Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	15, 16
Reasoning and Proving	8, 11, 12, 15–17
Reflecting	11, 12, 15–17
Selecting Tools and Computational Strategies	1, 2, 6–17
Connecting	1–17
Representing	15
Communicating	8, 11–13, 15–17

7.3 Present Value

Student Text Pages
436 to 443

Suggested Timing
75 min

Tools
- scientific calculator
- graphing calculator with TVM Solver application
- computer with spreadsheet software

Related Resources
- G–2 Placement
- T–1 Microsoft® *Excel*
- BLM 7–4 Section 7.3 Practice

Differentiated Instruction

- Add the definitions and formulas for *future value* and *present value* to the **word wall**.
- Use **placemat** to answer questions such as C2. Put a problem in the centre of the placemat. Write one equation in each quadrant, only one of which is correct. Have students explain why their equation is right or wrong.

Teaching Suggestions

- For the **Investigate**, students should work in pairs to discover that the compound interest formula can be applied to solve for an unknown principal when the future value and interest conditions are known. A simple algebraic manipulation of the compound interest formula.
- **Example 1** illustrates how present value can be calculated using the formula. It should be noted that the interest rate per compounding period, i, and the number of compounding periods, n, are calculated in exactly the same way as in the previous section, based on the compounding frequency. Method 1 illustrates a traditional approach of substituting and evaluating. Method 2 shows how the TVM Solver can be used. Demonstrate both methods as a means of checking and as a way to introduce the TVM Solver, which becomes an increasingly useful tool for the rest of the chapter.
- In **Example 2**, three methods are presented for determining the frequency of compounding, given all other information. The type of equation that results is very difficult to solve algebraically. Method 1 applies a combination of algebraic and graphical reasoning. Method 2 uses the TVM Solver and systematic trial. In Method 3, a spreadsheet is used to examine several possible cases. If needed, use **T–1 Microsoft® *Excel*** to support Method 3. If possible, students should see all three methods and discuss the relative merits and weaknesses of each.
- **Example 3** compares two borrowing options. Note that the interest rate alone is not sufficient for this purpose; the compounding period also affects the total interest charged. It may not be obvious why the option with the higher present value is the better one in this question. Some discussion about why this is so may be beneficial.

Investigate Answers (pages 436–437)

1. $A = 10\ 000$, $i = 0.06$, $n = 5$ years

2. a) $10\ 000 = P(1.06)^5$

b) Answers may vary. Sample answer: The unknown variable is P, and can be found by dividing 10 000 by $(1.06)^5$.

c) $P \doteq \$7472.58$

3. Answers may vary. Sample answer: If Eric's parents invest \$7472.58 now, they will have \$10 000 at the end of 5 years.

Communicate Your Understanding Responses (page 441)

C1 Answers and examples may vary. Sample answers:

a) As the principal increases, the amount increases.

b) The principal and the present value represent the same item.

c) The present value multiplied by the factor of $(1 + i)^n$ gives the future value.

C2 Answers may vary. Sample answer: Choice D; it correctly factors in the semi-annual nature of the payment of interest, both for the rate and the number of times interest is paid. In choice A, 284.42 should be divided by $(1 + 0.045)^8$. In choices B and C, the interest rate and number of compounding periods are incorrect.

Practise, Connect and Apply, Extend

- For **questions 1** to **4**, have students use a TVM Solver to check their answers.
- Students should note that in **questions 7** and **8**, the investment with the lower present value is better because it will cost less to achieve the desired financial goal.
- **Question 8** gives students the opportunity to reason through and reflect on two bank offers. They must select appropriate tools and connect concepts from this chapter with previously learned mathematical material to determine the best offer. They are required to use communicating skills to discuss from which bank Jacques should borrow the money and the reasons for their choice.
- For **question 12**, students should realize that increasing the compounding period increases the total interest earned, especially over long periods of time.
- As an extension of **question 13**, challenge students to research to find historical evidence that supports or denies their mathematical prediction of the past.
- Some students may find some of the financial terminology in **question 14** daunting. Direct students to the Connections box for clarification, as needed.
- **Question 16** provides an opportunity for students to begin to interpret a concept used in financial mathematics called the *time value of money*. Present and future values are all relative concepts. The value of a given investment can be thought of as a continuous function of time. This graph should cause some cognitive dissonance, because time goes backward as the graph progresses from left to right. This phenomenon is "restored" in **question 18** when students reflect the function in the vertical axis by having time moving forward from left to right.
- **Question 16** encourages students to use reasoning, reflecting, and connecting skills to write an equation that represents the present value of the given account as a function of time. They select appropriate tools and use connecting skills to represent the function graphically. Students reflect on and reason through a solution to find the meaning of the horizontal scale of the graph. They use communicating skills to describe the limits of the scale.
- Use **BLM 7–4 Section 7.3 Practice** for extra practice.

Ongoing Assessment

Achievement Check, question 17, on student text page 443.

Achievement Check, question 17, student text page 443

This performance task is designed to assess the specific expectations covered in Section 7.3. The following mathematical process expectations can be assessed.

- Reasoning and Proving
- Reflecting
- Connecting
- Selecting Tools and Computational Strategies
- Representing
- Communicating

Sample Solution

a) Use the formula for present value: $PV = \dfrac{FV}{(1 + i)^n}$.

$$FV = 1200$$
$$i = \frac{0.064}{2} = 0.032$$
$$n = 2.5 \times 2 = 5$$
$$PV = \frac{1200}{(1 + 0.032)^5} \doteq 1025.14$$

The amount that Tanya must invest today is $1025.14.

b) $FV = 1200$

$PV = 970$

Let i represent the semi-annual interest rate as a decimal.

$$n = 2.5 \times 2 = 5$$

Substitute $n = 5$ into the present value formula and solve for i.

$$PV = \frac{FV}{(1 + i)^n}$$
$$970 = \frac{1200}{(1 + i)^5}$$
$$(1 + i)^5 = \frac{1200}{970}$$
$$(1 + i)^5 = 1.237\ 113\ 402$$
$$1 + i = (1.237\ 113\ 402)^{\frac{1}{5}}$$
$$1 + i = 1.043\ 474\ 649$$
$$i = 0.043\ 474\ 649$$

The semi-annual interest is approximately 4.35%.

$4.35 \times 2 = 8.7$

Tanya needs to obtain an annual interest rate of 8.7%.

c) If Tanya has $970 today and can only obtain an interest rate of 6.4%, then the amount that she will have in 2.5 years is determined by the formula $FV = PV(1 + i)^n$.

Substitute $PV = 970$, $i = 0.032$, and $n = 5$ into the future value formula.

$$FV = PV(1 + i)^n = 970(1 + 0.032)^5 \doteq 1135.46$$

In 2.5 years Tanya will have $1135.46, so she will be short of $1200 by $64.54.

Answers may vary for the following options. Sample answers:

i) Wait a little longer and take the vacation when she has enough money.

ii) Borrow $64.54 so that she can take the vacation in 2.5 years.

iii) Invest the $970 that she has today and then add some more money into the account on a regular basis, say every 6 months, so that she will have enough money in 2.5 years.

iv) Invest in an account that has a more frequent compounding period.

Level 3 Notes

Look for the following:

- Determines the present value in part a) with considerable accuracy
- Determines the interest rate in part b) with considerable accuracy
- Provides two options in part c)
- Provides considerable justification or explanation of one or both of the options in part c)
- Communicates steps of solutions with considerable clarity and uses proper mathematical form most of the time

What Distinguishes Level 2

- Determines the present value in part a) with some accuracy
- Determines the interest rate in part b) with some accuracy
- Provides one option in part c)
- Provides some justification or explanation of the option in part c)
- Communicates steps of solutions with some clarity and uses proper mathematical form some of the time

What Distinguishes Level 4

- Determines the present value in part a) with a high degree of accuracy
- Determines the interest rate in part b) with a high degree of accuracy
- Provides more that two options in part c)
- Provides thorough justification or explanation of the options in part c)
- Communicates steps of solutions with a high degree of clarity and uses proper mathematical form consistently

Mathematical Process Expectations

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Reflecting	7, 8, 12, 16–19
Selecting Tools and Computational Strategies	1–19
Connecting	1–19
Representing	16
Communicating	7, 8, 12, 15–18