.

b) The vertical intercept is \$800, representing the initial amount of the investment.

c) Answers may vary. Sample answer: As time goes on, the slope of the curve increases.

Common Errors

- Some students may forget to adjust *i* and *n* when compounding occurs more frequently than once per year.

R_x This type of error is not immediately obvious when considering the reasonableness of the answer. Encourage students to (perhaps mentally) underline key words that indicate the compounding period, and to make sure that they take it into account before substituting into the compound interest formula.

Practise, Connect and Apply, Extend

- **Questions 1** and **2** require application of the compound interest formula without having to calculate *i* or *n* first. These are good warm-up questions that allow students to become familiar with using the formula.
- **Questions 3** to **5** provide practice for determining *i* and *n* when the compounding interval is not annual. These are good scaffolding questions to help prepare students for the contextual problems that follow.

- For **question 7**, students need to calculate the amount, using the compound interest formula, before calculating the interest.
- As students work through **question 8**, they should recognize that the amount of interest increases with the frequency of compounding, when all other factors are held equal. This question requires students to select tools and use connecting skills to compare interest charges under four given conditions and rank the scenarios from best to worst. Students then use reasoning skills to determine the effect of the different compounding periods on this loan and to communicate their findings.
- **Question 11** provides an opportunity to assess students' reasoning and communication skills. This question allows students to reason through and reflect on the two given options by using connecting skills and by selecting the appropriate tools to determine the best investment. They are required to choose the best investment and communicate their reason for choosing that investment.
- There are different methods students can use for solving **question 14**, such as using systematic trial, using graphing technology, and working backward.
- For extension work, have students explore the *rule of 72* in greater depth, using the TVM Solver of a graphing calculator.
- Use **BLM 7–3 Section 7.2 Practice** for extra practice.

Ongoing Assessment

Achievement Check, question 15, on student text page 435.

Achievement Check, question 15, student text page 435

This performance task is designed to assess the specific expectations covered in Sections 7.1 and 7.2. The following mathematical process expectations can be assessed.

- Reflecting
- Connecting
- Selecting Tools and Computational Strategies
- Representing
- Communicating

Sample Solution

a) *Option A*: For simple interest, use the formula $A = P + I$, where $I = Prt$.

$A = 800 + 800(0.10)t$

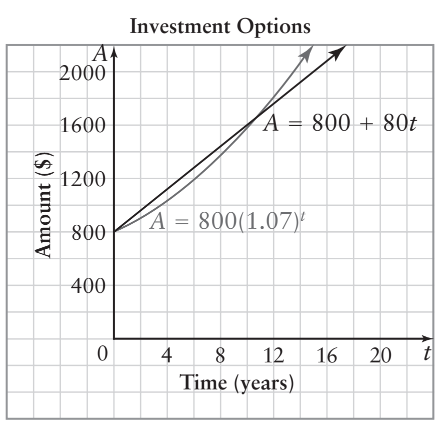
$= 800 + 80t$

Option B: For compound interest, use the formula $A = P(1 + i)^n$. Let $t = n$.

$A = 800(1 + 0.07)^t$

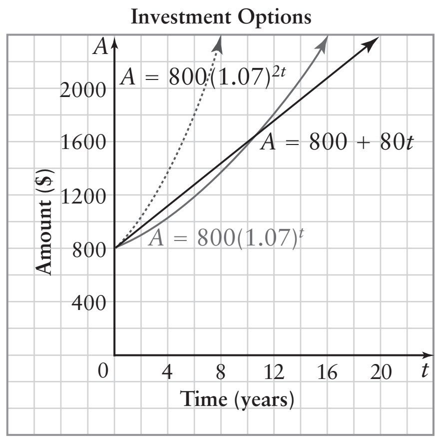
$= 800(1.07)^t$

b)



c) Option A is the better investment option if the money remains in the account less than approximately 11 years. If the investment is longer than 11 years, then Option B is better.

d) Option B would be better if the compounding period was more frequent. Even semi-annual compounding, represented by $A = 800(1.07)^{2t}$, results in a better rate of return, as shown by the graph.



Level 3 Notes

Look for the following:

- Determines the equations in part a) with considerable accuracy
- Graphs representing each equation are considerably accurate
- Graphs are mostly labelled
- Understanding of the conditions determining which option is better in part c) is mostly evident
- Understanding of the effects of a change in the compounding period in part d) is mostly evident
- Justification and explanations in parts c) and d) are mostly valid

What Distinguishes Level 2

- Determines the equations in part a) with some accuracy
- Graphs representing each equation are somewhat accurate
- Graphs are somewhat labelled
- Understanding of the conditions determining which option is better in part c) is somewhat evident
- Understanding of the effects of a change in the compounding period in part d) is somewhat evident
- Justification and explanations in parts c) and d) are somewhat valid

What Distinguishes Level 4

- Determines the equations in parts a) with a high degree of accuracy
- Graphs representing each equation are accurate
- Graphs are thoroughly and clearly labelled
- Understanding of the conditions determining which option is better in part c) is clearly evident
- Understanding of the effects of a change in the compounding period in part d) is clearly evident
- Justification and explanations in parts c) and d) are clearly valid

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15, 16
Reasoning and Proving	8, 11, 12, 15–17
Reflecting	11, 12, 15–17
Selecting Tools and Computational Strategies	1, 2, 6–17
Connecting	1–17
Representing	15
Communicating	8, 11–13, 15–17

7.3 Present Value

Student Text Pages
436 to 443

Suggested Timing
75 min

Tools
- scientific calculator
- graphing calculator with TVM Solver application
- computer with spreadsheet software

Related Resources
- G–2 Placement
- T–1 Microsoft® *Excel*
- BLM 7–4 Section 7.3 Practice

Differentiated Instruction
- Add the definitions and formulas for *future value* and *present value* to the **word wall**.
- Use **placemat** to answer questions such as C2. Put a problem in the centre of the placemat. Write one equation in each quadrant, only one of which is correct. Have students explain why their equation is right or wrong.

Teaching Suggestions

- For the **Investigate**, students should work in pairs to discover that the compound interest formula can be applied to solve for an unknown principal when the future value and interest conditions are known. A simple algebraic manipulation of the compound interest formula.
- **Example 1** illustrates how present value can be calculated using the formula. It should be noted that the interest rate per compounding period, i, and the number of compounding periods, n, are calculated in exactly the same way as in the previous section, based on the compounding frequency. Method 1 illustrates a traditional approach of substituting and evaluating. Method 2 shows how the TVM Solver can be used. Demonstrate both methods as a means of checking and as a way to introduce the TVM Solver, which becomes an increasingly useful tool for the rest of the chapter.
- In **Example 2**, three methods are presented for determining the frequency of compounding, given all other information. The type of equation that results is very difficult to solve algebraically. Method 1 applies a combination of algebraic and graphical reasoning. Method 2 uses the TVM Solver and systematic trial. In Method 3, a spreadsheet is used to examine several possible cases. If needed, use **T–1 Microsoft® *Excel*** to support Method 3. If possible, students should see all three methods and discuss the relative merits and weaknesses of each.
- **Example 3** compares two borrowing options. Note that the interest rate alone is not sufficient for this purpose; the compounding period also affects the total interest charged. It may not be obvious why the option with the higher present value is the better one in this question. Some discussion about why this is so may be beneficial.