7.4 Annuities

Student Text Pages
444 to 455

Suggested Timing
75–150 min

Tools
- scientific calculator
- graphing calculator with TVM Solver application
- TI-Nspire™ CAS graphing calculator
- computer with spreadsheet software

Related Resources
- T–1 Microsoft® *Excel*
- BLM 7–5 Section 7.4 Practice

Differentiated Instruction
- Add the terms *regular payments*, *annuity*, *ordinary annuity*, and *simple annuity* to the **word wall**. Include the formula for calculating the amount of an annuity.
- Use **cooperative task groups** to complete the Investigate.
- Use **jigsaw** to compare the effect of compounding periods, interest rates, and payments on the amount of an annuity. Assign the same problem to each group member. Assign each group member a different compounding period, interest rate, and payment for the problem. Have group members calculate and compare their results.

Teaching Suggestions

- Depending on the needs of the class, it may be worthwhile to spend two periods on this section. Monitor progress through the first example and practice questions. If needed, teach the remaining examples the following day.
- Have students work in pairs or small groups through the **Investigate**. Allow 5 to 10 min for the activity. Students should discover that the value of an annuity can be determined by adding the future values of all regular payments. When debriefing, get groups to explain how they solved this problem. It should emerge from the discussion that this type of sum can be represented by a geometric series, a topic that was studied in Chapter 6.
- After the **Investigate**, spend some time developing the formula for the amount of an annuity, which can be represented by either S_n (sum of a geometric series) or FV (future value of all regular payments). Some students may find the algebraic development somewhat daunting. Use a computer algebra system (CAS) to help. Students do not need to reproduce the algebraic development.
- In **Example 1,** attention should be paid to the careful development of the time line as a visual organizer. This will make it easier to understand the problem and to use the formula effectively. In Method 1, the formula is used to calculate the amount of the annuity. Method 2 uses the TVM Solver to solve for the amount of the annuity. Use the side comments beside the calculator screen to explain the various field entries.
- In **Example 2**, two methods are shown to solve for the regular payment, given the amount of the annuity. It should be noted that the second method leads to a variation of the formula that could be used directly for similar situations. However, emphasize the application of formulas to solve problems rather than the need to memorize them.
- In **Example 3**, the interest rate is the unknown. The equation that arises is very difficult to solve algebraically. Three methods of solution are presented, each using technology. Students should see all methods, if possible, and discuss the relative merits and shortcomings of each.
- In **Example 4**, students should see the effect of changing the frequency of deposits, given the same conditions, including the total amount being deposited. More frequent deposits leads to a greater future amount. Again, students should have the opportunity to compare the relative strengths of the technology tools used in this example. If needed, use **T–1 Microsoft®** ***Excel*** to support Method 2.
- Slightly different answers occur when using the two methods in **Example 4**, due to rounding and the calculation processes of the technology. While such discrepancies may be minor in most of the problems discussed in this chapter, these can become quite significant when dealing with significantly large figures. This example can also be done with a scientific calculator but, depending on the functions available, the keystroke input could be more time-consuming than the methods shown.

Investigate Answers (pages 444–445)

1. a) He will receive $500 four times.

b) Answers may vary. Sample answer: No, since interest will be paid on each based on how long they have been invested.

2. a) Answers may vary. Sample answer: Each $500 deposit can be thought of as a simple investment for the period of time the particular deposit will be in the account, and then these can be combined for a total.

$$\begin{aligned}\text{Total} &= \text{Year 1} + \text{Year 2} + \text{Year 3} + \text{Year 4}\\ &= 500(1.04)^3 + 500(1.04)^2 + 500(1.04) + 500\\ &\doteq 2123.23\end{aligned}$$

b) $2123.23

3. Answers may vary. Sample answer: This method would not be very efficient if the number of deposits was large, as calculating each value and then adding them together leaves room for mistakes.

Communicate Your Understanding Responses (page 453)

C1 a) Interest is compounded semi-annually. The annual interest is 8%, but in the time line, the $(1.04)^t$ indicates that for each payment period, 4% interest is paid. This means that interest is paid twice a year.

b) The annuity lasts for 3 years. The time line shows 6 payments, with payments made twice a year.

c) Answers may vary. Sample answer: The annuity can be represented as a geometric series. The sum of the terms must be taken and consecutive terms have a common ratio; both are characteristics of a geometric series.

d) $a = 400$, $r = 1.04$

C2 a) The duration is 5 years, as N = 5 and P/Y is 1, meaning that there are 5 payments that occur once per year.

b) The negative sign indicates that payments are being made, not received. There is an outflow of cash.

c) The future value is positive. It represents the future amount of the investment, which is a positive quantity.

Practise, Connect and Apply, Extend

- For **questions 1** to **3**, it is important for students to be able to both read and construct a time line for annuities to visualize what is happening over the term of the investment. A time line also helps to organize given information.
- **Question 3** enables students to select tools and to connect skills from previously learned mathematical concepts to draw a time line representing Carlo's annuity. They use reasoning skills to answer questions related to the time line that they have drawn.
- Students should be able to solve **questions 4** to **8** both algebraically and with technology (e.g., TVM Solver). Suggest that one method be used to solve and the other to check.
- **Questions 9** and **10** provide opportunities to assess reasoning and communication skills.
- **Question 10** requires students to select tools and use reasoning, reflecting, and connecting skills to determine which option Maurice should choose. Once students have chosen an option, they are required to justify their choice.
- For **question 11**, students should note the advantage of investing at more frequent intervals, with all other factors being equal.
- The graphical techniques encouraged in **questions 13** and **14** are very important because they are applicable to many different types of functions. They provide a way to obtain (approximate) solutions to equations that can be very difficult to solve algebraically.

Common Errors

- Some students may forget to use a negative sign when entering the regular payment in a TVM Solver, and therefore may become confused if a negative amount results.

R_x Students must understand the direction of cash flow when investing is opposite to when cash is withdrawn. So if the PMT is negative, the amount will be positive, and vice versa. It does not matter which is which, just that they are opposite.

- Point out in **question 14** that the amount of an annuity as a function of the number of compounding periods (payments) is exponential in nature. Consequently, the longer one can continues to invest, the "exponentially better" the payoff.
- Use **BLM 7–5 Section 7.4 Practice** for extra practice.

Mathematical Process Expectations

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectation	Selected Questions
Problem Solving	10, 13, 14
Reasoning and Proving	2, 3, 7, 9, 10, 12–14
Reflecting	9, 10, 12–14
Selecting Tools and Computational Strategies	2–14
Connecting	1–14
Representing	2, 3, 13, 14
Communicating	9–14

7.5 Present Value of an Annuity

Student Text Pages
456 to 463

Suggested Timing
75 min

Tools
- scientific calculator
- graphing calculator with TVM Solver application

Related Resources
- BLM 7–6 Section 7.5 Practice
- BLM 7–7 Section 7.5 Achievement Check Rubric

Teaching Suggestions

- Consider beginning the class with a discussion about why people save for retirement, and what happens to their savings after retirement. In the previous section, students learned that investing money at regular intervals will make it grow to a large amount in the future. In retirement savings, a large present value is typically used to finance regular withdrawals.
- Have students work in pairs or small groups to carry out the **Investigate**. Allow 10 min for this work.
- Students should see that the present value of an annuity is the amount of money required today in order to finance a series of regular withdrawals. This present value can be determined by determining the sum of the present value of all the withdrawals. Have groups explain how they solved this problem. This can lead into a guided discussion and the development of the formula for present value of an annuity, which follows.
- Use the simplified visual models in the middle of page 457 to help students understand the two types of annuity problems (amount versus present value), and their similarities and differences. When developing the formula, explain how the time line of a present value problem is similar to those used in the previous section. However, it should be noted that the time value arrows lead back to today, as opposed to forward to the end of the term. The geometric series that evolves represents a series of present value amounts (compared to future value amounts in the previous section).
- The remaining algebraic development is very similar to that used in the previous section, in fact with some steps left as an exercise (see **question 15**). Alternatively, the development could be written out by hand or by using a computer algebra system (CAS).