Answers

Advanced Functions 12 Study Guide and University Handbook

Chapter 1

1.1 Power Functions

1. **a)** 2 **b)** 3 **c)** 10 **d)** 1 **e)** 3 **f)** 4 **g)** 6 **h)** 1

2. **a)** line **b)** point **c)** line **d)** point **e)** point **f)** line **g)** line **h)** point
Even-degree power functions have line symmetry in the y-axis. Odd-degree power functions have point symmetry about the origin.

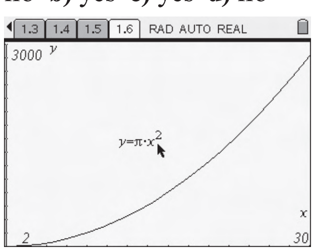
3. **a)** quadrant 2 to 1, even exponent, positive leading coefficient
b) quadrant 2 to 4, odd exponent, negative leading coefficient
c) quadrant 2 to 1, even exponent, positive leading coefficient
d) quadrant 2 to 4, odd exponent, negative leading coefficient
e) quadrant 2 to 4, odd exponent, negative leading coefficient
f) quadrant 2 to 1, even exponent, positive leading coefficient
g) quadrant 3 to 4, even exponent, negative leading coefficient
h) quadrant 3 to 1, odd exponent, positive leading coefficient

4. **a)** yes **b)** no **c)** no **d)** no **e)** yes **f)** yes **g)** no

5. Answers may vary. For example:
a) $y = x^3$ **b)** $y = 2x^3$ **c)** $y = -x$ **d)** $y = -x^2$

6. **a)** no **b)** yes **c)** yes **d)** no

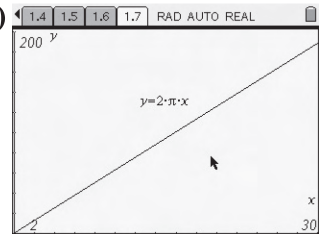
7. **a)**



b) The domain is $\{r \in \mathbb{R} \mid 0 \le r \le 30\}$
The range is approximately $\{A \in \mathbb{R} \mid 0 \le A \le 2827.4\}$
c) Similarities: The functions $A(r) = \pi r^2$ and $y = x^2$ are both quadratic, with positive leading coefficients. Both graphs pass through the origin (0, 0) and have one end that extends upward in quadrant 1.

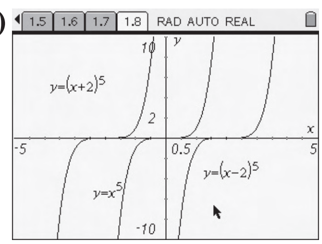
Differences: The graph of $A(r)$ has a restricted domain. Since the two functions are both quadratic power functions that have different leading coefficient, all points on each graph, other than (0, 0), are different.

8. **a)**



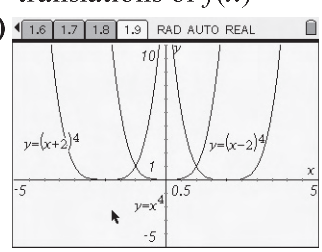
b) $\{r \in \mathbb{R} \mid 0 \le r \le 30\}$;
$\{C \in \mathbb{R} \mid 0 \le C \le 188.5\}$
c) Similarities: linear; positive leading coefficient; passes through the origin; one end extends upward in quadrant 1.
Differences: $C(r)$ has restricted domain; all points, other than (0, 0), are different.

9. **a)**

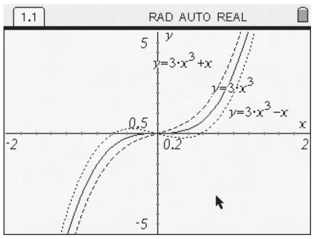


quadrant 3 to 1; degree 5; leading coefficient 1; same shape; same domain and range; $g(x)$ and $h(x)$ are horizontal translations of $f(x)$

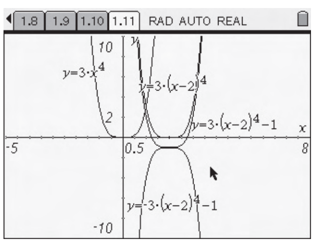
b)



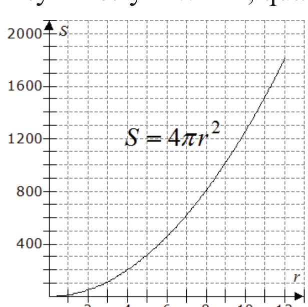
quadrant 2 to 1; degree 4; leading coefficient 1; same shape; same domain and range; $g(x)$ and $h(x)$ are horizontal translations of $f(x)$
c) They have the same end behaviour and shape. The function $y = (x + b)^n$ is a horizontal translation of $y = x^n$. If $b > 0$, there is a horizontal translation to the left. If $b < 0$, the horizontal translation is to the right.
d) Answers will vary.

10. a) 

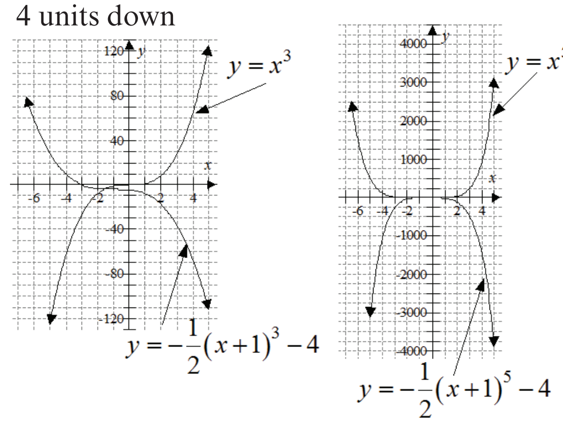
b) Similarities: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$, degree 3, positive leading coefficient 3, end behaviour: quadrant 3 to 1, point symmetry about the origin, y-intercept 0
Differences: $f(x) = 3x^3$ and $h(x) = 3x^3 + x$ and both have one x-intercept: 0; $g(x) = 3x^3 - x$ has three x-intercepts: 0, $\pm\frac{1}{\sqrt{3}}$.

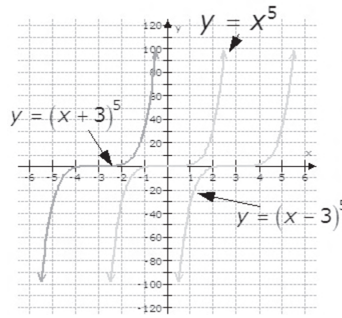
11. a) 

b) i) $\{x \in \mathbb{R}\}$; $\{y \in \mathbb{R}, y \geq 0\}$; line symmetry in the y-axis; quadrant 2 to 1
ii) $\{x \in \mathbb{R}\}$; $\{y \in \mathbb{R}, y \geq 0\}$; line symmetry in $x = 2$; quadrant 2 to 1
iii) $\{x \in \mathbb{R}\}$; $\{y \in \mathbb{R}, y \geq -1\}$; line symmetry in $x = 2$; quadrant 2 to 1
iv) $\{x \in \mathbb{R}\}$; $\{y \in \mathbb{R}, y \leq -1\}$; line symmetry in $x = 2$; quadrant 3 to 4

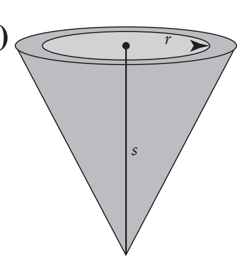
12. a) 

b) domain $\{x \in \mathbb{R}, 0 \leq r \leq 12\}$
range $\{s \in \mathbb{R}, 0 \leq s \leq 1809.6\}$
c) Similarities: both are quadratic functions, end behaviour to quadrant 1, positive leading coefficients
Differences: $s(r) = 4\pi r^2$ has restricted domain, $s(r) = 4\pi r^2$ is obtained by applying a vertical stretch by a factor of 4π of the graph of $y = x^2$, therefore other than (0, 0) all points are different.

13. a) $y = -\frac{1}{2}(x + 1)^5 - 4$ to $y = x^5$ Reflection in the x-axis, vertical by factor $\frac{1}{2}$, translation 1 unit to left and 4 units down
b) $y = -\frac{1}{2}(x + 1)^3 - 4$ to $y = x^3$
Reflection in the x-axis, vertical by factor $\frac{1}{2}$, translation 1 unit to left and 4 units down
c) 

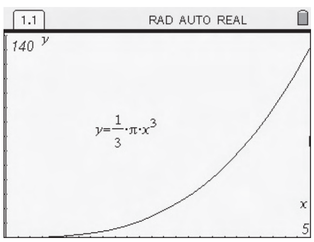
14. a) 

b) Similarities: quadrant 3 to 1, degree 5, leading coefficient 1, same domain and range, $y = (x + 3)^5$ and $y = (x - 3)^5$ are horizontal translations of $y = x^5$
Differences: different point symmetry. The graph of $y = (x + 3)^5$ is obtained by translating the graph of $y = x^3$ three units left, whereas the graph of $y = (x - 3)^5$ is obtained by translating the graph of $y - x^3$ three units right.

15. a) 

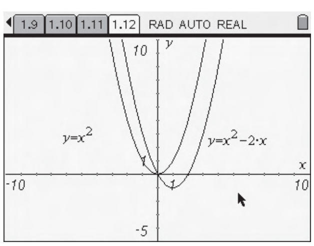
The volume of this conical reservoir can be described with the function $V(r) = \frac{1}{3}\pi r^2 s$, where r is the radius in metres and s is the depth of water in metres. The maximum diameter of the cone is 10 m; therefore, the maximum radius is 5 m. The maximum depth of water in the cone is also 5 m. As the reservoir fills with water, the radius and the depth of water increase at the same rate. Therefore, we can substitute r for s in the function for volume. The result is a function with only one variable r: $V(r) = \frac{1}{3}\pi r^3$.

b)



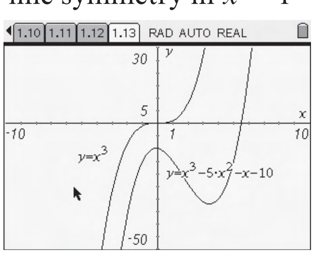
c) The domain is $r \in [0, 5]$. The range is approximately $V \in [0, 130.9]$.

16. a)



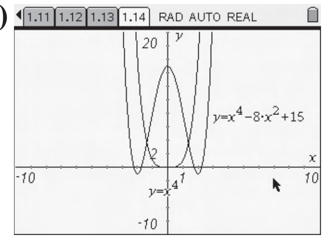
degree 2
Similarities: domain $\{x \in \mathbb{R}\}$, end behaviour: quadrant 2 to 1, positive leading coefficient
Differences: $f(x) = x^2$: range $\{y \in \mathbb{R}, y \geq 0\}$, line symmetry in the y-axis
$g(x) = x^2 - 2x$: range $\{y \in \mathbb{R}, y \geq -1\}$, line symmetry in $x = 1$

b)



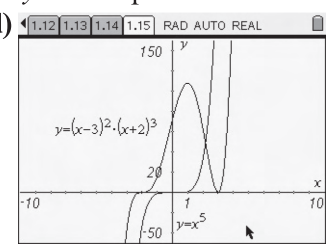
degree 3
Similarities: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$, end behaviour: quadrant 3 to 1, positive leading coefficient
Differences: different point symmetry, x-intercepts, y-intercepts

c)



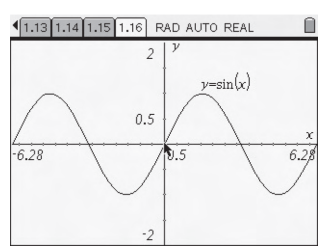
degree 4
Similarities: domain $\{x \in \mathbb{R}\}$, end behaviour: quadrant 2 to 1, positive leading coefficient, line symmetry in the y-axis
Differences: range, x-intercepts, y-intercepts

d)



degree 5
Similarities: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$, end behaviour: quadrant 3 to 1, positive leading coefficient
Differences: x-intercepts, y-intercepts, $f(x) = x^5$ has point symmetry about origin, $g(x) = (x - 3)^2 (x + 2)^3$ has no symmetry

17. Answers may vary. For example:



$y = \sin x$ is a trigonometric function, not a polynomial function.

1.2 Characteristics of Polynomial Functions

1. **a)** degree 1, leading coefficient −3
   b) degree 2, leading coefficient 1
   c) degree 5, leading coefficient −2
   d) degree 4, leading coefficient 6
   e) degree 3, leading coefficient 5.25
2. **a)** 5 **b)** 3 **c)** 4 **d)** 4
3. Graph a): **a)** positive **b)** quadrant 3 to 1 **c)** no symmetry **d)** local maximums 2, local minimums 2; no minimum or maximum points
   Graph b): **a)** negative **b)** quadrant 2 to 4 **c)** no point symmetry about origin **d)** local maximum 1, local minimum 1; no minimum or maximum points
   Graph c): **a)** positive **b)** quadrant 2 to 1 **c)** no symmetry **d)** local maximum 1, local minimum 2; minimum point 1, no maximum point
   Graph d): **a)** negative **b)** quadrant 3 to 4 **c)** no symmetry **d)** local maximum 2, local minimum 1; no minimum point, maximum point 1
4. **a) i)** quadrant 2 to 1 **ii)** fourth **iii)** 24
   b) i) quadrant 2 to 4 **ii)** first **iii)** −7
   c) i) quadrant 2 to 4 **ii)** seventh **iii)** −15120
   d) i) quadrant 3 to 4 **ii)** second **iii)** −12
   e) i) quadrant 3 to 4 **ii)** fourth **iii)** −12
   f) i) quadrant 3 to 1 **ii)** third **iii)** 162
5. a) and iii); b) and iv); c) and ii); d) and i)
6. Construct a finite difference table. Determine the finite differences until they are constant.

a)

x	y	1st Differences	2nd Differences	3rd Differences	4th Differences
−3	140				
−2	37	37 − 140 = −103			
−1	8	8 − 37 = −29	−29 − (−103) = 74		
0	5	5 − 8 = −3	−3 − (−29) = 26	2 − 6 − 74 = −48	
1	4	4 − 5 = −1	−1 − (3) = 2	2 − 26 = −24	−24 − (−48) = 24
2	5	5 − 4 = 1	1 − (−1) = 2	2 − 2 = 0	0 − (−24) = 24
3	32	32 − 5 = 27	27 − 1 = 26	26 − 2 = 24	24 − 0 = 24

i) Since the 4th differences are constant, the table of values represents a quartic function.
ii) Since the 4th differences are positive, the leading coefficient is positive.
iii) The value of the leading coefficient is the value of a such that
$24 = a[n \times (n - 1) \times \ldots \times 2 \times 1]$.
Substitute $n = 4$:
$24 = a(4 \times 3 \times 2 \times 1)$
$24 = 24a$
$a = 1$

b)

x	y	1st Differences	2nd Differences	3rd Differences
−3	0			
−2	−4	−4 − 0 = −4		
−1	0	0 − (−4) = 4	4 − (−4) = 8	
0	6	6 − 0 = 6	6 − 4 = 2	2 − 8 = −6
1	8	8 − 6 = 2	2 − 6 = −4	−2 = −6
2	0	0 − 8 = −8	−8 − 2 = −10	−10 − (−4) = −6
3	−24	−24 − 0 = −24	−24 − (−8) = −16	−16 − (−10) = −6

i) Since the 3rd differences are constant, the table of values represents a cubic function.
ii) Since the 3rd differences are negative, the leading coefficient is negative.
iii) The value of the leading coefficient is the value of a such that
$-6 = a[n \times (n - 1) \times \ldots \times 2 \times 1]$.
Substitute $n = 3$:
$-6 = a(3 \times 2 \times 1)$
$-6 = 6a$
$a = -1$

c)

x	y	1st Differences	2nd Differences
−3	36		
−2	16	16 − 36 = −20	
−1	4	4 − 16 = −12	−12 − (−20) = 8
0	0	0 − 4 = −4	−4 − (−12) = 8
1	4	4 − 0 = 4	4 − (−4) = 8
2	16	16 − 4 = 12	12 − 4 = 8
3	36	36 − 16 = 20	20 − 12 = 8

i) Since the 2nd differences are constant, the table of values represents a quadratic function.

ii) Since the 2nd differences are positive, the leading coefficient is positive.

iii) The value of the leading coefficient is the value of a such that
$8 = a[n \times (n-1) \times \ldots \times 2 \times 1]$.
Substitute $n = 2$:
$8 = a(2 \times 1)$
$8 = 2a$
$a = 4$

7. a) The function $P(x)$ is a polynomial of degree 4; therefore, it is a quartic function.

b) Since the degree of this polynomial is 4, then the 4th finite differences are constant. The value of the constant finite difference is $a[n \times (n-1) \times \ldots \times 2 \times 1]$, where a is the leading coefficient and n is the degree.
Substitute $n = 4$ and $a = 0.65$:
Constant finite difference $=$
$0.65(4 \times 3 \times 2 \times 1) = 15.6$

c) The function is quartic with positive leading coefficient. The graph extends from quadrant 2 to quadrant 1.

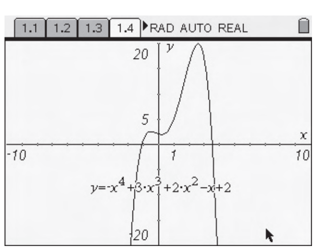
d) Since x represents the number, in hundreds, of flying discs sold, x cannot be a negative number. Therefore, the domain is $\{x \in \mathbb{R}, x \geq 0\}$.

e) The x-intercepts of the graph represent the break-even point.

f) $P(x) = 0.65x^4 - 3.5x^2 - 12$
$P(5) = 0.65(5)^4 - 3.5(5)^2 - 12$
$= 306.75$
The profit from the sale is $306,750.

8 a) degree 4; leading coefficient is negative; end behaviour: quadrant 3 to 4

b)

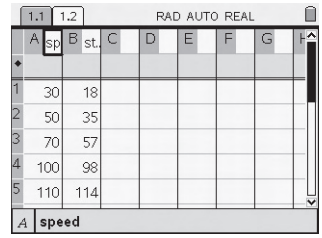


c) −24

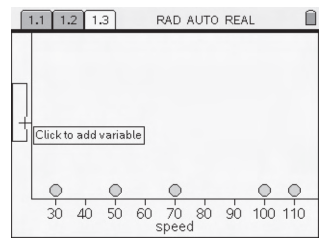
9. Unrestricted domain and range because they extend from quadrant 2 to 4 or quadrant 3 to 1; no maximum points, no minimum points; at least one x-intercept, at least one y-intercept

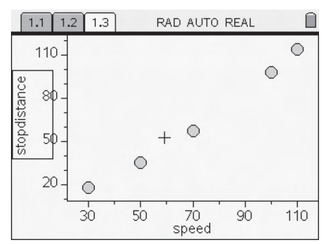
10. TI-*n*spire graphing calculator was used to solve this problem.

a) Open Lists and Spreadsheets application and enter the data into a list.

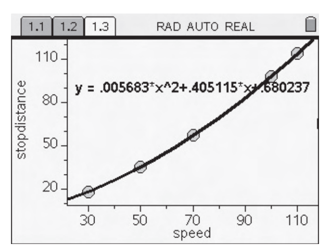


Open Data and Statistics application and add *speed* variable to the x-axis and *stopdistance* variable to the y-axis.

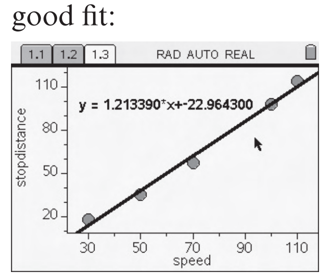




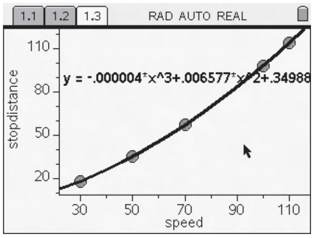
b) Click on Menu button; choose 3: Actions; choose 5: Regression; choose 4: Show Quadratic. The result is a quadratic model of the data:
$y = 0.005683x^2 + 0.45115x + 0.680237$



Linear function does not seem like a good fit:



Cubic function adds a cubic term with leading coefficient so small that it does not make a big difference compared with the quadratic function:



11. Polynomial functions used to draw graphs will vary. For example: **a)** $y = x^5$ **b)** $y = (x - 3)^3(x + 1)^2$ **c)** $y = x(x - 1)(x + 1)(x - 2)(x + 2)$ **d)** $y = (x - 1)^5$

12. Odd degree n: minimum 1 x-intercept, maximum n x-intercepts
Even degree n: minimum 0 x-intercepts, maximum n x-intercepts

13. a) The surface area of the silo (not including the top and bottom) can be represented as:
$A(r) =$ (circumference of the base) $\times$ (height)
$A(r) = (2\pi r) \times (h)$
where r is the radius and h is the height of the silo
Since $\frac{r}{h} = \frac{1}{6}$, then $h = 6r$
Substitute $6r$ for h:
$A(r) = (2\pi r) \times (6r)$
$A(r) = 12\pi r^2$

b) $V(r) =$ (area of the base) $\times$ (height)
$V(r) = (\pi r^2) \times (h)$
Substitute $6r$ for h:
$V(r) = (\pi r^2) \times (6r)$
$V(r) = 6\pi r^3$

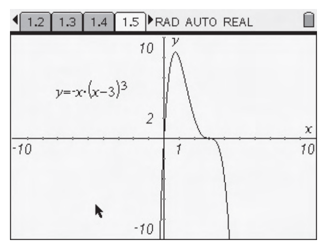
c) $A(r) = 12\pi r^2$: domain $\{r \in R, r > 0\}$, range $\{A \in R, A > 0\}$, ends in quadrant 1
$V(r) = 6\pi r^3$: domain $\{r \in R, r > 0\}$, range $\{V \in R, V > 0\}$, ends in quadrant 1

1.3 Equations and Graphs of Polynomial Functions

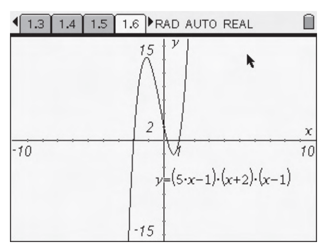
1. a) i) 3, positive **ii)** quadrant 3 to 1 **iii)** 0, 2, −3
b) i) 4, negative **ii)** quadrant 3 to 4 **iii)** 2, 4 (order 2), 6
c) i) 4, positive **ii)** quadrant 2 to 1 **iii)** $\frac{1}{2}$, −3 (order 3)
d) i) 4, negative **ii)** quadrant 3 to 4 **iii)** 4, −1 (order 2), 2

2. a) i) −7, −0.5, 3.5 **ii)** positive: $-7 < x < -0.5$, $x > 3.5$; negative: $x < -7$, $-0.5 < x < 3.5$ **iii)** no roots of order 2 or order 3
b) i) −6 (order 2), 5 (order 2) **ii)** positive: no intervals; negative: $x < -6$, $-6 < x < 5$, $x > 5$ **iii)** order 2: yes, order 3: no
c) i) 0 (order 3), 4 **ii)** positive: $0 < x < 4$; negative: $x < 0$, $x > 4$ **iii)** order 2: no, order 3: yes

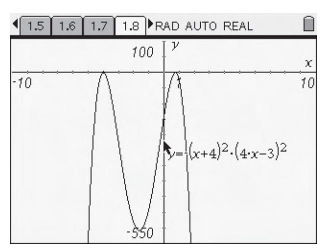
3. a) i) 0 (order 1), 3 (order 3) **ii)** $\frac{1}{5}$ (order 1), −2 (order 1), 1 (order 1) **iii)** −4 (order 2), $\frac{3}{4}$ (order 2)
b) i) neither **ii)** neither **iii)** neither **iv)** neither
c) i)



ii)



iii)



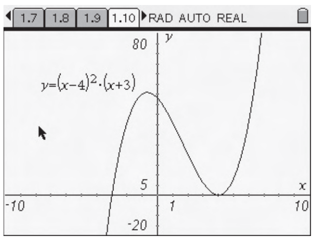
4. a) i) Since the exponent of each term is even, $y = -x^4 + 3x^2$ is an even function.
ii) Since the function is even, it has line symmetry about the y-axis.
b) i) Since the exponent of each term is odd, $y = -6x + 5x^3$ is an odd function.
ii) Since the function is odd, it has point symmetry about the origin.

c) i) Some exponents in $y = x^4 - x^2 + 4x + 2$ are even and some are odd, so the function is neither even nor odd.

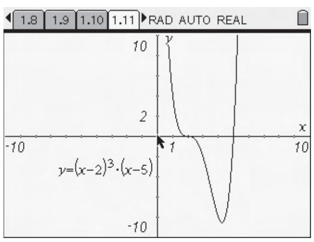
ii) Since the function is neither even nor odd, there is no line symmetry about the y-axis, and no point symmetry about the origin.

5. Answers will vary. For example:

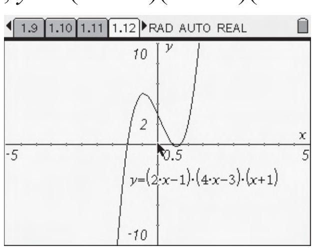
a) $y = (x - 4)^2 (x + 3)$; neither



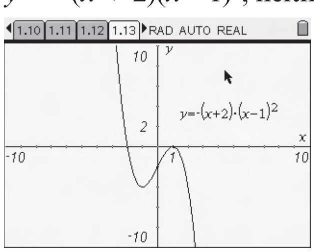
b) $y = (x - 2)^3 (x - 5)$; neither



c) ; $y = (2x - 1)(4x - 3)(x + 1)$; neither



d) $y = -(x + 2)(x - 1)^2$; neither



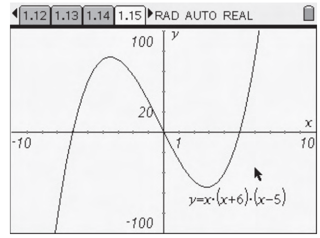
6. a) line symmetry **b)** neither

7. a) $f(x) = -x(x + 3)(x + 1)(x - 2)$

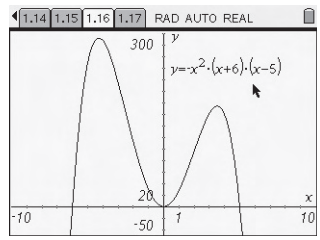
b) $f(x) = x^2 (x + 1)(x - 2)(x - 5)^3$

8. Answers may vary. For example:

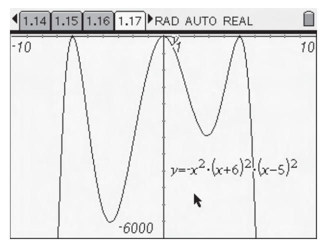
a) $y = x(x + 6)(x - 5)$



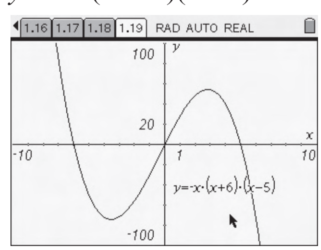
b) $y = -x^2 (x + 6)(x - 5)$



c) $y = -x^2 (x + 6)^2 (x - 5)^2$



d) $y = -x(x + 6)(x - 5)$

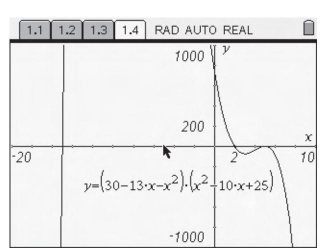


9. a) $f(x) = (30 - 13x - x^2)(x^2 - 10x + 25)$

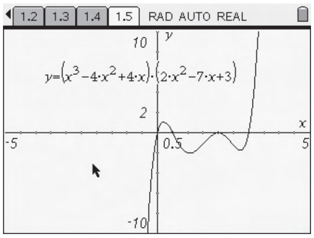
$f(x) = (2 - x)(15 + x)(x - 5)(x - 5)$

$x = 2$ or $x = -15$ or $x = 5$ (*order* 2)

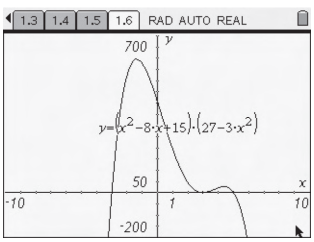
Verify the answers using a graphing calculator:



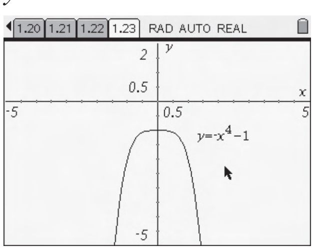
b) $g(x) = (x^3 - 4x^2 + 4x)(2x^2 - 7x + 3)$
$g(x) = x(x^2 - 4x + 4)(2x^2 - 7x + 3)$
$g(x) = x(x - 2)^2 (2x - 1)(x - 3)$
$x = 0$ or $x = 2$ (*order* 2) or $x = \frac{1}{2}$ or $x = 13$
Verify the answers using a graphing calculator:



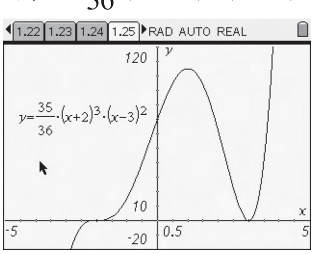
c) $h(x) = (x^2 - 8x + 15)(27 - 3x^2)$
$h(x) = 3(x - 5)(x - 3)(9 - x^2)$
$= -3(x - 5)(x - 3)^2 (x + 3)$
$x = 5$ or $x = 3$ (*order* 2) or $x = -3$
Verify using a graphing calculator:



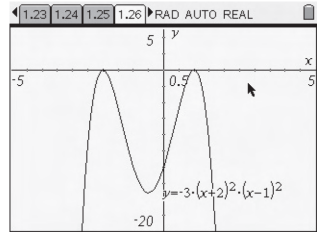
10. Answers may vary. For example:
$y = -x^4 - 1$



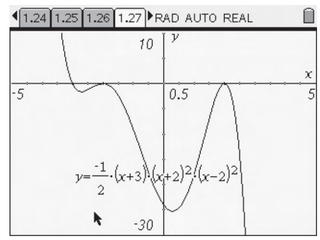
11. a) $y = \frac{35}{36}(x + 2)^3 (x - 3)^2$; neither



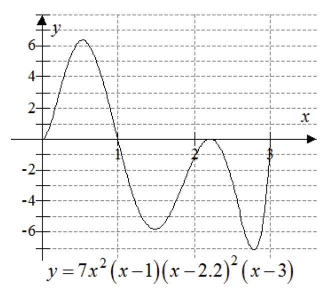
b) $y = -3(x + 2)^2 (x - 1)^2$; neither



c) $y = -\frac{1}{2}(x + 3)(x + 2)^2 (x - 2)^2$; neither



12. Answers may vary. For example:
$y = 7x^2(x - 1)(x - 2.2)^2 (x - 3)$



1.4 Transformations

1. a) $a = 5, k = 1, d = 1, c = 0$; vertical stretch by a factor of 5 and a reflection in the x-axis; translation 1 unit to the right

b) $a = 3, k = 1, d = 0, c = 5$; vertical stretch by a factor of 3; translation 5 units up

c) $a = -1, k = 3, d = \frac{2}{3}, c = 0$; reflection in the x-axis; horizontal compression by a factor of $\frac{1}{3}$ translation $\frac{2}{3}$ units to the right

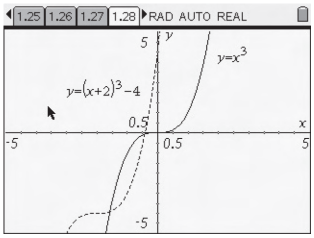
d) $a = \frac{1}{3}, k = 1, d = 2, c = 0$; vertical compression by a factor of $\frac{1}{3}$; translation 2 units to the right

e) $a = \frac{3}{5}, k = 6, d = 1, c = -4$; vertical compression by a factor of $\frac{3}{5}$; horizontal compression by a factor of $\frac{1}{6}$; translation 1 unit right and 4 units down

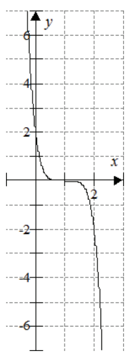
f) $a = 7, k = -1, d = 0, c = 1$; vertical stretch by a factor of 7; reflection in the y-axis; translation 1 unit up

2. **a)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$ **b)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 5\}$ **c)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \leq 0\}$
d) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 0\}$ **e)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$ **f)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

3.

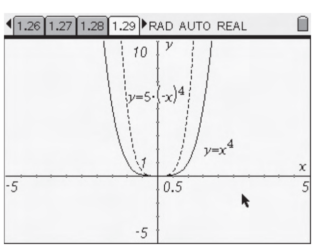


4. a) and i); b) and iii); c) and ii); d) and iv)

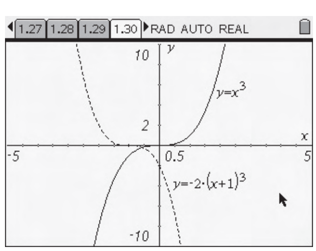


5. **a)** $y = 5(-x)^4$; vertical stretch by a factor of 5; reflection in the y-axis
b) $y = -2(x + 1)^3$; vertical stretch by a factor of 2 and a reflection in the x-axis; translation 1 unit to the left
c) $y = 5[-(x + 1)]^4 + 3$; vertical stretch by a factor of 5; reflection in the y-axis; translation 1 unit to the left and 3 units up

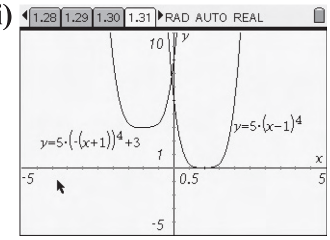
6. **a) i)**



ii)

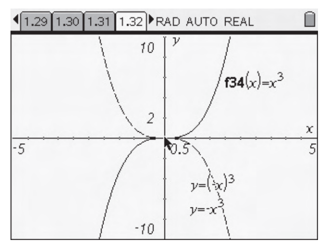


iii)

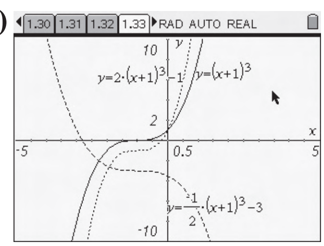


b) i) $y = x^4$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 0\}$; vertex: (0, 0), axis of symmetry: $x = 0$
$y = 5(-x)^4$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 0\}$; vertex (0, 0), axis of symmetry: $x = 0$
ii) $y = x^3$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$
$y = -2(x + 1)^3$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$
iii) $y = 5(x - 1)^4$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 0\}$; ver tex: (1, 0), axis of symmetry: $x = 1$
$y = 5[-(x + 1)]^4 + 3$: $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 3\}$, vertex: (−1, 3), axis of symmetry: $x = -1$

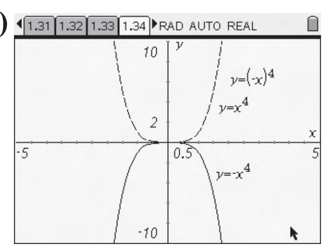
7. **a) i)**



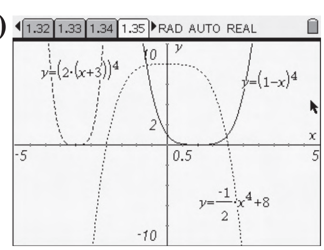
ii)



iii)



iv)



b)

Value of c in $y = a[k(x - d)]^n + c$	Transformation of the Graph of $y = x^n$
$c > 0$	Translation c units up
$c < 0$	Translation $\|c\|$ units down
Value of d in $y = a[k(x - d)]^n + c$	
$d > 0$	Translation d units right
$d < 0$	Translation $\|d\|$ units left
Value of a in $y = a[k(x - d)]^n + c$	
$a > 1$	Vertical stretch by a factor of a
$0 < a < 1$	Vertical compression by a factor of a
$-1 < a < 0$	Vertical compression by a factor of $\|a\|$ and a reflection in the x-axis
$a < -1$	Vertical stretch by a factor of $\|a\|$ and a reflection in the x-axis
Value of k in $y = a[k(x - d)]^n + c$	**Transformation of the Graph of $y = x^n$**
$k > 1$	Horizontal compression by |a| factor of $\frac{1}{k}$
$0 < k < 1$	Horizontal stretch by a factor of $\frac{1}{k}$
$-1 < k < 0$	Horizontal stretch by a factor of $\left\|\frac{1}{k}\right\|$ and a reflection in the y-axis
$k < -1$	Horizontal compression by a factor of $\left\|\frac{1}{k}\right\|$ and a reflection in the y-axis

8. **a)** $a = \frac{1}{3}$, $k = -2$, $d = -4$, $c = -10$

b) The function $y = x^4$ is vertically compressed by a factor of $\frac{1}{3}$, horizontally compressed by a factor of $\frac{1}{2}$ and a reflection in the y-axis, translated left | 4 units, translated down 10 units.

c) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, y \geq -10\}$, vertex: $(-4, -10)$, axis of symmetry: $x = -4$

d) Apply transformation represented by a first, then k, then d and c, or apply k first, then a, then c and d.

9. **a) i)** $c = 4$, $d = 2$; $y = (x - 2)^3 + 4$

ii) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

b) i) $k = \frac{1}{5}$, $d = 3$; $y = [\frac{1}{5}(x - 3)]^4$

ii) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, y \geq 0\}$, vertex: $(3, 0)$, axis of symmetry: $x = 3$

c) i) $a = 2$, $k = -1$, $c = -2$, $d = -4$; $y = 2[-(x + 4)]^5 - 2$

ii) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

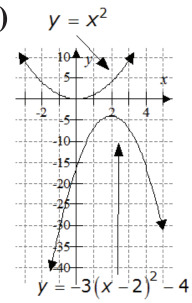
d) i) $a = -1$, $k = -\frac{1}{3}$, $c = 3$, $d = -1$; $y = -[-\frac{1}{3}(x + 1)]^6 + 3$

ii) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, y \leq 3\}$, vertex: $(-1, 3)$, axis of symmetry: $x = -1$

10. **a) i)** vertical stretch by factor 3, reflection in x-axis, translation 2 units right and 4 down

ii) $y = -3(x - 2)^2 - 4$

iii)

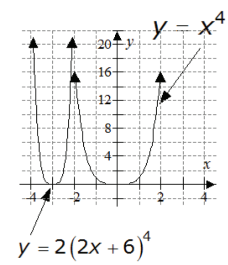


iv) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} | y \leq -4\}$

b) i) vertical stretch by factor 2, horizontal compression by factor $\frac{1}{2}$, translation 3 units to left

ii) $y = 2(2x + 6)^4$

iii)

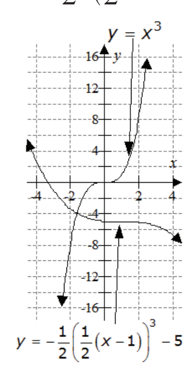


iv) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \leq 0\}$

c) i) vertical compression by factor $\frac{1}{2}$, reflection in x-axis, horizontal stretch by factor 2, translation 1 unit to right and 5 down

ii) $y = \frac{-1}{2}\left(\frac{1}{2}(x - 1)\right)^3 - 5$

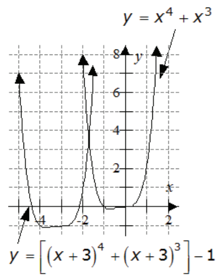
iii)



iv) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

11. a) horizontal shift three to the left and one down

b)



c) For $y = x^4 + x^3$, the x-intercepts are 0, −1
For $y = [(x + 3)^4 + (x + 3)^3] - 1$, the x-intercepts are −4.4, −2.2

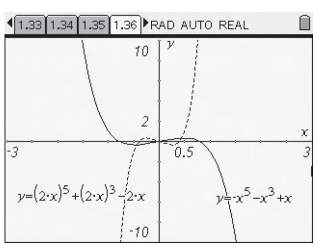
d) For $y = x^4 + x^3$, domain = $\{x|x\in\}$, range = $\{y|y\in\mathbb{R}, y \geq -0.1\}$
For $y = [(x + 3)^4 + (x + 3)^3] - 1$, domain = $\{x|x\in\}$, range = $\{y|y\in\mathbb{R}, y \geq -1.1\}$

12. a) $y = [-3((x-2)-2)((x-2)+3)((x-2)+5)^2] + 1$
$y = -3(x-4)(x+1)(x+3)^2 + 1$

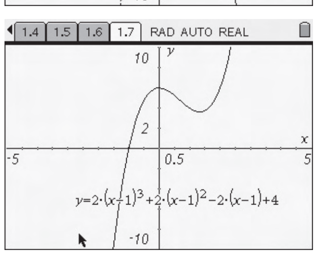
b) $y = \frac{1}{3}[-3((-x)-4)((-x)+1)((-x)+3)^2 + 1]$
$y = -(-x-4)(1-x)(3-x)^2 + \frac{1}{3}$

13. a) reflection in the x-axis; horizontal compression by a factor of $\frac{1}{2}$

b)



14. a)



b) $y = 2(x-1)^3 + 2(x-1)^2 - 2(x-1) + 4$
Vertical stretch by a factor of 2 and translation of 1 unit to the right were applied to the original function.
$y = 2(x)^3 + 2(x)^2 - 2(x) + 4$
$y = 2(x^3 + x^2 - x + 2)$
$y = x^3 + x^2 - x + 2$
Original function: $y = x^3 + x^2 - x + 2$

1.5 Slopes of Secants and Average Rates of Change

1. c), e), and f)

2. A: constant and negative, B: zero, C: constant and positive

3. A: −3, B: 0, C: $\frac{1}{3}$

4. **a)** −2 **b)** −3.8 **c)** −4 **d)** −2.7

5. 7.4

6. a) Average rate of change =
$$\frac{\Delta Index}{\Delta Year} = \frac{128.8 - 120.5}{2004 - 2003} = 8.3$$
Average rate of change =
$$\frac{\Delta Index}{\Delta Year} = \frac{137.8 - 128.8}{2005 - 2004} = 9$$
Average rate of change =
$$\frac{\Delta Index}{\Delta Year} = \frac{144.2 - 137.8}{2006 - 2005} = 6.4$$
Average rate of change =
$$\frac{\Delta Index}{\Delta Year} = \frac{150.1 - 144.2}{2007 - 2006} = 5.9$$

b) The greatest average rate of change was between 2004 and 2005. The least was between 2005 and 2006. The average rate of change of the new housing price index is positive but decreasing; the increase of prices of new houses is slowing down.

c) The average rate of change of the New Housing Price Index over the 4-year period, between 2003 and 2007, is 7.4. The average rate of change is positive; the prices of new houses are still increasing during that period. From part b) we see that the increase of prices is slowing down. This implies that the housing market in the St. Catharines-Niagara area is still doing well.

7. a) Average rate of change
$$= \frac{\Delta WaterAmount}{\Delta Time}$$
$$= \frac{[150000 - 7500(10) + (10)^2] - [150000 - 7500(5) + (5)^2]}{10 - 5}$$
$$= \frac{75100 - 112525}{5}$$
$= -7485$ gallons per minute

b) Average rate of change
$$= \frac{\Delta WaterAmount}{\Delta Time}$$
$$= \frac{[150000 - 7500(10) + (10)^2] - [150000 - 7500(9) + (9)^2]}{10 - 9}$$
$$= \frac{75100 - 82581}{1}$$
$= -7481$ gallons per minute

c) Average rate of change

$= \frac{\Delta Water Amount}{\Delta Time}$

$= \frac{[150000 - 7500(10) + (10)^2] - [150000 - 7500(9.99) + (9.99)^2]}{10 - 9.99}$

$= \frac{75100 - 75174.8001}{0.01}$

$= -7480.01$ gallons per minute

$\cong -7480$ gallons per minute

The estimated rate of change at which the water runs out after exactly 10 minutes is −7480 gallons per minute.

8. a) i) 14.1 **ii)** 13.5 **iii)** 13.4 **iv)** 12.9 **v)** 12.6
b) 12.6 **c)** same

9. a) i) constant and positive **ii)** constant and negative **iii)** zero for first 4 months and then constant and positive
b) i) $\frac{1}{4}$ **ii)** $-\frac{5}{12}$ **iii)** $\frac{11}{12}$

10. a) Marshall: 5.75 m/s; Teagan: 5.25 m/s
b) Marshall: 9.8 m/s; Teagan: 2.5 m/s

11. a) i) Average rate of change

$= \frac{\Delta cost}{\Delta time}$

$= \frac{(4500 + 1530)(4) - 0.04(4)^3) - (4500 + 1530(0) - 0.04(0^3)}{4 - 0}$

= \$1529.36 per year

ii) $\frac{(4500 + 1530(7) - 0.004(7)^3) - (4500 + 1530(4) - 0.004(4^3)}{7 - 4}$

= \$1526.28 per year

iii) $= \frac{(4500 + 1530(9) - 0.004(9)^3) - (4500 + 1530(7) - 0.004(7^3)}{9 - 7}$

= 1522.28 per year

b) The production became more efficient over time as average cost decreased.

12. a) i) Average rate of change

$= \frac{\Delta height}{\Delta time}$

$= \frac{(120 - 4.9(4)^2) - (120 - 4.9(1)^2)}{4 - 1}$

$= -24.5$ m/s

ii) $= \frac{(120 - 4.9(6)^2) - (120 - 4.9(4)^2)}{6 - 4}$

$= -49$ m/s

iii) $= \frac{(120 - 4.9(7)^2) - (120 - 4.9(6)^2)}{7 - 6}$

$= -63.7$ m/s

b) The average rate of change represents the average speed of the ball.
c) The ball is falling faster as times passes.

13. a) i) Average rate of change

$= \frac{\Delta population}{\Delta years}$

$= \frac{15\,596 - 13\,500}{1996 - 1991}$

= 419.2 people per year

ii) $\frac{16\,039 - 15\,596}{2001 - 1996}$

= 88.6 people per year

iii) $\frac{17\,290 - 16\,039}{2006 - 2001}$

= 250.2 people per year

iv) $\frac{17\,290 - 13\,500}{2006 - 1991}$

= 252.7 people per year

b) Answers may vary.

14. a) approximately 0.18ºC/h
b) between hours 12 and 14
c) Answers may vary. Sample answer: between hours 2 and 4 and between 4 and 6

15. a) The average rate of change is the change of distance with respect to a change in speed, i.e., the time it takes for a vehicle to stop.

b) i) Average rate of change $= \frac{\Delta Distance}{\Delta Speed}$

Since distance is measured in metres and speed in km/h, divide the distance by 1000 in order to change the distance into km.

$= \frac{(d(30) - d(20))/1000}{30 - 20}$

$= \frac{([0.01(30)^2 - 0.25(30) + 10] - [0.01(20)^2 - 0.25(20) + 10])/1000}{10}$

$= \frac{(11.5 - 9)/1000}{10}$

= 0.00025 hours

ii) Average rate of change $= \frac{\Delta Distance}{\Delta Speed}$

Since distance is measured in m and speed in km/h, divide the distance by 1000 to change the distance to km.

$= \frac{(d(50) - d(40))/1000}{50 - 40}$

$= \frac{([0.01(50)^2 - 0.25(50) + 10] - [0.01(40)^2 - 0.25(40) + 10])/1000}{10}$

$= \frac{(22.5 - 16)/1000}{10}$

= 0.00065 hours

iii) Average rate of change $= \dfrac{\Delta \text{Distance}}{\Delta \text{Speed}}$

Since distance is measured in m and speed in km/h, divide the distance by 1000 to change the distance to km.

$= \dfrac{(d(70) - d(60))/1000}{70 - 60}$

$= \dfrac{([0.01(70)^2 - 0.25(70) + 10] - [0.01(60)^2 - 0.25(60) + 10])/1000}{10}$

$= \dfrac{(41.5 - 31)/1000}{10}$

$= 0.00105$ hours

iv) Average rate of change $= \dfrac{\Delta \text{Distance}}{\Delta \text{Speed}}$

Since distance is measured in m and speed in km/h, divide the distance by 1000 to change the distance to km.

$= \dfrac{(d(90) - d(80))/1000}{90 - 80}$

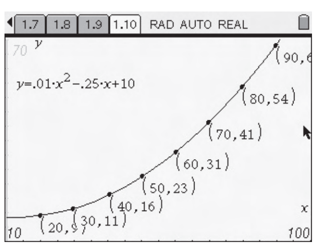
$= \dfrac{([0.01(90)^2 - 0.25(90) + 10] - [0.01(80)^2 - 0.25(80) + 10])/1000}{10}$

$= \dfrac{(68.5 - 54)/1000}{10}$

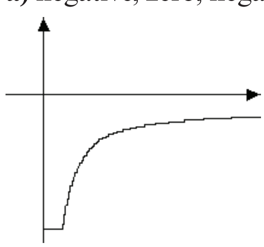
$= 0.00145$ hours

c) As the speed increases, the average rate of change is positive and increasing.

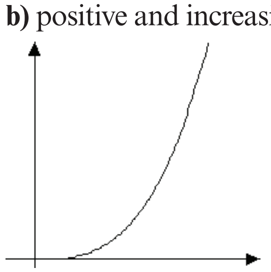
d)



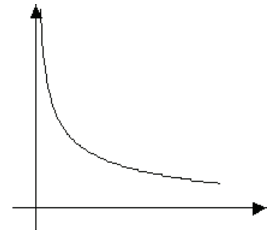
16. a) negative, zero, negative



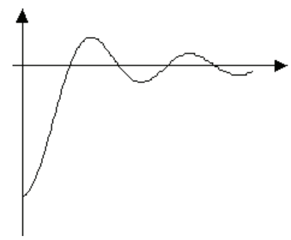
b) positive and increasing

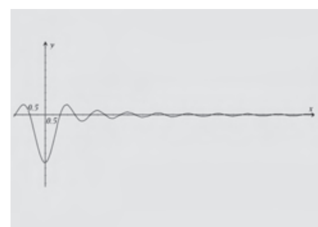


c) positive and decreasing



d) changes periodically from negative to zero to positive, etc.





1.6 Slopes of Tangents and Instantaneous Rates of Change

1. **a) i)** negative **ii)** −1 **b) i)** zero **ii)** 0
c) i) positive **ii)** ≅1.8

2. **a)** 2, 6, 18, 54, 162, 486 **b)** −0.02, −0.028, −0.034, −0.04, −0.044, −0.05

3. **Method 1: Points on the Tangent Line**
Estimate the slope of a tangent at the point (4, 7) on the graph by sketching an approximate tangent line through that point and then selecting a second point on that line. Select the point (7, 4).

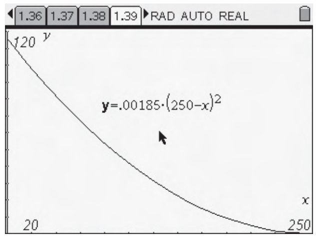
$m = \dfrac{4 - 7}{7 - 4} = \dfrac{-3}{3} = -1$

The instantaneous rate of change at the point (4, 7) on the graph is −1.

Method 2: Graph and Two Points
Estimate the instantaneous rate of change from the graph by finding the slope of a secant passing through the given point (4, 7) and another point close to (4, 7) on the curve. Select the point (6, 4).

$m = \dfrac{4 - 7}{6 - 4} = -\dfrac{3}{2} = -1.5$

4. a) −0.65 mm/s **b) i)** −0.93 mm/s **ii)** −0.70 mm/s **iii)** −0.48 mm/s **iv)** −0.26 mm/s

c) 

The slope of the graph is negative and increasing.

5. a)

Interval	ΔP	Δt	$\frac{\Delta P}{\Delta t}$
$9 \le t \le 10$	$[0.2(10)^2 + 500] - [0.2(9)^2 + 500]$ $= 520 - 516.2$ $= 3.8$	$10 - 9 = 1$	$\frac{3.8}{1} = 3.8$
$9.9 \le t \le 10$	$[0.2(10)^2 + 500] - [0.2(9.9)^2 + 500]$ $= 520 - 519.602$ $= 0.398$	$10 - 9.9$ $= 0.1$	$\frac{0.398}{1}$ $= 3.98$
$9.99 \le t \le 10$	$[0.2(10)^2 + 500] - [0.2(9.99)^2 + 500]$ $= 520 - 519.96002$ $= 0.03998$	$10 - 9.99$ $= 0.01$	$\frac{0.03998}{0.01}$ $= 3.998$
$10 \le t \le 10.1$	$[0.2(10.1)^2 + 500] - [0.2(10)^2 + 500]$ $= 520.402 - 520$ $= 0.402$	$10.1 - 10$ $= 0.1$	$\frac{0.402}{0.1}$ $= 4.02$
$10 \le t \le 10.01$	$[0.2(10.01)^2 + 500] - [0.2(10)^2 + 500]$ $= 520.04002 - 520$ $= 0.04002$	$10.01 - 10$ $= 0.01$	$\frac{0.04002}{0.01}$ $= 4.002$
$10 \le t \le 10.001$	$[0.2(10.001)^2 + 500] - [0.2(10)^2 + 500]$ $= 520.0040002 - 520$ $= 0.0040002$	$10.001 - 10$ $= 0.001$	$\frac{0.0040002}{0.001}$ $= 4.0002$

b) Time intervals before and after 10 years decrease in size.

c) As the size of time intervals decreases, the average rate of change approaches 4. Therefore, the instantaneous rate of change at 10 years is approximately 4. This value represents increase in population at 10 years.

6. a) 1101 **b)** Answers will vary. For example: −9805, 1132 **c)** On average, the number of births per year is increasing.

7. a) The warmest day occurred on day 7, since the thickness of the ice was the smallest.

b) Rate of change $= m_{1,2} = \dfrac{7.6 - 8.9}{2 - 1}$

$= \dfrac{-1.3}{1} = -1.3$

c) Instantaneous rate of change $= \dfrac{\Delta T}{\Delta d}$

$= \dfrac{T(1) - T(0.999)}{1 - 0.999}$

$= \dfrac{[0.1(0.999)^3 + 1.2(0.999)^2 - 5.4(0.999) + 12] - [-0.1(1)^3 + 1.2(1)^2 - 5.4(1) + 12]}{0.001}$

$= \dfrac{7.7033009 - 7.7}{0.001}$

$= 3.3009$ cm/day $\cong 3.3$ cm/day

d) Average rate of change

$= \dfrac{T(1 + h) - T(1)}{(1 + h) - 1} = \dfrac{T(1 + h) - T(1)}{h}$

e) i) Average rate of change

$= \dfrac{T(1 + 0.1) - T(1)}{(1 + 0.1) - 1} = \dfrac{T(1.1) - T(1)}{0.1}$

$= \dfrac{[-0.1(1)^3 + 1.2(1)^2 - 5.4(1) + 12] - [0.1(1.1)^3 + 1.2(1.1)^2 - 5.4(1.1) + 12]}{0.1}$

$= \dfrac{7.7 - 7.3789}{0.1} = 3.2$ cm/day

ii) Average rate of change

$= \dfrac{T(1 + 0.01) - T(1)}{(1 + 0.01) - 1} = \dfrac{T(1.01) - T(1)}{0.01}$

$= \dfrac{[-0.1(1)^3 + 1.2(1)^2 - 5.4(1) + 12] - [-0.1(1.01)^3 + 1.2(1.01)^2 - 5.4(1.01) + 12]}{0.01}$

$= \dfrac{7.7 - 7.6670899}{0.01}$

$= 3.29101$ cm/day $\cong 3.3$ cm/day

iii) Average rate of change

$$= \frac{T(1 + 0.001) - T(1)}{(1 + 0.001) - 1} = \frac{T(1.001) - T(1)}{0.001}$$

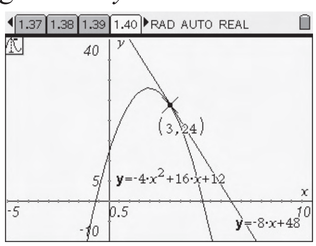
$$= \frac{[-0.1(1)^3 + 1.2(1)^2 - 5.4(1) + 12] - [0.1(1.001)^3 + 1.2(1.001)^2 - 5.4(1.001) + 12]}{0.001}$$

$$= \frac{7.7 - 7.6967009}{0.001}$$

$= 3.2991$ cm/day $\cong 3.3$ cm/day

f) The instantaneous rate of change of the thickness after one day is approximately 3.3 cm/day.

8. −8; tangent line: $y = -8x - 48$



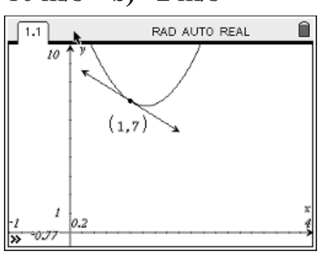
9. **a)** 250

b) Dollars per system sold

c) This tells us that the rate of change in profit is increasing at a rate of \$250/unit when 1000 systems are sold.

10. **a)** 10 m/s **b)** −2 m/s

c)



d) The particle is moving away from the origin between 1 and 4 s. It was moving towards the origin at 1 s.

11. **a)** Average rate of change $= \frac{P(10 + h) - P(10)}{(10 + h) - 10}$

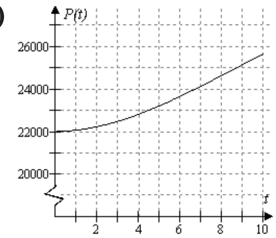
$$= \frac{[-2(10 + h)^3 + 55(10 + h)^2 + 15(10 + h) + 22000] - [-2(10)^3 + 55(10)^2 + 15(10) + 22000]}{h}$$

b) **i)** 482 people/year

ii) 440 people/year

c) 515 people/year

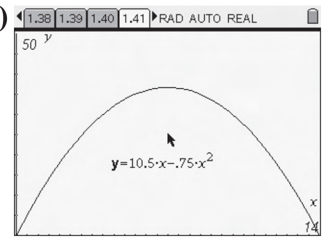
d)



e) Answers may vary. For example: Yes because the population is growing.

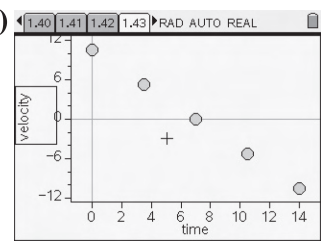
f) No, the population is shrinking in 30 years.

12. **a)**



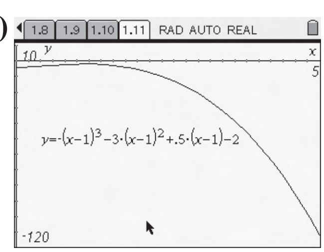
Rachel is running on a track. At the beginning she is running away from the start/finish position, and then she changes direction and runs back toward the start/finish line.

b)



c) At time $t = 0$ s, Rachel is at the start line at position $s = 0$ m. At $t = 3.5$ s she is running away from the start position. At $t = 7$ s she is at the farthest position $s = 36.75$ m, where she changes direction and starts running back toward the finish line. At $t = 14$ s she is back at the finish line with position $s = 0$ m.

13. **a)**



The depth of the fish is increasing.

b)

Interval	Δd	Δt	$\frac{\Delta d}{\Delta t}$
$3 \le t \le 4$	$[-(4-1)^3 - 3(4-1)^2 + 0.5(4-1) - 2] - [-(3-1)^3 - 3(3-1)^2 + 0.5(3-1) - 2]$ $= [-(3)^3 - 3(3)^2 + 0.5(3) - 2] - [-(2)^3 - 3(2)^2 + 0.5(2) - 2]$ $= (-54.5) - (-21)$ $= -33.5$	$4 - 3$ $= 1$	$\frac{-33.5}{1}$ $= -33.5$
$3.5 \le t \le 4$	$[-(4-1)^3 - 3(4-1)^2 + 0.5(4-1) - 2] - [-(3.5-1)^3 - 3(3.5-1)^2 + 0.5(3.5-1) - 2]$ $= [-(3)^3 - 3(3)^2 + 0.5(3) - 2] - [-(2.5)^3 - 3(2.5)^2 + 0.5(2.5) - 2]$ $= (-54.5) - (-35.125)$ $= -19.375$	$4 - 3.5$ $= 0.5$	$\frac{-19.375}{0.5}$ $= -38.75$
$3.9 \le t \le 4$	$[-(4-1)^3 - 3(4-1)^2 + 0.5(4-1) - 2] - [-(3.9-1)^3 - 3(3.9-1)^2 + 0.5(3.9-1) - 2]$ $= [-(3)^3 - 3(3)^2 + 0.5(3) - 2] - [-(2.9)^3 - 3(2.9)^2 + 0.5(2.9) - 2]$ $= (-54.5) - (-50.169)$ $= -4.331$	$4 - 3.9$ $= 0.1$	$\frac{-4.331}{0.1}$ $= -43.31$
$3.99 \le t \le 4$	$[-(4-1)^3 - 3(4-1)^2 + 0.5(4-1) - 2] - [-(3.99-1)^3 - 3(3.99-1)^2 + 0.5(3.99-1) - 2]$ $= [-(3)^3 - 3(3)^2 + 0.5(3) - 2] - [-(2.99)^3 - 3(2.99)^2 + 0.5(2.99) - 2]$ $= (-54.5) - (-54.056199)$ $= -0.443801$	$4 - 3.99$ $= 0.01$	$\frac{0.443801}{0.01}$ $= -44.3801$

c) As the time t approaches 4 s, the average rate of change approaches −44.4 m/s. The fish's velocity at $t = 4$ s is −44.4 m/s.

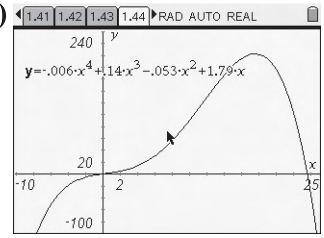
$d(4) = -(4-1)^3 - 3(4-1)^2 + 0.5(4-1) - 2$
$= -(3)^3 - 3(3)^2 + 0.5(3) - 2$
$= -54.5\ m$

At time $t = 4$ s, the fish is swimming at depth a of 54.5 m.

Chapter 1 Challenge Questions

C1. −4

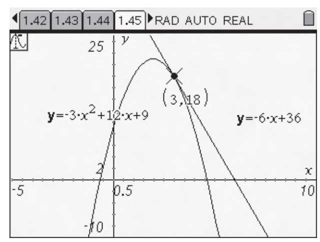
C2. a)



b) 0, 23.498; concentration of dye is zero at $t = 0$ s and $t \cong 23.5$ s **c)** 9.3

C3. a) 250 m, 2250 m, 9000 m
b) distance increases by 9 times

C4. −6, $y = -6x + 36$



C5. 8.44 m by 10.44 m **C6.** \$165/unit
C7. 25 times **C8.** (3, 7)
C9. 4.45; $0 < x < 6.83$

Chapter 2

2.1 The Remainder Theorem

1. a) $\frac{3x^3 - 2x^2 - 8}{x + 5} = 3x^2 - 17x + 85 + \frac{-433}{x + 5}$
b) $x \neq -5$
c) $(3x^2 - 17x + 85)(x + 5) - 433$

2. a) $\frac{3x^4 + 2x^2 - 6x + 1}{x + 1} =$
$3x^3 - 3x^2 + 5x - 11 + \frac{12}{x + 1}$
b) $x \neq -1$
c) $(3x^3 - 3x^2 + 5x - 11)(x + 1) + 12$

3. a) $\frac{2x^2 - x + 5}{x + 3} = 2x - 7 + \frac{26}{x + 3}, x \neq -3$
b) $\frac{x^3 - x - 10}{x + 4} =$
$x^2 - 4x + 15 + \frac{-70}{x + 4}, x \neq -4$
c) $\frac{x^3 + x^2 - 4x + 4}{x - 2} =$
$x^2 + 3x + 2 + \frac{8}{x - 2}, x \neq 2$

d) $\dfrac{3x^4 + 2x^2 - 6x + 1}{x} =$

$3x^3 + 2x - 6 + \dfrac{1}{x}, x \neq 0$

e) $\dfrac{4x^3 - 10x^2 + 6x - 18}{2x - 5} =$

$2x^2 + 3 + \dfrac{-3}{2x - 5}, x \neq \dfrac{5}{2}$

f) $\dfrac{2x^3 - x^2 + 8x + 4}{2x - 1} = x^2 + 4 + \dfrac{8}{2x - 1}$

g) $\dfrac{x^5 - 10x^4 + 20x^3 - 5x - 95}{x + 10} =$

$x^4 - 20x^3 + 220x^2 - 2200x + 21995$

$+ \dfrac{-220045}{x + 10}$

4. **a)** -24 **b)** 7 **c)** -3 **d)** 0

5. $(y + 3) \times (2y + 3) \times (y - 4)$

6. **a)** 0 **b)** -15 **c)** 0 **d)** -15

7. **a)** 9 **b)** 0 **c)** -36 **d)** 8

8. **a)** 12 **b)** -10 **c)** 19 **d)** $\dfrac{8}{3}$

9. **a)** By the remainder theorem, when $P(x)$ is divided by $x + 2$, the remainder is $P(-2)$.

Solve $P(-2) = 26$.

$(-2)^3 + k(-2)^2 - 4(-2) + 2 = 26$

$-8 + 4k + 8 + 2 = 26$

$4k = 24$

$k = 6$

b) $P(-1) = (-1)^3 + 6(-1)^2 - 4(-1) + 2$

$= -1 + 6 + 4 + 2$

$= 11$

$P(1) = (1)^3 + 6(1)^2 - 4(1) + 2$

$= 1 + 6 - 4 + 2$

$= 5$

10. **a)** 37 **b)** Answers may vary.

11. **a)** 3 **b)** -11, 165 **c)** 1 and $2\sqrt{2}$

12. **a)** 0 **b)** $-\dfrac{1}{2}$ **c)** 1 and $2\sqrt{2}$

13. Use the division statement $p(x) = (x + 3)Q(x) + R$ to find $p(x)$.

$p(x) = (x + 3)(x^2 - 3x + 5) + 6$

$= x^3 - 3x^2 + 5x + 3x^2 - 9x + 15 + 6$

$= x^3 - 4x + 21$

14. $x^3 + 2x^2 - 15x + 10$

15. **a)** $Q(t) = 0.45t$

$R = 3.3t$

$\dfrac{N(t)}{t - 2} = Q(t) + \dfrac{R}{t - 2}, t \neq 2$

b) yes, as time increases, number of tickets sold also increases

16. **a)** 0 **b)** $(x - 3)$ is a factor of the polynomial **c)** $(x + 2)(2x + 5)(x - 3)$

17. $k = 7$ **18.** $a = 1, b = -2$

19. **a)** $c = 5$ **b)** Answers may vary.

20. $k = -5$

21. $4\dfrac{40}{81}$ **22.** $m = 3, n = 3$

23. Let $P(x) = -x^3 - vx^2 + 2x + w$

By the remainder theorem, when $P(x)$ is divided by $x + 2$, the remainder is $P(-2)$.

Solve $P(-2) = 6$.

$-(-2)^3 - v(-2)^2 + 2(-2) + w = 6$

$8 - 4v - 4 + w = 6$

$-4v + w = 2$

By the remainder theorem, when $P(x)$ is divided by $x - 3$, the remainder is $P(3)$.

Solve $P(3) = 119$.

$-(3)^3 - v(3)^2 + 2(3) + w = 119$

$-27 - 9v + 6 + w = 119$

$-9v + w = 140$

Solve the system of equations for v and w:

$-4v + w = 2$ // equation 1

$-9v + w = 140$ // equation 2

equation 1 $-$ equation 2:

$5v = -138$

$v = -\dfrac{138}{5}$

Substitute $v = -\dfrac{138}{5}$ in equation 1 and solve for w:

$-4\left(-\dfrac{138}{5}\right) + w = 2$

$\dfrac{552}{5} + w = 2$

$w = -\dfrac{542}{5}$

24. **a)** no **b)** no **c)** no **25.** $a = -5, b = 10$

2.2 The Factor Theorem

1. **a)** $(x + 4)$ **b)** $(x - 2)$ **c)** $(5x - 2)$ **d)** $(3x + 4)$

2. **a)** yes **b)** yes **c)** no

3. **a)**

-7	1	20	91
+		-7	-91
×	1	13	0

b)

3	1	-9	27	-28
+		3	-18	27
×	1	-6	9	-1

4. a) $\pm1, \pm2; (x-1)(x^2-2x+2)$
b) $\pm1; (x+1)(x^2+x+1)$
c) $\pm1, \pm3, \pm5, \pm15; (x+3)(x^2+4x+5)$

5. a) $(x+2)(x-2)(x-1)$ **b)** $(x-2)^2(x+2)$
c) $(x+1)(x+1)(x+1)$
d) $(x+2)(x-1)(x^2+x+1)$
e) $(x-6)(3x^3+1)$ **f)** $(x-1)(x+1)(x^2+1)$

6. a) $\pm1, \pm\frac{1}{2}, \pm3, \pm\frac{3}{2}; (x-1)(x-3)(2x-1)$

b) $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}\ \pm3, \pm\frac{3}{2}, \pm\frac{3}{4};$
$(x+1)(2x+1)(2x-3)$

c) $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm3, \pm\frac{3}{2}, \pm\frac{3}{4};$
$(x-1), (2x-3)(2x+1)$

d) $\pm1, \pm\frac{1}{3}; (x-1)(x+1)(3x-1)$

7. a)

−7	1	−1	−56
+		−7	56
×	1	−8	0

b)

3	1	−9	27	−28
+		3	−18	27
×	1	−6	9	−1

c)

1	2	0	−2	−3
+		2	2	0
×	2	2	0	−3

d)

−2	1	0	−8	0	16
+		−2	4	8	−16
×	1	−2	−4	8	0

e)

$\frac{3}{2}$	2	−7	−10	26
+		3	−6	−24
×	2	−4	−16	2

f)

$\frac{1}{3}$	9	0	2	−1
+		3	1	1
×	9	3	3	0

8. a) $(x-1)(2x^2+x+2)$ **b)** $(x+4)(x+1)(2x-1)$ **c)** cannot be factored
d) $(x+3)(x-3)(2x+1)^2$ **e)** cannot be factored **f)** $(x-1)(2x-1)(3x+1)$
g) $(x-4)(2x+1)(x-1)^2$

9. $k=9$ **10.** $k=3$

11. Let $p(x)=2x^3-(k+1)x^2+6kx+11$
Use the factor theorem to solve $p(1)=0$.
$$2(1)^3-(k+1)(1)^2+6k(1)+11=0$$
$$2-k-1+6k+11=0$$
$$5k=-12$$
$$k=-\frac{12}{5}$$

12. a) Factor by grouping terms:
$$\begin{aligned}p(x)&=x^3+2x^2-9x-18\\&=x^2(x+2)-9(x+2)\\&=(x+2)(x^2-9)\\&=(x+2)(x-3)(x+3)\end{aligned}$$
So, $p(x)=x^3+2x^2-9x-18$
$=(x+2)(x-3)(x+3)$

b) $(x-1)(2x+1)(2x-3)$

c) Let $p(x)=6x^3+x^2-31x+10$.
Use the rational zero theorem:
Let b represent the factors of the constant term 10, which are $\pm1, \pm2, \pm5, \pm10$.
Let a represent the factors of the leading coefficient 6, which are $\pm1, \pm2, \pm3, \pm6$.
The possible values of $\frac{b}{a}$ are $\pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm2, \pm\frac{2}{3}, \pm5, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm10, \pm\frac{10}{3}$
Test the values of $\frac{b}{a}$ for x to find the zeros.
Substitute $x=2$ to test
$p(2)=6(2)^3+(2)^2-31(2)+10$
$=48+4-62+10=0$
$x=2$ is a zero and $x-2$ is a factor.
Use synthetic division method to determine the other factors.

2	6	1	−31	10
+		12	26	−10
×	6	13	−5	0

$6x^2+13x-5$ can be factored further to
$6x^2+13x-5=(2x+5)(3x-1)$
So, $p(x)=6x^3+x^2-31x+10$
$=(x-2)(2x+5)(3x-1)$

d) $(2x+3)(2x^2-3x+7)$ **e)** $(x+3)(3x-1)(x-2)$ **f)** $(x+1)^2(x+3)(x-4)$
g) $x(x+2)(x^2+x-3)$

13. a) $(2x - 1)(4x^2 + 2x + 1)$ **b)** $64(2x - y)(4x^2 + 2xy + y^2)$ **c)** $\left(x - \frac{1}{3}\right)\left(x^2 + \frac{1}{3}x + \frac{1}{9}\right)$
d) $(1 + 5x)(1 - 5x + 25x^2)$
e) $\frac{1}{8}\left(x + \frac{1}{2}\right)\left(x^2 - \frac{1}{2}x + \frac{1}{4}\right)$ **f)** $5(3x + 5y)(9x^2 - 15xy + 25y^2)$

14. $Q(x) = 3x - 6$; $R = -5x^2 + 12x - 13$

15. a) $14(1)^{99} - 65(1)^{56} + 51 = 0$
b) $(1)^5 + 1 = 2$

16. $f(x) = 32x^4 - 128x^3 - 54x^2 + 243x - 108$

$f(1) = 32 - 128 - 54 + 243 - 108$
$= -15$ therefore, $x - 1$ is not a factor

$f(2) = 32(2)^4 - 128(2)^3 - 54(2)^2 + 243(2) - 108$
$= -350$ $x - 2$ is not a factor

$f(3) = 32(3)^4 - 128(3)^3 - 54(3)^2 + 243(3) - 108$
$= -729$ $x - 3$ is not a factor

$f(4) = 32(4)^4 - 128(4)^3 - 54(4)^2 + 243(4) - 108$
$= 0$ $x - 4$ is a factor

$f(-1) = 32 + 128 - 54 - 243 - 108$
$= -245$ $x + 1$ is not a factor

$f(-2) = 32(-2)^4 - 128(-2)^3 - 54(-2)^2 + 243(-2) - 108$
$= 726$ $x + 2$ is not a factor

$f(-3) = 32(-3)^4 - 128(-3)^3 - 54(-3)^2 + 243(-3) - 108$
$= 4725$ $x + 3$ is not a factor

$f(-4) = 32(-4)^4 - 128(-4)^3 - 54(-4)^2 + 243(-4) - 108$
$= 14440$ $x + 4$ is not a factor

Somewhere between $f(-1)$ and $f(-2)$ there must be a zero since the value of $f(x)$ changes from a negative for $x = -1$ to a positive for $x = -2$. Using the remainder theorem for a value between $x = -1$ and $x = -2$, the remainder for $x = -\frac{3}{2}$:

$f(x) = 32x^4 - 128x^3 - 54x^2 + 243x - 108$

$$f\left(-\frac{3}{2}\right) = 32\left(-\frac{3}{2}\right)^4 - 128\left(-\frac{3}{2}\right)^3 - 54\left(-\frac{3}{2}\right)^2 + 243\left(-\frac{3}{2}\right) - 108$$

$= 0$ $2x + 3$ is also factor

Dividing the polynomial by $(x - 4)(2x + 3)$ or $2x^2 - 5x - 12$:

$$\begin{array}{r} 16x^2 - 24x + 9 \\ 2x^2 - 5x - 12\,\overline{)\,32x^4 - 128x^3 - 54x^2 - 243x - 108} \\ \underline{-(32x^4 - 80x^3 - 192x^2)} \\ -48x^3 + 138x^2 - 243x \\ \underline{-(-48x^3 + 120x^2 + 288x)} \\ 18x^2 - 45x - 108 \\ \underline{-(18x^2 - 45x - 108)} \\ 0 \end{array}$$

$32x^4 - 128x^3 - 54x^2 - 243x - 108$
$= (x - 4)(2x + 3)(16x^2 - 24x + 9)$
$= (x - 4)(2x + 3)(4x - 3)^2$

17. $a^n - a^n = 0$

18. $(a - a)^3 + (a - b)^3 + (b - a)^3 =$
$0 + (a - b)^3 - (a - b)^3 = 0$

2.3 Polynomial Equations

1. a) $x = 0, x = 1, x = -3$ **b)** $x = -2, x = -5$ **c)** $x = \frac{5}{2}, x = -3, x = \frac{1}{4}$ **d)** $x = \frac{7}{2}, x = \frac{4}{3}, x = -6$ **e)** $x = 3, x = -\frac{1}{2}, x = \frac{2}{15}$
f) $x = -8, x = 9, x = \frac{1}{3}$ **g)** $x = 2, x = \pm\frac{4}{5}$

2. a) $x = -4, x = -3, x = 0, x = 2$
b) $x = -1, x = 0, x = 1$ **c)** $x = -2, x = 4$

3. a) $x = 9, x = -9, x = -3$ **b)** $x = 5, x = -3, x = 2$ **c)** $x = \frac{1}{3}, x = 2, x = -2, x = -8$ **d)** $x = 3, x = -3, x = 1$ **e)** $x = -\frac{1}{2}, x = \frac{5}{2}, x = 2, x = -2$ **f)** $x = 10, x = -10$
g) $x = -1, x = \frac{4}{3}, x = -\frac{1}{2}, x = \frac{1}{3}$

4. a) $0, \pm\frac{7}{4}$ **b)** $0, -6$ **c)** $0, 9, 7$ **d)** $0, \pm 11$
e) $0, 3$ **f)** $\pm 2\sqrt{2}$ **g)** $\pm 3, \pm\sqrt{3}$

5. Since the zeros are -2 and 1, the factors for a cubic function are $(x + 2)$ and $(x - 1)$. Since there is a local minimum at $(1, 0)$, $(x - 1)$ will be of order 2. An equation for a cubic function is
$y = (x - 1)^2(x + 2)$.

6. $y = (x + 4)(x - 2)^2(x - 1)$

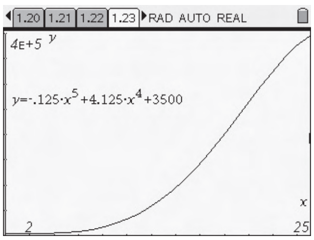
7. a) $x = 5, x = -2$ **b)** $x = -2, x = 2, x = 3$
c) $x = -4, x = 3, x = 4$ **d)** $x = -1, x = 2, x = 3$ **e)** $x = -4, x = -3, x = -1$
f) $x = -3, x = -1, x = 4$ **g)** $x = -3, x = -1, x = 2, x = 3$ **8.** same **9. a)** $x = -2, 3$ **b)** $x = -3, -1, 2, 3$ **c)** $x = -3, -2, 2$
d) $x = -3, -2, -1, 1, 2$

10. a) $x = 3.4, x = 0.6$ **b)** $x = 2, x = 0.4, x = -2.4$ **c)** $x = 2, x = 5.2, x = -0.2$ **d)** $x = -2$ **e)** $x = -2$ **f)** $x = -0.3, x = 2.9$

11. a) 3.9 bacteria/h, −38.1 bacteria/h **b)** growth rate never reaches 20 bacteria per hour during the first 100 hours of the experiment **c)** 14.5 h **d)** 26.8 h; decline in bacteria population **e)** 8.2 h

12. The box does not reach a volume of 9600 cm ^{3}.

13. a)



b) 1905 represents year 0. Substitute 0 for x in $f(x)$:

$f(0) = -0.125(0)^5 + 4.125(0)^4 + 3500$
$= 3500$

The population of wolves was 3500.

c) 1920 represents year 15. Substitute 15 for x in $f(x)$:

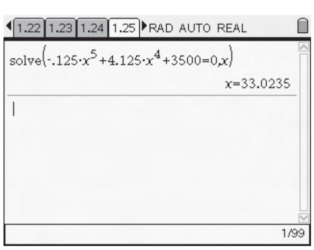
$f(15) = -0.125(15)^5 + 4.125(15)^4 + 3500$
$= -94921.875 + 208828.125 + 3500$
$= 117406.25$
$\cong 117407$

The population of wolves was 117 407.

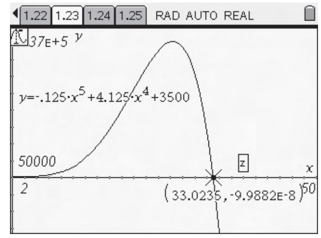
d) Solve $f(x) = 0$

$-0.125x^5 + 4.125x^4 + 3500 = 0$

Use a graphing calculator to solve the equation. Use the solve() function to solve $-0.125x^5 + 4.125x^4 + 3500 = 0$



Or, use the graph from part a) to determine the zero of the function $y = -0.125x^5 + 4.125x^4 + 3500$



The wolf population became zero at approximately $x = 33$, in 1938.

14. $y = ax^3 + bx^2 + cx + d$

Point (0, −1) *is on the graph. Substitute* 0 *for* x *and* −1 *for* y.

$a(0)^3 + b(0)^2 + c(0) + d = -1$; $\boldsymbol{d = -1}$

Point (1, 0) is on the graph. Substitute 1 for x and 0 for y.

$0 = a(1)^3 + b(1)^2 + c(1) + d$

$0 = a + b + c + d$

Substitute −1 for d.

$a + b + c - 1 = 0$

$a + b + c = 1$ // Equation 1

Point (−1, −4) is on the graph.

Substitute −1 for x and −4 for y.

$a(-1)^3 + b(-1)^2 + c(-1) + d = -4$

$-a + b - c + d = -4$

Substitute −1 for d.

$-a + b - c - 1 = -4$

$-a + b - c = -3$ // Equation 2

Point (2, 5) is on the graph. Substitute 2 for x and 5 for y.

$a(2)^3 + b(2)^2 + c(2) + d = 5$

$8a + 4b + 2c + d = 5$

Substitute −1 for d.

$8a + 4b + 2c - 1 = 5$

$8a + 4b + 2c = 6$ // Equation 3

Solve the system of equations:

$a + b + c = 1$ // Equation 1
$-a + b - c = -3$ // Equation 2
$8a + 4b + 2c = 6$ // Equation 3

Equation 1 + Equation 2:

$2b = -2$; $\boldsymbol{b = -1}$

Substitute −1 for b in Equation 1:

$a - 1 + c = 1$; $a + c = 2$

Substitute −1 for b in Equation 3:

$8a + 4(-1) + 2c = 6$

$8a + 2c = 10$; $4a + c = 5$

Solve the system of equations:

$a + c = 2$ // Equation 4
$4a + c = 5$ // Equation 5

Equation 5 − Equation 4:

$3a = 3$; $a = 1$

Substitute 1 for a in Equation 4:
$1 + c = 2; c = 1$
Substitute $a = 1, b = -1, c = 1, d = -1$ in $y = ax^3 + bx^2 + cx + d$ to get the equation of the cubic function whose graph passes through the points (1, 0), (0, −1), (−1, −4), (2, 5): $y = x^3 - x^2 + x - 1$

15. $(x + 4)(x + 3)(x + 1)(x - 1)(x - 3)$

16. a) $x = 0, x = \frac{1 \pm i\sqrt{19}}{2}$ **b)** $x = 0, x = \frac{9}{2}$
c) $x = 0, x = \pm 4$ **d)** $x = \pm i, x = \pm 3i$
e) $x = \pm\sqrt{2 + \sqrt{3}}, x = \pm\sqrt{2 - \sqrt{3}}$

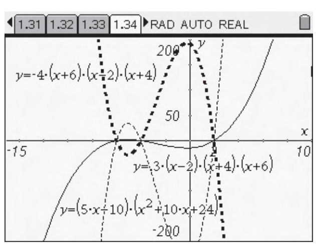
2.4 Families of Polynomial Functions

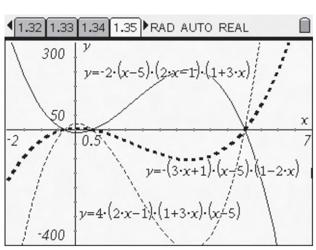
1. a) $y = k(x - 5)(x - 8), k \in R$
b) Answers may vary. Sample answer:
$y = (x - 5)(x - 8)$; $y = 2(x - 5)(x - 8)$
c) $y = -3(x - 5)(x - 8)$

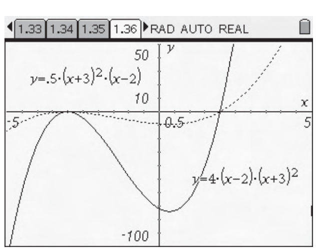
2. a) −2, 3, 8
b) Answers may vary. Sample answer:
$y = (x + 2)(x - 3)(x - 8)$;
$y = 2(x + 2)(x - 3)(x - 8)$
c) $y = 4(x + 2)(x - 3)(x - 8)$

3. a) b) f) $y = k(x + 4)(x - 3)$
c) d) e) $y = k(2x - 1)(x - 5)$

4. a), e), and **h)** belong to one family; **b), d),** and **g)** belong to another family; **c)** and **f)** belong to a third family



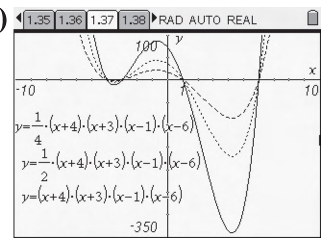




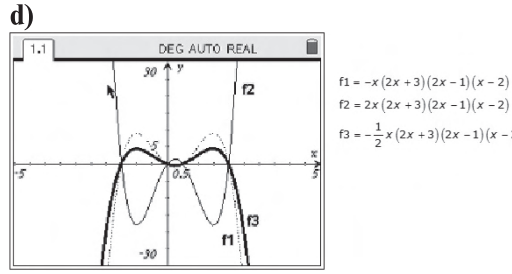
5. a) $y = k(x - 3)(x + 2)$ **b)** $y = kx(x - 1)(x - 5)$ **c)** $y = k(x + 3)(x - 1)(x - 6)$
d) $y = kx(x + \sqrt{3})(x - \sqrt{3})(3x - 2)$

6. a) $y = k(x + 5)(x + 1)(x - 2)$
b) Answers may vary. Sample answer:
$y = (x + 5)(x + 1)(x - 2)$;
$y = 2(x + 5)(x + 1)(x - 2)$
c) $y = (x + 5)(x + 1)(x - 2)$

7. a) $y = k(x + 4)(x + 3)(x - 1)(x - 6)$
b) Answers may vary. Sample answer:
$y = (x + 4)(x + 3)(x - 1)(x - 6)$;
$y = \frac{1}{2}(x + 4)(x + 3)(x - 1)(x - 6)$
c) $y = \frac{1}{4}(x + 4)(x + 3)(x - 1)(x - 6)$
d)



8. a) Since the zeros are $\frac{3}{2}$, 0, $\frac{1}{2}$, and 2, the factors for the family of quartic functions are $(2x + 3)$, x, $(2x - 1)$, and $(x - 2)$. An equation for this family is $y = kx(2x + 3)(2x - 1)(x - 2)$, where $k \in R$.
b) Use any two values for k to write two members of the family.
For $k = 2$, $y = 2x(2x + 3)(2x - 1)(x - 2)$
For $k = -1$, $y = -x(2x + 3)(2x - 1)(x - 2)$
c) Since the graph passes through (−1, 4.5), substitute $x = -1$ and $y = 4.5$ into
$y = kx(2x + 3)(2x - 1)(x - 2)$
$4.5 = k(-1)[2(-1) + 3][2(-1) - 1][(-1) - 2]$
$4.5 = k(-1)(1)(-3)(-3)$
$4.5 = -9k; k = \frac{-1}{2}$
The equation is
$y = \frac{-1}{2x}(2x + 3)(2x - 1)(x - 2)$
d)



9. a) From the graph, the x-intercepts are -4, -2, 2, and 3.
The corresponding factors are $(x + 4)$, $(x + 2)$, $(x - 2)$, and $(x - 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 4)(x + 2)(x - 2)(x - 3)$
Select a point that the graph passes through, such as A(1, 90).
Substitute $x = 1$ and $y = 90$ into the equation to solve for k.
$90 = k(1 + 4)(1 + 2)(1 - 2)(1 - 3)$
$90 = k(5)(3)(-1)(-2)$
$90 = 30k; k = 3$
The equation is
$y = 3(x + 4)(x + 2)(x - 2)(x - 3)$.
b) From the graph, the x-intercepts are $-\frac{5}{2}$, 0 (order 3), and 2.
The corresponding factors are $(2x + 5)$, x^3, $(x - 2)$.
An equation for the family of polynomial functions with these zeros is
$y = kx^3(2x + 5)(x - 2)$
Select a point that the graph passes through, such as A(-2, -32).
Substitute $x = -2$ and $y = -32$ into the equation to solve for k.
$-32 = k(-2)^3[2(-2) + 5][(-2) - 2]$
$-32 = k(-8)(1)(-4)$
$-32 = 32k; k = -1$
The equation is $y = -x^3(2x + 5)(x - 2)$

10. a) $y = k(x^4 - 8x^3 - 2x^2 + 120x - 175)$
b) $y = 0.3(x^4 - 8x^3 - 2x^2 + 120x - 175)$

11. $y = (x + 4)^2(2x + 1)(x - 7)$
$y = (x + 4)(2x + 1)(x - 7)$
similarities: same domain, share three roots; differences: different range, degree, y-intercepts

12. a) Since the zeros are $1 \pm \sqrt{5}$, -3, and $\frac{1}{3}$, the factors for the family of quartic functions are $(x - 1 - \sqrt{5})$, $(x - 1 + \sqrt{5})$, $(x + 3)$, and $(3x - 1)$. An equation for this family is $y = k(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})(x + 3)(3x - 1)$, where $k \in R$
b) Since the graph passes through the point $(-2, -7)$, substitute $x = -2$ and $y = -7$ into the equation to solve for k.
$-7 = k[(-2)-1 - \sqrt{5}][(-2)-1 + \sqrt{5}]((-2) + 3)(3(-2) - 1)$,
$-7 = k(-3 - \sqrt{5})(-3 + \sqrt{5})(1)(-7)$
$-7 = k(4)(1)(-7)$
$-7 = -28k; k = \frac{1}{4}$
The equation is
$y = \frac{1}{4}(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})(x + 3)(3x - 1)$

13. Since the zeros are $1 \pm \sqrt{5}$, -3, and $\frac{1}{3}$, the factors for the family of functions are $(x - 1 - \sqrt{5})$, $(x - 1 + \sqrt{5})$, $(x + 3)$, and $(3x - 1)$. To determine the equation for a family of functions with the same zeros, but with a higher degree, one or more of the zeros must be of order 2 or higher. An equation for this family is
$y = k(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})(x + 3)^2(3x - 1)^3$, where $k \in R$.

14. a) $y = k(x + 1.5)^2(x - 7)$ b) 3
c) $y = -0.5(x + 1.5)^2(x - 7)$
d) $y = 0.5(x - 1.5)^2(x + 7)$
e) $y = 0.5(x + 1.5)^2(x - 7)$

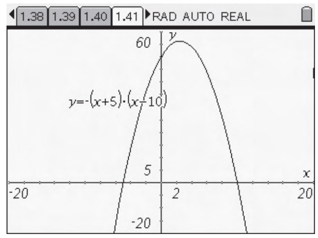
15. a) $(x - 2)$ is a factor b) Answers may vary. For example: As k increases, the height of the relative minimum to the right of $x = 0$ decreases and moves to the right and the relative maximum between $x = 0$ and $x - 2$ decreases and moves to the right. However, the y-intercept and the minimum at $x = 2$ remains constant. c) $k < -4$ or $k > 4$
d) Answers may vary. For example: $k = -2$

16. a) $y = k(x - 25)(x - 50)(x - 200)^2(x - 350)$
b) $y = k(0.5x - 25)(0.5x - 50)(0.5x - 200)^2(0.5x - 350)$

2.5 Solve Inequalities Using Technology

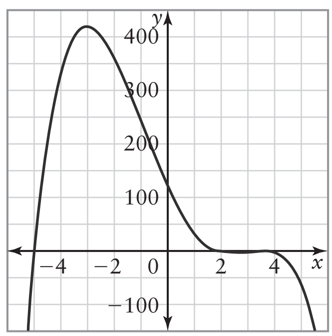
1. a) $x > 1$ b) $x \le 2.5$ c) $2.5 < x < 6$
d) $0 \le x \le 3$, $x > 10$
2. a) $x < -2$, $-2 < x < 0$, $x > 0$ b) $x < -3$, $-3 < x < 1$, $1 < x < 2$, $x > 2$ c) $x < 1$, $1 < x < 5$, $5 < x < 6$, $6 < x < 10$, $x > 10$
d) $x < -14$, $-14 < x < -12$, $-12 < x < -0.5$, $-0.5 < x < 21$, $x > 21$
3. a) i) 0, 3 ii) $0 < x < 3$ iii) $x < 0$, $x > 3$
b) i) -2, 2 ii) $x > 2$ iii) $x < -2$, $-2 < x < 2$
c) i) -5, 2, 6 ii) $-5 < x < 2$, $x > 6$
iii) $x < -5$, $2 < x < 6$ d) i) -1.5, 0
ii) $x < -1.5$, $-1.5 < x < 0$ iii) $x > 0$

4. a) Answers may vary. For example:



b) $-(x + 5)(x - 10) > 0$;
$-(x + 5)(x - 10) < 0$

5. a) Answers may vary. For example:



For the required quartic function, the function must exist below the x-axis for the intervals $x \le -5$, $2 \le x \le 3$, and $x \ge 4$, and must exist above the x-axis for $-5 < x < 2$ and $3 < x < 4$. From this information, the x-intercepts are $x = -5, 2, 3$, and 4. Therefore, the function must be $f(x) = -(x + 5)(x - 2)(x - 3)(x - 4)$, with the negative required to have the needed behaviour in the intervals.

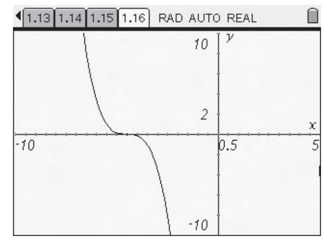
b) $f(x) \le 0$, for $x \le -5$, $2 \le x \le 3$ and $x \ge 4$
$f(x) > 0$, for $-5 < x < 2$ and $3 < x < 4$

6. a) $3 \le x \le 10$ **b)** $x < -12$, $x > -8$
c) $x \le -2$, $2 \le x \le 3$ **d)** $-4 \le x \le 3$, $x \ge 4$
e) $x < -1$, $2 < x < 3$ **f)** $-4 \le x \le -3$, $x \ge -1$ **g)** $-3 < x < -1$, $2 < x < 3$

7. a) $-1.5 \le x \le 1$, $x \ge 2$ **b)** $x < -\sqrt{2}$, $\frac{1}{3} < x < \sqrt{2}$ **c)** $1-\sqrt{2} \le x \le 0.5$, $x \ge 1+\sqrt{2}$
d) $x \le -0.5$, $1 \le x \le 1.5$ **e)** $-3<x<1.5$, $x>2$ **f)** $-3 \le x \le -1.5$, $x \ge 0.5$
g) $x < -2.5$, 1 __ 3 $< x < 2$

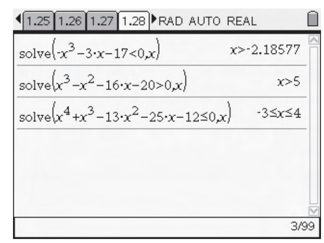
8. a) $x < -0.32$, $x > 6.32$ **b)** $-8.67 < x < 0.17$
c) $-3.60 \le x \le -2.32$, $x \ge 1.92$
d) no solution **e)** $x < -1.34$, $-0.23 < x < 6.57$ **f)** $x > -1.24$ **g)** $-2.10 < x < 1.59$

9. a) Answers may vary. For example:
$y = -(x + 4.5)^3$



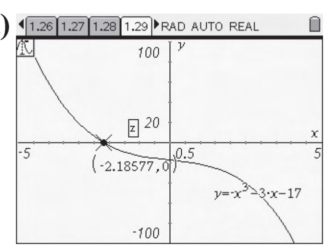
b) The family of functions $y = -k(x + 4.5)^3$ also satisfies this criterion.

10. a) $x > -2.2$ **c)** $x > 5$ **e)** $-3 \le x \le 4$
Use a graphing calculator to solve the three inequalities. Use the solve() function.



Or graph the corresponding polynomial functions. Find the zeros, and then find the values of x that satisfy the inequalities.

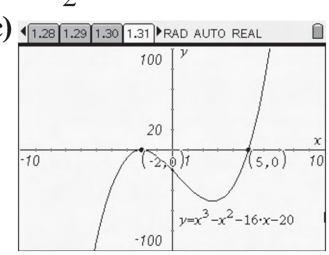
a)



$x > -2.2$

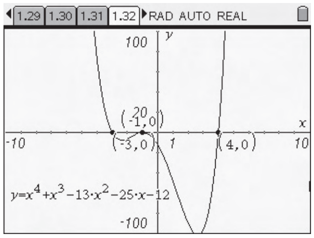
b) $\frac{5 - \sqrt{29}}{2} \le x \le 2$ or $x \ge \frac{5 + \sqrt{29}}{2}$

c)



$x > 5$

d) $\frac{-3 - \sqrt{13}}{4} \le x \le 0$ or $x \ge \frac{-3 + \sqrt{13}}{4}$

e) 

$-3 \le x \le 4$

f) $-1.0785 < x < 1.2906$

11. approximately 2.3 s

12. a) A rectangular prism cannot have dimensions less than or equal to zero; therefore, the restrictions on x are: $\{x \in R, x > 3\}$

b) $V(x) = \frac{1}{3}(\text{base}) \times (\text{height})$

$V(x) = \frac{1}{3}(2x - 1)(x - 3)(3x - 4)$

Solve $V(x) \ge 4$

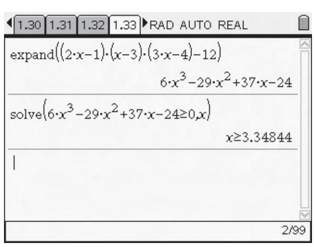
$\frac{1}{3}(2x - 1)(x - 3)(3x - 4) \ge 4$

$(2x - 1)(x - 3)(3x - 4) \ge 12$

$6x^3 - 29x^2 + 37x - 12 \ge 12$

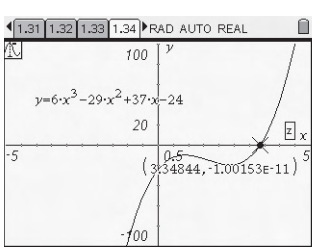
$6x^3 - 29x^2 + 37x - 24 \ge 0$

Use a graphing calculator to solve the above inequality. Use the solve() function.



$x \ge 3.3$

Graph the corresponding polynomial function. Find the zeros, and then find the values of x that satisfy the inequality.



$x \ge 3.3$

13. a) $V(x) = \frac{2}{3}x^3$ **b)** $1 \le x \le 4.48$ and $\frac{2}{3} \le V(x) \le 60$

14. a) $(x + 2.5)(x - 3.5)(x - 5) > 0$;
$-(x + 2.5)(x - 3.5)(x - 5) < 0$
b) $(x + 2\sqrt{2})(x + \sqrt{2})(x - \sqrt{2}) < 0$;
$-(x + 2\sqrt{2})(x + \sqrt{2})(x - \sqrt{2}) > 0$

15. a) $x < -3$, $x > 2$ **b)** $x < -5$
c) $-4 < x < -1$,
$-\frac{3}{4} < x < \frac{2}{3}$, $2 < x < 3$

16. Answers may vary. For example:
a) $(x + 3)(x - 2) > 0$; $(x + 3)^3(x - 2) > 0$
b) $(x + 5)^3 < 0$; $(x + 5)^5 < 0$
c) $(x + 4)(x + 1)(4x + 3)(3x - 2)(x - 2)(x - 3) < 0$; $(x + 4)(x + 1)(4x + 3)(3x - 2)(x - 2)(x - 3)^3 < 0$

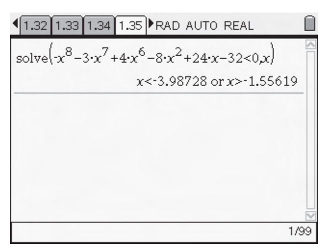
17. Answers may vary. For example:
$y = x^2 + 1$ has two complex roots, $x = \pm i$
$x \in R$ is a solution to $x^2 + 1 > 0$.
There is no solution to $x^2 + 1 < 0$.

18. a) $x < 0.22$, $x > 2.28$ **b)** $x < -1.34$, $-0.32 < x < 1.16$ **c)** $x < 0.92$
d) $x \le -2.66$, $-1.21 \le x \le 1.87$
e) $0.77 < x < 1.31$

19. a) $x > 2.7$ **b)** $x < 0.5$ **c)** $-3.4 \le x \le 0.5$, $x \ge 2.9$ **d)** $1.3 \le x \le 2.8$

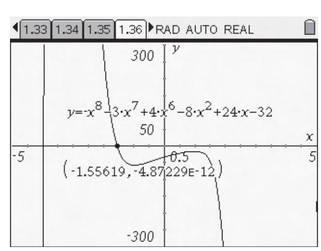
20. Answers will vary. For example:
$8x^4 - 68x^3 + 34x^2 + 425x - 525 > 0$

21. Use a similar graphing calculator to solve the inequality. Use the solve() function.



$x < -3.99$ or $x > -1.56$

Graph the corresponding polynomial function. Find the zeros, and then find the values of x that satisfy the inequality.



$x < -3.99$ or $x > -1.56$

22. $x \le -0.60$, $0.54 \le x \le 3$.

23. Translate the model approximately 15 units up.

2.6 Solve Factorable Inequalities Algebraically

1. a) $x \le 13$ b) $x \le 6$ c) $x < -3$ d) $x \ge -11$ e) $x < -1$ f) $x \le \frac{1}{12}$
2. a) $-4 \le x \le 2$ b) $-3 \le x \le 2$ c) $x < -\frac{2}{3}, x > \frac{3}{4}$ d) $\frac{1}{2} < x < \frac{5}{6}$
3. a) $-4 < x < -2, x > 1$ b) $x \le -4, -2 \le x \le 1$ c) $x \le -3, \frac{1}{2} \le x \le \frac{3}{5}$ d) $x \le \frac{1}{4}$
4. a) $x \le -4, x \ge 6$ b) $5 - \sqrt{46} \le x \le 5 + \sqrt{46}$ c) $-\frac{3}{2} < x < \frac{1}{3}$ d) $\frac{-5 - \sqrt{97}}{4} \le x \le \frac{-5 + \sqrt{97}}{4}$ e) $-4 < x < -2, x > 4$ f) $x \le -4, -1 \le x \le 1$
5. a) $-4 \le x \le 1$ b) $x \le -5, 0 \le x \le 4$ c) $x < -1, x > 2$ d) $x \ge 3$ e) $x < -4, -\frac{4}{3} < x < 1$ f) no solution
6. a) $x \ge 2$; As the line of the inequality moves down from $y = 0$ to $y = -2$ to finally $y = -10$ the portions of the function $y = x^3 - 3x - 2$ that satisfy the inequality also shifts. For the inequality in a) the line $y = 0$ intersects the curve $y = x^3 - 3x - 2$ twice, the inequality in b) has the line $y = -2$ intersecting the curve $y = x^3 - 3x - 2$ three times and in c) the line $y = -10$ intersects the curve $y = x^3 - 3x - 2$ only once. b) $-\sqrt{3} \le x \le 0, x \ge \sqrt{3}$ c) $x \ge -2.49$
7. $A(x) = \text{length} \times \text{width}$
   Let width = x and length = $x + 6$
   $A(x) = (x + 6)x = x^2 + 6x$
   Solve $A(x) \ge 630$
   $x^2 + 6x \ge 630$
   $x^2 + 6x - 630 \ge 0$
   Use quadratic formula to find the zeros of the polynomial $p(x) = x^2 + 6x - 630$
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   $$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-630)}}{2(1)}$$
   $$x = \frac{-6 \pm \sqrt{2556}}{2}$$
   $$x = \frac{-6 \pm 6\sqrt{71}}{2}$$
   $$x = -3 \pm 3\sqrt{71}$$
   The solution to the inequality is:
   $x \le -3 - 3\sqrt{71}$ or $x \ge -3 + 3\sqrt{71}$
   Or approximately: $x \le -28.3$ or $x \ge 22.3$
   Since width or length of a rectangle cannot be negative, the possible widths of a rectangle are $x \ge -3 + 3\sqrt{71}$ cm or approximately $x \ge 22.3$ cm.
8. a) Factor the polynomial function.
   Let $p(x) = x^3 - x^2 - 34x - 56$
   Find a value $x = b$ such that $p(b) = 0$
   By the integral zero theorem, test factor of -56, that is $\pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$
   Substitute $x = -2$ to test
   $p(-2) = (-2)^3 - (-2)^2 - 34(-2) - 56$
   $= -8 - 4 + 68 - 56 = 0$
   $x = -2$ is a zero and $x + 2$ is a factor.
   Use synthetic division method to determine the other factors.

−2	1	−1	−34	−56
+		−2	6	56
×	1	−3	−28	0

   $x^2 - 3x - 28$ can be further factored to $(x + 4)(x - 7)$
   So $p(x) = x^3 - x^2 - 34x - 56$
   $= (x + 2)(x + 4)(x - 7)$
   Use intervals $x \le -4$, $-4 \le x \le -2$, $-2 \le x \le 7$, and $x \ge 7$
   Test arbitrary values of x for each interval.
   The solution to the inequality $(x + 2)(x + 4)(x - 7) \ge 0$ is:
   $-4 \le x \le -2$ or $x \ge 7$

   b) $x < -2, -\frac{4}{3} < x < 1$

   c) Factor the polynomial function.
   Let $p(x) = 3x^3 + 4x^2 - 5x - 2$
   Use the rational zero theorem to determine the values that should be tested.
   Let b represent the factors of the constant term -2, which are $\pm 1, \pm 2$.
   Let a represent the factors of the leading coefficient 3, which are $\pm 1, \pm 3$.
   The possible values of $\frac{b}{a}$ are $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}$.
   Test the values of $b - a$ for x to find the zeros.
   Substitute $x = 1$ to test

$p(1) = 3(1)^3 + 4(1)^2 - 5(1) - 2$
$= 3 + 4 - 5 - 2 = 0$
$x = 1$ is a zero and $x - 1$ is a factor.
Use synthetic division method to determine the other factors.

1	3	4	−5	−2
+		3	7	2
×	3	7	2	0

$3x^2 + 7x + 2$ can be factored further to $(x + 2)(3x + 1)$

So $p(x) = 3x^3 + 4x^2 - 5x - 2 = (x - 1)(x + 2)(3x + 1)$

Use intervals $x < -2$, $-2 < x < -\frac{1}{3}$, $-\frac{1}{3} < x < 1$, and $x > 1$.

Test arbitrary values of x for each interval.

The solution to the inequality $(x - 1)(x + 2)(3x + 1) > 0$ is:
$-2 < x < -\frac{1}{3}$ or $x > 1$

d) $x \le -1, x \ge 1$
e) $x^4 - 13x^2 + 36 \le 0$
Let $w = x^2$
Substitute w for x^2:
$w^2 - 13w + 36 \le 0$
$(w - 4)(w - 9) \le 0$
Substitute x^2 for w:
$(x^2 - 4)(x^2 - 9) \le 0$
$(x - 2)(x + 2)(x - 3)(x + 3) \le 0$
Use intervals $x \le -3$, $-3 \le x \le -2$, $-2 \le x \le 2$, $2 \le x \le 3$, $3 \ge x$
Test arbitrary values of x for each interval
The solution to the inequality $(x - 2)(x + 2)(x - 3)(x + 3) \le 0$ is:
$-3 \le x \le -2$ or $2 \le x \le 3$
f) $x = 1$

9. $0.75 \le x \le 3.25$

10. Let length $l = 12 + x$, width $w = 6 + x$, and height $h = 2 + x$.
Surface area of a rectangular reflecting pool is:
$SA(x) = 2lh + 2wh + lw$
$= 2(12 + x)(2 + x) + 2(6 + x)(2 + x) + (12 + x)(6 + x)$
$= 48 + 28x + 2x^2 + 24 + 16x + 2x^2 + 72 + 18x + x^2$
$= 5x^2 + 62x + 144$
Solve $SA(x) \le 440$
$5x^2 + 62x + 144 \le 440$
$5x^2 + 62x + 144 - 440 \le 0$
$5x^2 + 62x - 296 \le 0$
Find zeros using quadratic formula

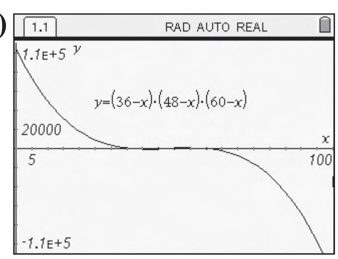
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-62 \pm \sqrt{62^2 - 4(5)(-296)}}{2(5)}$$

$$x = \frac{-62 \pm \sqrt{9764}}{10}$$

$x \cong 3.68$ or $x \cong -16.08$
Dimensions of a reflecting pool cannot be less than or equal to zero; therefore, $x \cong 3.68$ m.
The maximum dimensions of the larger reflecting pool are approximately:
$l = 15.68$ m, $w = 9.68$ m, and $h = 5.68$ m

11. a) $0.3 \le x \le 4.5$, $x \ge 7.2$ **b)** domain: $\{x \in R, 0 \le x \le 14\}$; range: $\{S \in R, 0 \le S \le 896\}$ **c)** The lemonade sales model predicts over 600 000 sales toward the end of summer, which is not very reasonable. The model works only for the first two weeks of summer.

12. a) $V(x) = (36 - x)(48 - x)(60 - x)$
b)


c) $(36 - x)(48 - x)(60 - x) = 62208$
d) $x \cong 7.17$ cm

13. $x < -2$, $-2 < x < -1$, $1 < x < 2$, $x > 2$; $x \le -1$, $x \ge 1$

14. The inequality has a real solution for all $s \in R$ and all $t \in R$.

15. $x < -6.43$

16. $-3 \le x \le -1 - \sqrt{2}$, $-1 + \sqrt{2} \le x \le 1$

Chapter 2 Challenge Questions

C1. a) yes, only if $b = 0$, otherwise no
b) yes **c)** yes, only if $b = 0$, otherwise no
d) yes, only if $b = 0$, otherwise no

C2. $x \pm 4i$ **C3.** $10\sqrt{2} \times 5\sqrt{2}$

C4. a) $p(r_1) = 0$, $p(r_2) = 0$, $p(r_3) = 0$

b) $p(x) = a(x - r_1)(x - r_2)(x - r_3)$
$= ax^3 - a(r_1 + r_2 + r_3)x^2 +$
$a(r_1 r_2 + r_2r_3 + r_1r_3)x - a(r_1r_2r_3)$
$= ax^3 + bx^2 + cx + d$

c) $r_1r_2 + r_2r_3 + r_1r_3 = \frac{c}{a}$

C5. a) $x^2 - 8x - 24 = 0$ **b)** $x^3 - 12x^2 + 36x = 0$

C6. $a = 28, b = \frac{57}{14}$

C7. a) two real positive zeros: $x = 2, x = -1.5,$ $x = 0.25$

b) two real positive zeros: $x = -3, x = 0.5,$ $x = 1$

C8. $x < -3, \frac{1}{2} < x < 5; x \neq \frac{1}{2}, x \neq -3$

C9. no solution

C10. a) $x \leq -2$ or $x \geq 3$ **b)** $-\frac{1}{2} < x < 2$

C11. a) $x < -4$ or $x > \frac{5}{3}$ **b)** $-\frac{2}{3} \leq x \leq 1$

Chapter 3

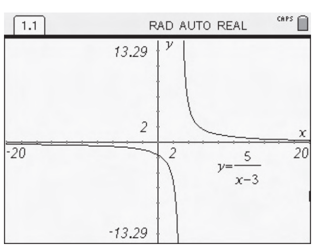
3.1 Reciprocal of a Linear Function

1. a) $x = 2, y = 0$ **b)** $x = -7, y = 0$ **c)** $x = 5,$ $y = 0$ **d)** $x = 9, y = 0$ **e)** $x = \frac{4}{3}, y = 0$ **f)** $x = -\frac{1}{7}, y = 0$

2. a) $-\frac{1}{2}$ **b)** $\frac{3}{7}$ **c)** $\frac{4}{5}$ **d)** $\frac{2}{9}$ **e)** $\frac{1}{4}$ **f)** 5

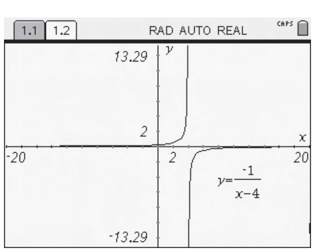
3. a) $y = \frac{5}{x - 5}$ **b)** $y = \frac{-4}{x + 2}$ **c)** $y = \frac{1}{2x + 1}$ **d)** $y = \frac{-2}{x - 3}$

4. a)



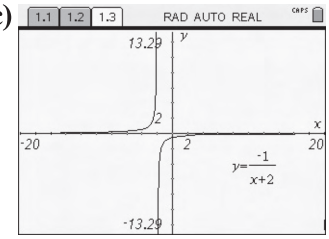
$\{x \in R, x \neq 3\}, \{y \in R, y \neq 0\},$
$x = 3, y = 0$

b)



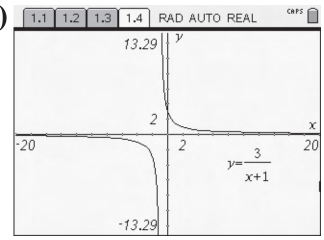
$\{x \in R, x \neq 4\}, \{y \in R, y \neq 0\},$
$x = 4, y = 0$

c)



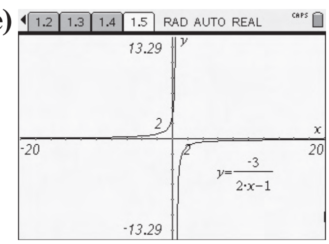
$\{x \in R, x \neq -2\}, \{y \in R, y \neq 0\},$
$x = -2, y = 0$

d)



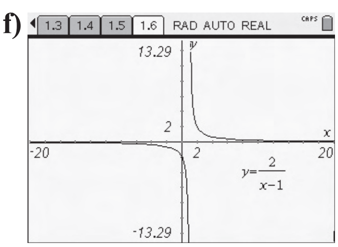
$\{x \in R, x \neq -1\}, \{y \in R, y \neq 0\},$
$x = -1, y = 0$

e)



$\left\{x \in R, x \neq \frac{1}{2}\right\}, \{y \in R, y \neq 0\},$
$x = \frac{1}{2}, y = 0$

f)

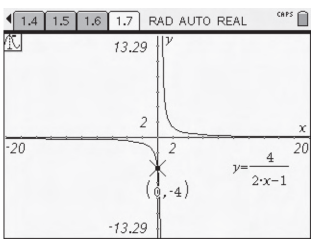


$\{x \in R, x \neq 1\}, \{y \in R, y \neq 0\},$
$x = 1, y = 0$

5. Answers may vary. Sample answer:

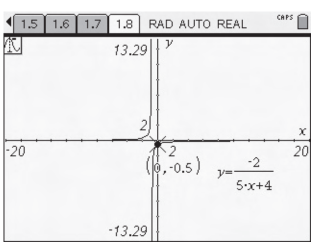
Vertical Asymptote	Comparison of Two Pairs of Points to the Left of Vertical Asymptote	Comparison of Two Pairs of Points to the Right of Vertical Asymptote
$x = \frac{1}{2}$	**Points:** $\left(-3, \frac{3}{7}\right), \left(-2, \frac{3}{5}\right)$ **Rate of change:** $\frac{6}{35}$	**Points:** $\left(3, -\frac{3}{5}\right), \left(4, -\frac{3}{7}\right)$ **Rate of change:** $\frac{6}{35}$
	Points (closer to asymptote): $(-1, 1), (0, 3)$ **Rate of change:** 2	**Points (closer to asymptote):** $(1, -3), (2, -1)$ **Rate of change:** 2
Conclusion (Is rate of change increasing or decreasing?)	increasing	decreasing

6. a)



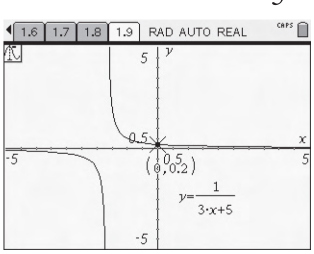
y-intercept: $y = -4$; $\left\{x \in R, x \neq \frac{1}{2}\right\}$, $\{y \in R, y \neq 0\}$, $x = \frac{1}{2}$, $y = 0$

b)



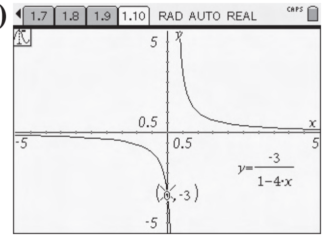
y-intercept: $y = -\frac{1}{2}$; $\left\{x \in R, x \neq -\frac{4}{5}\right\}$, $\{y \in R, y \neq 0\}$, $x = -\frac{4}{5}$, $y = 0$

c)



y-intercept: $y = \frac{1}{5}$; $\left\{x \in R, x \neq -\frac{5}{3}\right\}$, $\{y \in R, y \neq 0\}$, $x = -\frac{5}{3}$, $y = 0$

d)



y-intercept: $y = -3$; $\left\{x \in R, x \neq \frac{1}{4}\right\}$, $\{y \in R, y \neq 0\}$, $x = \frac{1}{4}$, $y = 0$

7. a) decreasing: $\left(-\infty, \frac{1}{2}\right)$, increasing: $\left(\frac{1}{2}, \infty\right)$

b) increasing: $\left(-\infty, -\frac{4}{5}\right)$, decreasing: $\left(-\frac{4}{5}, \infty\right)$

c) decreasing: $\left(-\infty, -\frac{5}{3}\right)$, increasing: $\left(-\frac{5}{3}, \infty\right)$

d) decreasing: $\left(-\infty, \frac{1}{4}\right)$, increasing: $\left(\frac{1}{4}, \infty\right)$

8. a) as $x \to \frac{1}{2}^-$, $f(x) \to -\infty$

as $x \to \frac{1}{2}^+$, $f(x) \to +\infty$

as $x \to +\infty$, $f(x) \to 0$ from above

as $x \to -\infty$, $f(x) \to 0$ from below

b) as $x \to -\frac{4}{5}^-$, $g(x) \to +\infty$

as $x \to -\frac{4}{5}^+$, $g(x) \to -\infty$

as $x \to +\infty$, $g(x) \to 0$ from below

as $x \to -\infty$, $g(x) \to 0$ from above

c) as $x \to -\frac{5}{3}^-$, $h(x) \to -\infty$

as $x \to -\frac{5}{3}^+$, $h(x) \to +\infty$

as $x \to +\infty$, $h(x) \to 0$ from above

as $x \to -\infty$, $h(x) \to 0$ from below

d) as $x \to \frac{1}{4}^-$, $k(x) \to -\infty$

as $x \to \frac{1}{4}^+$, $k(x) \to +\infty$

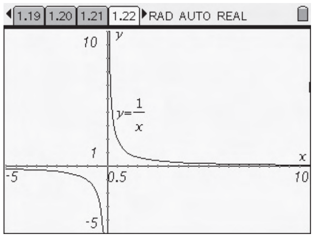
as $x \to +\infty$, $k(x) \to 0$ from above

as $x \to -\infty$, $k(x) \to 0$ from below

9. a) $y = \frac{2}{x + 4}$ **b)** $y = \frac{6}{3x - 1}$

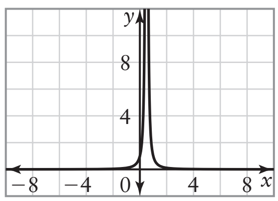
10. a) 1.47 h, 1.25 h, 1.05 h

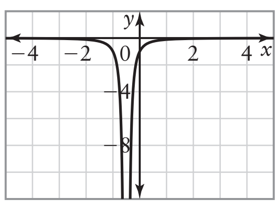
b) $t(v) = \frac{d}{v}$; d is distance in km, t is time in h, v is speed in km/h.

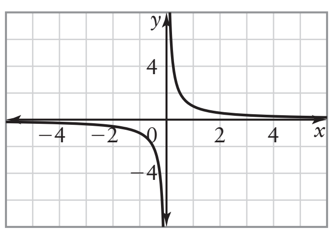
c) 

d) As speed increases, time decreases.
As speed decreases, time increases.

11. a) Answers may vary. Sample answers:

i) 

ii) 

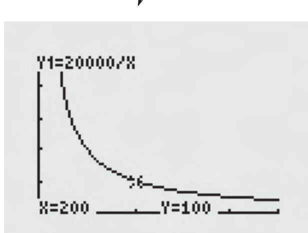
iii) 

b) Answers may vary. Sample answers:

i) $y = \dfrac{1}{(2x - 1)^2}$ **ii)** $y = -\dfrac{1}{(2x + 1)^2}$

iii) $y = \dfrac{1}{x}$

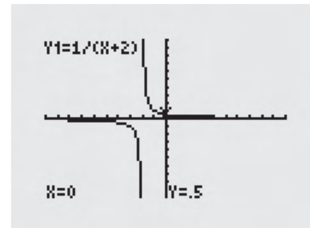
12. a) $P(V) = \dfrac{20\,000}{V}$

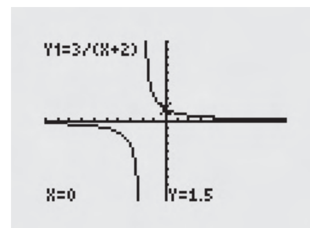
b) 

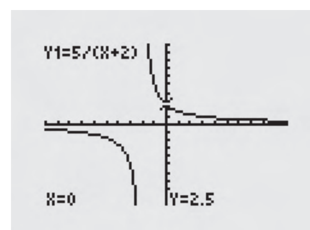
c) $\dfrac{800}{3}$ kPa

d) As the volume increases, the rate of change of the pressure decreases.

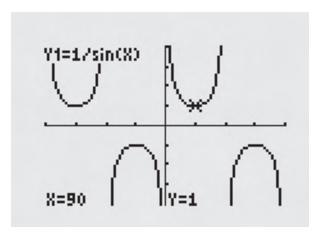
13. a) $0 < b < 1$, vertical compression by factor of b; $b > 1$, vertical stretch by factor of b

b) i) 

ii) 

iii) 

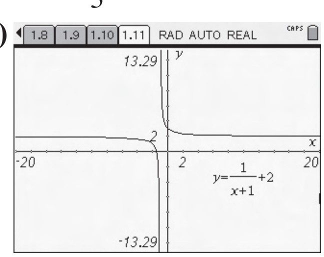
14. $\{x \in \mathbb{R}, x \neq k180°, k \in \mathbb{Z}\}$, $\{y \in \mathbb{R}, y \leq -1, y \geq 1\}$, vertical asymptotes: $x = k180°, k \in \mathbb{Z}$

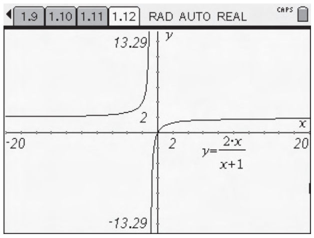


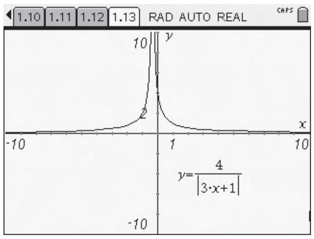
15. a) $\{x \in R, x \neq -1\}$, $\{y \in R, y \neq 2\}$, $x = -1, y = 2$

b) $\{x \in R, x \neq -1\}$, $\{y \in R, y \neq 2\}$, $x = -1, y = 2$

c) $\left\{x \in R, x \neq -\dfrac{1}{3}\right\}$, $\{y \in R, y \neq 0\}$, $x = -\dfrac{1}{3}, y = 0$

16. a) 

b) 

c) 

17. 1 **18.** The reciprocal of a linear function $y = kx - c$ is a rational function of the form $y = \dfrac{1}{kx - c}$.

19. Similarities: domain $\{x \in R, x \neq 1\}$; range $\{y \in R, y \neq 0\}$; vertical asymptote: $x = 1$; horizontal asymptote: $y = 0$; no x-intercepts
Differences:

	$f(x) = \dfrac{1}{1-x}$		$g(x) = \dfrac{1}{x-1}$	
Intervals	$x < 1$	$x > 1$	$x < 1$	$x > 1$
Sign of Function	+	−	−	+
Sign of Slope	+	+	−	−
Change in Slope	+	−	−	+
y-intercept	1		−1	

20. Similarities: range $\{y \in R, y \neq 0\}$; share one vertical asymptote: $x = -2$; horizontal asymptote: $y = 0$; no x-intercepts
Differences:

	$f(x) = \dfrac{1}{x+2}$	$g(x) = \dfrac{1}{(x-2)(x+2)}$
Domain	$\{x \in R, x \neq -2\}$	$\{x \in R, x \neq -2, x \neq 2\}$
Vertical Asymptotes	$x = -2$	$x = -2, x = 2$
y-intercept	$\frac{1}{2}$	$-\frac{1}{4}$
Maximum	None	$\left(0, -\frac{1}{4}\right)$

21. $y = \dfrac{1}{(x+2)(x-4)}$

3.2 Reciprocal of a Quadratic Function

1. **a)** $x = 3, x = -4, \{x \in R, x \neq 3, x \neq -4\}$
b) $x = -3, \{x \in R, x \neq -3\}$
c) $x = -2, x = -6, \{x \in R, x \neq -2, x \neq -6\}$
d) $x = -3, x = 3, \{x \in R, x \neq -3, x \neq 3\}$

2. **a)**

$x \to$	$f(x) \to$
1^-	$-\infty$
1^+	$+\infty$
-3^-	$+\infty$
-3^+	$-\infty$
$-\infty$	0 from above
$+\infty$	0 from above

b)

$x \to$	$f(x) \to$
$-\frac{1}{2}^-$	$+\infty$
$-\frac{1}{2}^+$	$-\infty$
2^-	$-\infty$
2^+	$+\infty$
$-\infty$	0 from above
$+\infty$	0 from above

3. **a) i)** 1 **ii)** none **iii)** $y = 0$ **iv)** $x = 1$ **b) i)** $-\frac{1}{6}$ **ii)** none **iii)** $y = 0$ **iv)** $x = -3, x = 2$ **c) i)** -1.67 **ii)** none **iii)** $y = 0$ **iv)** $x = \frac{1}{2}$, $x = 3$

4. **a)** $y = \dfrac{1}{(x-1)^2}$ **b)** $y = \dfrac{1}{(x+3)(x-2)}$
c) $y = -\dfrac{5}{(2x-1)(x-3)}$

5. **a)**

Intervals	$x < 1$	$x > 1$
Sign of Function	+	+
Sign of Slope	+	−
Change in Slope	+	+

b)

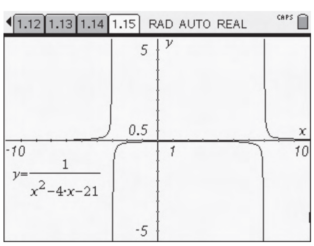
Intervals	$x < -3$	$-3 < x < -\frac{1}{2}$	$x = -\frac{1}{2}$	$-\frac{1}{2} < x < 2$	$x > 2$
Sign of Function	+	−	−	−	+
Sign of Slope	+	+	0	−	−
Change in Slope	+	−		−	+

c)

Intervals	$x < \frac{1}{2}$	$\frac{1}{2} < x < \frac{7}{4}$	$x = \frac{7}{4}$	$\frac{7}{4} < x < 3$	$x > 3$
Sign of Function	–	+	+	+	–
Sign of Slope	–	–	0	+	+
Change in Slope	–	+		+	–

6. **a) i)** $x = 7, x = -3, y = 0$
ii) no x-intercepts, y-intercept: $-\frac{1}{21}$
iii)

Intervals	$x < -3$	$-3 < x < 2$	$x = 2$	$2 < x < 7$	$x > 7$
Sign of Function	+	–	–	–	+
Sign of Slope	+	+	0	–	–
Change in Slope	+	–		–	+

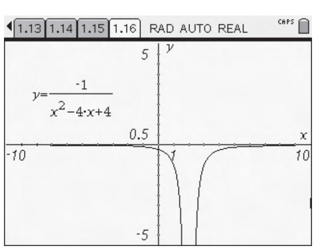
iv)


v) $\{x \in R, x \neq 7, x \neq -3\}$, $\{y \in R, y \neq 0\}$

b) i) $x = 2, y = 0$ **ii)** no x-intercepts, y-intercept: $-\frac{1}{4}$

iii)

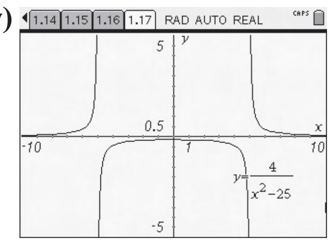
Intervals	$x < 2$	$x > 2$
Sign of Function	–	–
Sign of Slope	–	+
Change in Slope	–	–

iv)


v) $\{x \in R, x \neq 2\}$, $\{y \in R, y \neq 0\}$

c) i) $x = -5, x = 5, y = 0$ **ii)** no x-intercepts, y-intercept: $-\frac{4}{25}$
iii)

Intervals	$x < -5$	$-5 < x < 0$	$x = 0$	$0 < x < 5$	$5 > 2$
Sign of Function	+	–	–	–	+
Sign of Slope	+	+	0	–	–
Change in Slope	+	–		–	+

iv)


v) $\{x \in R, x \neq 5, x \neq -5\}$, $\{y \in R, y \neq 0\}$

7. $y = \frac{1}{(x + 2)(x - 7)}$

8. **a) i)** Since the function approaches the x-axis as x approaches both positive and negative infinity, there is a horizontal asymptote at $y = 0$.
ii) Since there is a horizontal asymptote at $y = 0$ that the function never crosses, there are no x intercepts. From the graph, the y- intercepts of (0, 0.1) can be seen.
iii) There is no restriction on x, so the domain is $x \in R$. The y values must be larger than zero, and can take on any values larger than zero, up to and including the y intercept of 0.1. Therefore, the range is $0 < y \leq 0.1, y \in R$
iv)

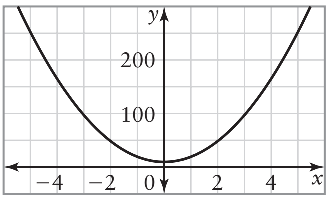
Intervals	$x < 0$	$x = 0$	$x > 0$
Sign of Function	+	+	+
Sign of Slope	+	0	–

The function increases from negative infinity to the y-intercept and then decreases to positive infinity. Therefore, the interval of increase is $x < 0$ and the interval of decrease is $x > 0$.

b) There is no vertical asymptote because there is no value that x can take on that makes the function undefined.

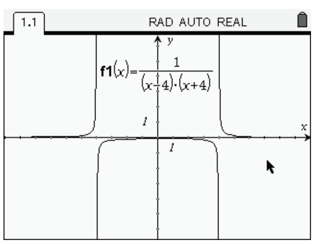
c) Answers may vary. Sample answer:
$y = \dfrac{1}{10(x^2 + 1)}$

9. a) In the form $g(x) = \dfrac{1}{f(x)}$, $f(x) = 10(x^2 + 1)$.



b) Answers may vary. Sample answer: From this graph, the domain for both $f(x)$ and $g(x)$ will be the same (that is, $x \in R$). They both do not have a vertical asymptote or x-intercepts. When $f(x)$ is increasing, $g(x)$ is decreasing and when $f(x)$ is decreasing, $g(x)$ is increasing.

10. a)

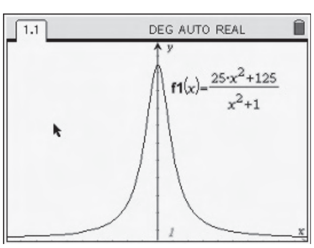


b) $y = \dfrac{1}{(x - 4)(x + 4)}$

11. a) $\{t \in R\}$, $\{P(t) \in R, 25 < P(t) \leq 125\}$

b) $y = 25$, no vertical asymptotes

c)



d)
$m_1 = -50$ $\quad m_6 \cong -0.88$
$m_2 = -16$ $\quad m_7 \cong -0.56$
$m_3 = -6$ $\quad m_8 \cong -0.38$
$m_4 \cong -2.77$ $\quad m_9 \cong -0.27$
$m_5 \cong -1.48$

The rate of change is negative and therefore blood pressure is decreasing at $0 < t < 10$.

3.3 Rational Functions of the Form $f(x) = \dfrac{ax + b}{cx + d}$

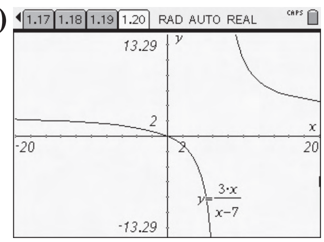
1. a) $x = 7$ **b)** $x = \dfrac{2}{3}$ **c)** $x = -4$ **d)** $x = -11$

2. a) $\{x \in R, x \neq 7\}$ **b)** $\left\{x \in R, x \neq \dfrac{2}{3}\right\}$
c) $\{x \in R, x \neq -4\}$ **d)** $\{x \in R, x \neq -11\}$

3. a) $y = 1$ **b)** $y = -5$ **c)** $y = -1$ **d)** $y = \dfrac{3}{4}$

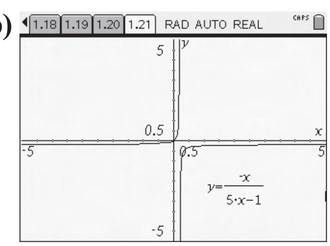
4. a) $\{y \in R, y \neq 1\}$ **b)** $\{y \in R, y \neq -5\}$
c) $\{y \in R, y \neq -1\}$ **d)** $\left\{y \in R, y \neq \dfrac{3}{4}\right\}$

5. a)



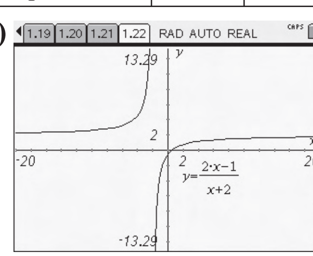
Intervals	$x < 0$	$0 < x < 7$	$x < 7$
Sign of Function	+	–	+
Sign of Slope	–	–	–
Change in Slope	–	–	+

b)

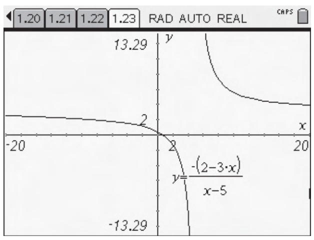


Intervals	$x < 0$	$0 < x < \frac{1}{5}$	$x < \frac{1}{5}$
Sign of Function	–	+	–
Sign of Slope	+	+	+
Change in Slope	+	+	–

c)



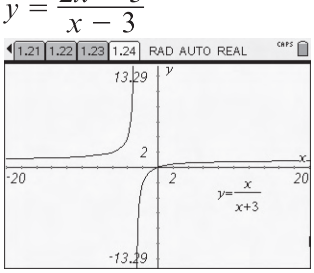
Intervals	$x < -2$	$-2 < x < \frac{1}{2}$	$x > \frac{1}{2}$
Sign of Function	+	−	+
Sign of Slope	+	+	+
Change in Slope	+	−	−

d) 

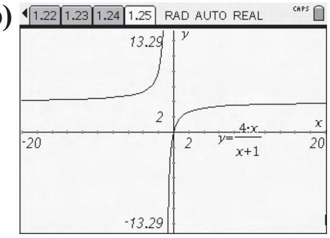
Intervals	$x < \frac{2}{3}$	$\frac{2}{3} < x < 5$	$x > 5$
Sign of Function	+	−	+
Sign of Slope	−	−	−
Change in Slope	−	−	+

6. **a) i)** 1 **ii)** $\frac{1}{2}$ **iii)** $y = 1$ **iv)** $x = 2$
v) $\{x \in R, x \neq 2\}, \{y \in R, y \neq 1\}$
b) i) 2 **ii)** $\frac{2}{3}$ **iii)** $y = -1$ **iv)** $x = -3$
v) $\{x \in R, x \neq -3\}, \{y \in R, y \neq -1\}$
c) i) $\frac{3}{2}$ **ii)** 1 **iii)** $y = 2$ **iv)** $x = 3$
v) $\{x \in R, x \neq 3\}, \{y \in R, y \neq 2\}$

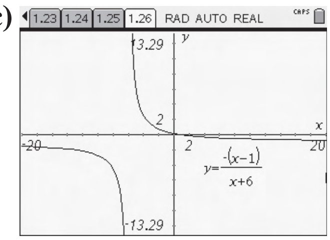
7. **a)** $y = \frac{x-1}{x-2}$ **b)** $y = -\frac{x-2}{x+3}$
c) $y = \frac{2x-3}{x-3}$

8. **a)** 

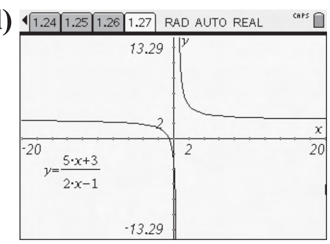
$y = 1, x = -3, \{x \in R, x \neq -3\}, \{y \in R, y \neq 1\}$

b) 

$y = 4, x = -1, \{x \in R, x \neq -1\}, \{y \in R, y \neq 4\}$

c) 

$y = -1, x = -6, \{x \in R, x \neq -6\}, \{y \in R, y \neq -1\}$

d) 

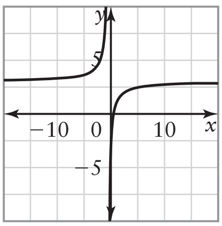
$y = \frac{5}{2}, x = \frac{1}{2}, \left\{x \in R, x \neq \frac{1}{2}\right\}, \left\{y \in R, y \neq \frac{5}{2}\right\}$

9. For the required function, the numerator must generate $x = 5$ when $y = 0$ is substituted. The numerator must be $x - 5$. The denominator must be equal to zero when $x = -\frac{8}{3}$, to give the vertical asymptote at this value, so the denominator must be $3x + 8$. The function must be: $f(x) = \frac{x-5}{3x+8}$. When $x = 0$, the y-intercept is the required value of $-\frac{5}{8}$, as well as the function having a horizontal asymptote at $y = \frac{1}{3}$.

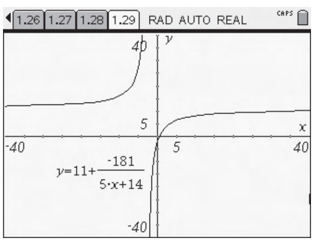
10. **a)**
$$\begin{array}{r} 3 \\ 2x+1 \overline{)6x-3} \\ \underline{6x+3} \\ -6 \end{array} \qquad f(x) = \frac{6x-3}{2x+1}$$
$$= 3 - \frac{6}{2x+1}$$

b) Answers may vary. Sample answer: For small values of x, the function $f(x)$ behaves according to $y = -\frac{6}{2x + 1}$, and as x increases to positive and negative infinity, the function behaves according to $y = 3$.

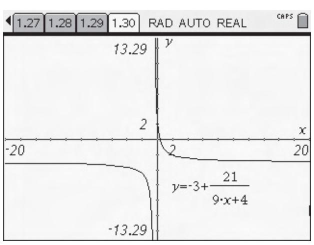
c)



11. a) $f(x) = 11 + \frac{-181}{5x + 14}$

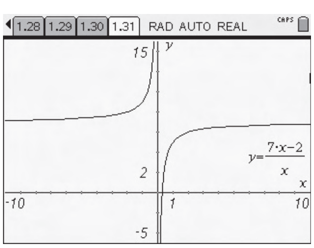


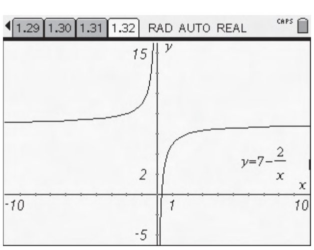
b) $g(x) = -3 + \frac{21}{9x + 4}$



12. a) $\{x \in R, x \neq 0\}$ **b)** $x = 0, y = 7$

c)

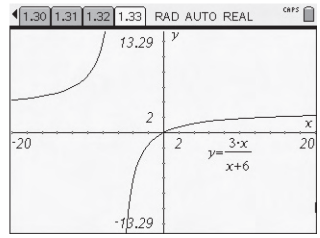




d) Answers may vary.

13. a) $x = -6$ **b)** $\{x \in R, x \neq -6\}$
c) $y = 3$ **d)** $\{y \in R, y \neq 3\}$

e)



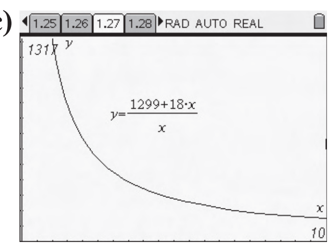
f)

Intervals	$x < -6$	$-6 < x < 0$	$x > 0$
Sign of Function	+	−	+
Sign of Slope	+	+	+
Change in Slope	+	−	−

g) i) $m_{-5} = 18$, $m_{15} = 0.04$, slope is positive and decreasing **ii)** $m_{-7} = 18$, $m_{-15} = 0.22$, slope is positive and increasing

14. a) \$147.90 **b)** $C(t) = \frac{1299 + 18t}{t}$

c)

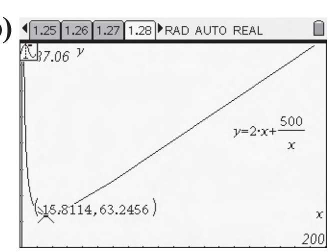


$t \in (0, 10)$, $C \in (0, 1317)$

d) vertical asymptote: $t = 0$; horizontal asymptote: $C = 0$ **e)** The longer the television lasts, the average annual costs approaches 0. **f)** no **g)** negative and increasing

15. a) $L(x) = 2x + \frac{500}{x}$, where L is length of rope in metres, x is width of enclosed area in metres.

b)

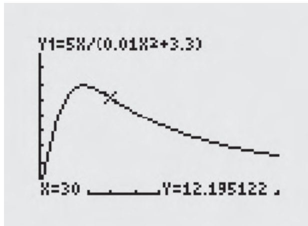


$\{x \in R, 0 < x < 500\}$

c) approximately 15.8 m by 31.6 m

16. vertical stretch by factor of 3. translation 5 units to the right; translation 8 units down

17. a)



b) The maximum concentration is about 13.8 mg/mL, which occurs at 18.2 min.

c) Increasing the "5" vertically stretches the graph. Decreasing the "5" vertically compresses the graph. Changing the sign of the "5" flips the graph about the x-axis. Increasing the "0.01" vertically compresses the graph and shifts the maximum to the left. Decreasing the "0.01" vertically stretches the graph and shifts the maximum to the right.

3.4 Solve Rational Equations and Inequalities

1. **a)** $x = 0$ **b)** $x = \frac{5}{6}$ **c)** $x = 5$ **d)** $x = -3, x = 6$

2. **a)** $x = -\frac{1}{13}$ **b)** $x = -1$ **c)** $x = \frac{4}{3}$ **d)** $x = \frac{4}{5}$ **e)** $x = 3$ **f)** $x = 3$ **g)** $x = 0, x = \frac{-1 \pm \sqrt{97}}{2}$

3. **a)** -0.33 **b)** -1.27 **c)** $-0.04, 8.37$ **d)** $-1.78, -0.66, 0.76, 1.68$

4. **a)** $\left(-\infty, -\frac{1}{3}\right), (2, +\infty)$ **b)** $(-\infty, -4.5], (-3, +\infty)$ **c)** $\left(-\infty, -\frac{3}{5}\right), \left[\frac{1}{2}, +\infty\right)$ **d)** $(-\infty, -2), \left(-\frac{1}{2}, 1\right)$ **e)** $(-\infty, -2], (0, 4]$ **f)** $(-3, 1), [5, +\infty)$ **g)** $(-1, 3), (3, +\infty)$ **h)** $(-1-\sqrt{3}, -2), (-1+\sqrt{3}, 1)$

5. **a)** $[-4, 1), [1.5, 5)$ **b)** $(-\infty, 1), (2, 3)$ **c)** $(-4, -1), [4, 5)$ **d)** $(-\infty, -6), (-4, 0), (12, +\infty)$ **e)** $(-\infty, -60], (-2, 2], (14, +\infty)$ **f)** $(-1, -0.83), [0.5, 4.83)$

6. $y = \frac{x}{(x+2)(2x-1)}$; $x = -2, x = \frac{1}{2}$ are vertical asymptotes

7. $(-\infty, -9), (-1, 1)$

8. **a)** $x = 2, x = 6$ **b)** $x = \frac{23}{15}$ **c)** $x = \frac{5}{3}$ **d)** $x = \frac{5}{13}$ **e)** $x \in R, x \neq 1$ **f)** $x = -\frac{1}{2}, x = 3$

9. **a)**
$$\frac{2}{3x} + \frac{5}{6x} > \frac{3}{4}$$
$$\frac{\mathbf{4}}{\mathbf{6}x} + \frac{5}{6x} > \frac{3}{4}$$
$$\frac{9}{6x} > \frac{3}{4}$$
$$36 > 18x$$
$$2 > x$$
But $x = 0$ is a vertical asymptote, where the sign of the expression may change.
Test $x = -1$:
$$\frac{2}{3x} + \frac{5}{6x} > \frac{3}{4}$$
$$\frac{2}{3(-1)} + \frac{5}{6(-1)} > \frac{3}{4}$$
$$-\frac{2}{3} - \frac{5}{6} > \frac{3}{4}$$
$$-\frac{9}{6} > \frac{3}{4}$$
which is not a valid statement. So the solution is $0 < x < 2$.

b)
$$5 + \frac{1}{x} > \frac{16}{x}$$
$$\frac{5x}{x} + \frac{1}{x} > \frac{16}{x}$$
$$5x + 1 > 16$$
$$5x > 15$$
$$x > 3$$
But at $x = 0$, there is a vertical asymptote where the expression could change signs.
Test $x = -1$:
$$5 + \frac{1}{-1} > \frac{16}{-1}$$
$$5 - 1 > -16$$
$$4 > -16$$
which is a valid statement. Therefore, the solution is $x < 0$ or $x > 3$.

c)
$$1 + \frac{5}{x-1} \leq \frac{7}{6}$$
$$\frac{x-1}{x-1} + \frac{5}{x-1} \leq \frac{7(x-1)}{(x-1)}$$
$$x + 4 \leq 7x - 7$$
$$11 \leq 6x$$
$$\frac{11}{6} \leq x$$
But at $x = 1$, there is a vertical asymptote where the expression could change signs.
Test $x = 36$:

$1 + \frac{5}{36 - 1} \leq \frac{7}{6}$

$1 + \frac{1}{7} \leq \frac{7}{6}$

$\frac{8}{7} \leq \frac{7}{6}$

which is a valid statement. Therefore, the solution is $x < 1$ or $x \geq 31$.

d) $\frac{1}{2x + 1} + \frac{1}{x + 1} > \frac{8}{15}$

$\frac{x + 1}{(2x + 1)(x + 1)} + \frac{2x + 1}{(2x + 1)(x + 1)} > \frac{8(2x + 1)(x + 1)}{15(2x + 1)(x + 1)}$

$\frac{3x + 2}{(2x + 1)(x + 1)} > \frac{8(2x + 1)(x + 1)}{15(2x + 1)(x + 1)}$

$\frac{15(3x + 2)}{(2x + 1)(x + 1)} > \frac{8(2x + 1)(x + 1)}{(2x + 1)(x + 1)}$

$\frac{45x + 30}{(2x + 1)(x + 1)} > \frac{8(2x^2 + 3x + 1)}{(2x + 1)(x + 1)}$

$\frac{45x + 30}{(2x + 1)(x + 1)} > \frac{16x^2 + 24x + 8}{(2x + 1)(x + 1)}$

$0 > \frac{16x^2 - 21x - 22}{(2x + 1)(x + 1)}$

The expression $16x^2 - 21x - 22$ is equal to zero when:

$x = \frac{21 \pm \sqrt{(-21)^2 - 4(16)(-22)}}{2(16)}$

$= \frac{21 \pm \sqrt{1849}}{32}$

$= \frac{21 \pm 43}{32}$

$x = \frac{21 - 43}{32}$ $\quad$ $x = \frac{21 + 43}{32}$

$x = -\frac{11}{16}$ $\quad$ $x = 2$

As well, there are vertical asymptotes at $x = -1$ and $x = -\frac{1}{2}$, which sets up the following intervals to test:

Interval	Test Value	Sign of Factors	Sign of Function
$x < -1$	$x = -2$	$\frac{(+)}{(-)(-)}$	+
$-1 < x < -\frac{11}{16}$	$x = -0.7$	$\frac{(+)}{(-)(+)}$	−

d) continued

Interval	Test Value	Sign of Factors	Sign of Function
$-\frac{11}{16} < x < -\frac{1}{2}$	$x = -0.6$	$\frac{(-)}{(-)(+)}$	+
$-\frac{1}{2} < x < 2$	$x = 0$	$\frac{(-)}{(+)(+)}$	−
$x > 2$	$x = 3$	$\frac{(+)}{(+)(+)}$	+

Therefore, the solution is $-1 < x < -\frac{11}{16}$ or $-\frac{1}{2} < x < 2$.

e) $\frac{x^2 + 3x + 2}{x^2 - 9} \geq 0$

$\frac{(x + 2)(x + 1)}{(x - 3)(x + 3)} \geq 0$

The zeros are at $x = -1$ and $x = -2$ and the vertical asymptotes are at $x = 3$ and $x = -3$, so the intervals to test are:

Interval	Test Value	Sign of Factors	Sign of Function
$x < -3$	$x = -4$	$\frac{(-)(-)}{(-)(-)}$	+
$- < x < -2$	$x = -.5$	$\frac{(-)(-)}{(-)(+)}$	−
$-2 < x < -1$	$x = -1.5$	$\frac{(+)(-)}{(-)(+)}$	+
$-1 < x < 3$	$x = 0$	$\frac{(+)(+)}{(-)(+)}$	−
$x > 3$	$x = 4$	$\frac{(+)(+)}{(+)(+)}$	+

Therefore, the solution is $-3 \leq x \leq -2$ or $-1 \leq x \leq 3$.

f) $\frac{(-2x - 10)(3 - x)}{(x^2 + 5)(x - 2)^2} < 0$

The zeros are at $x = -5$ and $x = 3$ and a vertical asymptote is at $x = 2$, so the intervals to test are:

Interval	Test Value	Sign of Factors	Sign of Function
$x < -5$	$x = -6$	$\frac{(+)(+)}{(+)(+)}$	+
$-5 < x < 2$	$x = 0$	$\frac{(-)(+)}{(+)(+)}$	−
$2 < x < 3$	$x = 2.5$	$\frac{(-)(+)}{(+)(+)}$	−
$x > 3$	$x = 4$	$\frac{(-)(-)}{(+)(+)}$	+

Therefore, the solution is $-5 < x < 2$ or $2 < x < 3$.

10. $\frac{x^2 - 2x + 5}{x^2 - 4x + 4} < 0; \frac{x^2 - 2x + 5}{(x - 2)^2} < 0$
In this inequality, the denominator will always be positive. Therefore, the sign on the expression will be determined by the sign in the numerator. Solve for zeros in the numerator:
$b^2 - 4ac$
$= (-2)^2 - 4(1)(5)$
$= 4 - 20 = -16$
which means that there are no real roots. Therefore, the numerator will either always be positive or always be negative. Testing any value for x (say $x = 0$), we find the numerator will always be positive. Since both the numerator and denominator will always be greater than zero, the expression can never be less than zero, so there is no real solution to the inequality.

11. 1st inequality: $(-\infty, -3)$, $(-1.83447, 4)$;
2nd inequality: $(-3, -1.83447)$, $(4, +\infty)$

12. approximately 6.1 km/h

13. approximately 18.63 m

14. $\left(-\infty, -\frac{25}{3}\right)$, $(5, +\infty)$

15. $x + \frac{x^2 - 5}{x^2 - 1} = \frac{x^2 + x + 2}{x + 1}$

$$x + \frac{x^2 - 5}{(x - 1)(x + 1)} = \frac{x^2 + x + 2}{x + 1}$$

$$\frac{x(x - 1)(x + 1)}{(x - 1)(x + 1)} + \frac{x^2 - 5}{(x - 1)(x + 1)} = \frac{(x^2 + x + 2)(x - 1)}{(x + 1)(x - 1)}$$

$$\frac{x(x^2 - 1)}{(x - 1)(x + 1)} + \frac{x^2 - 5}{(x - 1)(x + 1)} = \frac{x^3 + x^2 + 2x - x^2 - x - 2}{(x + 1)(x - 1)}$$

$$\frac{x^3 - x}{(x - 1)(x + 1)} + \frac{x^2 - 5}{(x - 1)(x + 1)} = \frac{x^3 + x^2 + 2x - x^2 - x - 2}{(x + 1)(x - 1)}$$

$$\frac{x^3 - x}{(x - 1)(x + 1)} + \frac{x^2 - 5}{(x - 1)(x + 1)} - \frac{x^3 + x^2 + 2x - x^2 - x - 2}{(x + 1)(x - 1)} = 0$$

$$\frac{x^3 - x + x^2 - 5 - x^3 - x^2 - 2x + x^2 + x + 2}{(x + 1)(x - 1)} = 0$$

$$\frac{x^2 - 2x - 3}{(x + 1)(x - 1)} = 0$$

$$\frac{(x + 1)(x - 3)}{(x + 1)(x - 1)} = 0$$

$$\frac{x - 3}{x - 1} = 0$$

$$x = 3$$

16. The equality has no solution because of the restriction on x (i. e. $x \neq 3$).

17. more than 7 years old

18. **a)** approximately $x = -1.60$, $x = 0.51$, $x = 1.33$ **b)** $(-3, -2.11)$, $[0.25, 1.86]$, $(2, +\infty)$

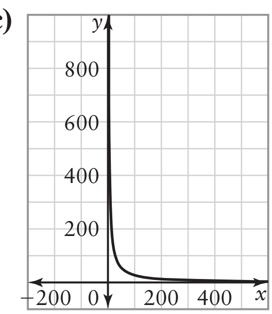
19. $\frac{-3}{x - 3}$ and $\frac{-6}{x + 3}$ **20.** 10

3.5 Making Connections with Rational Functions Equations

1. **a)** $T = 30 + 273 = 303$ K

$$n = \frac{PV}{8.314T} = \frac{(10130.0)(50)}{8.314(303)} = 210.06 \text{ moles}$$

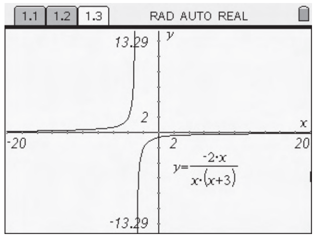
b) $V = \frac{8.314nT}{P} = \frac{8.314(0.050)(400)}{(202.6)} = 0.82$ L

c)


d) From the graph, it can be seen that there is a vertical asymptote at $V = 0$ and a horizontal asymptote at $P = 0$

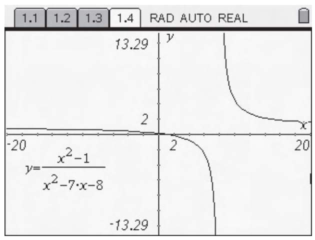
e) Since the pressure and volume are inversely proportional to each other, as the volume increases, the pressure would decrease.

2. a)



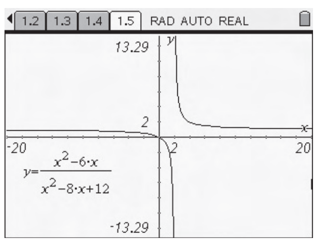
hole at $\left(0, -\frac{2}{3}\right)$

b)



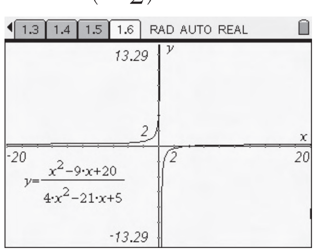
hole at $\left(-1, \frac{2}{9}\right)$

c)



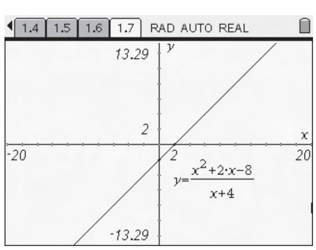
hole at $\left(6, \frac{3}{2}\right)$

d)



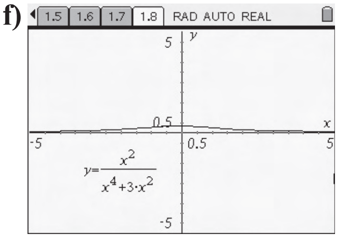
hole at $\left(5, \frac{1}{19}\right)$

e)



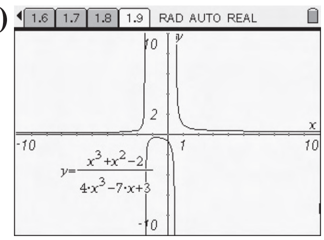
hole at $(-4, -6)$

f)



hole at $\left(0, \frac{1}{3}\right)$

g)



hole at $(1, 1)$

3. a) $F = \frac{6.67 \times 10^{-11}\, m_1 m_2}{r^2}$

$= \frac{6.67 \times 10^{-11}(5.98 \times 10^{24})(80)}{(6.39 \times 10^{6})^2}$

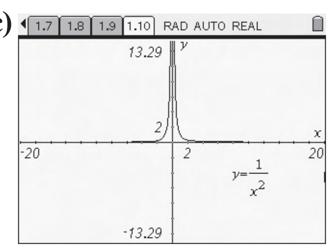
$= 781.5$ N

b) $F = \frac{6.67 \times 10^{-11}\, m_1 m_2}{r^2}$

$= \frac{6.67 \times 10^{-11}(85)(85)}{(1)^2}$

$= 4.82 \times 10^{-7}$ N

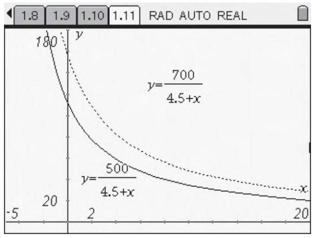
c)



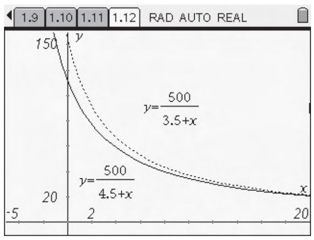
$r \neq 0$, $F > 0$

d) Since the relation is an inverse square relation, as the distance increases, the force rapidly decreases.

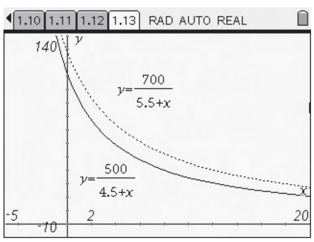
e) As the mass of the objects decreases, the force of attraction between the objects also decreases. This is a direct proportionality.

4. **a)** 

vertical stretch by factor of $\frac{7}{5}$

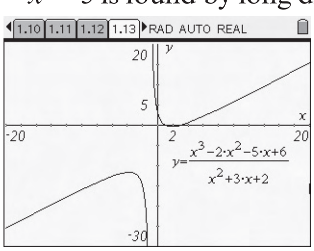
b) 

translation of 1 unit to the right

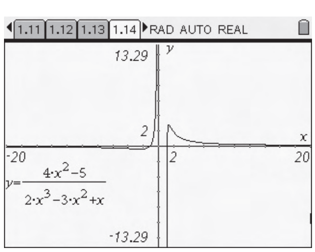
c) 

vertical stretch by factor of $\frac{7}{5}$; translation of 1 unit to the left

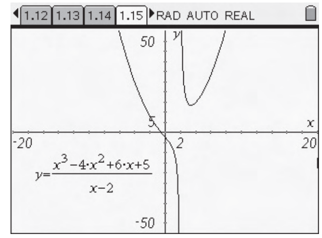
5. **a)** $x = -1$ **b)** x-intercept: 1, 3; y-intercept: 3 **c)** hole at $(-2, -15)$ **d)** Oblique asymptote $y = x - 5$ is found by long division.

e) 

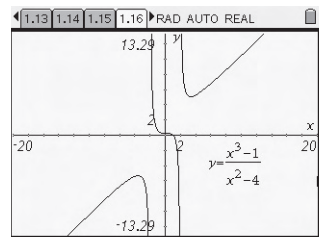
6. **a)** vertical asymptote: $x = 0$, $x = 1$, $x = \frac{1}{2}$; x-intercept: $\pm\frac{\sqrt{5}}{2}$; y-intercept: none; horizontal asymptote: $y = 0$



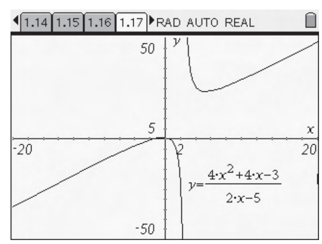
b) vertical asymptote: $x = 2$; x-intercept: -0.578; y-intercept: $-\frac{5}{2}$



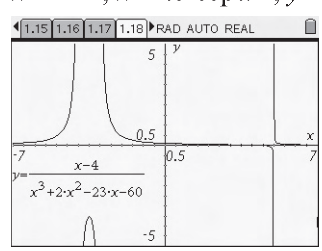
c) vertical asymptote: $x = -2$, $x = 2$; x-intercept: 1; y-intercept: $\frac{1}{4}$; oblique asymptote: $y = x$



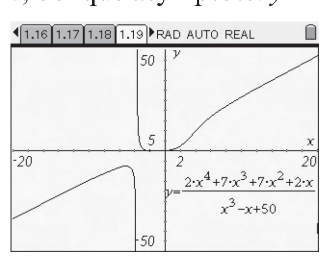
d) vertical asymptote: $x = \frac{5}{2}$; x-intercept: $\frac{1}{2}$, $-\frac{3}{2}$; y-intercept: $\frac{3}{5}$; oblique asymptote: $y = 2x + 7$



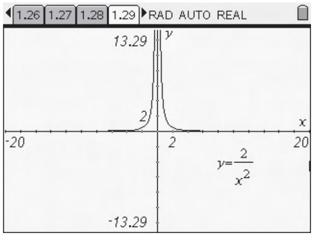
e) vertical asymptote: $x = -3$, $x = 5$, $x = -4$; x-intercept: 4; y-intercept: $\frac{1}{15}$



f) vertical asymptote: $x = -3.774$; x-intercept: -2, -1, -0.5, 0; y-intercept: 0; oblique asymptote: $y = 2x + 7$.



7. **a)** $m = \dfrac{2K}{v^2}$

b) 

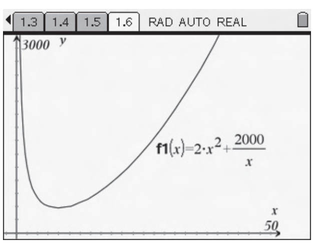
c) 1.5 kg

8. **a)** vertical asymptote: $x = -35$; horizontal asymptote: $C = 0.5$; as the number of litres of 0.5-molar solution is increasing, the concentration of the mixture approaches the 0.5

b) $C(x) = \dfrac{402.5}{x + 35} + 0.5$; vertical stretch by factor of 402.5; translation of 35 units to the left translation of 0.5 units up;

c) $C(x) = \dfrac{600 + 3x}{50 + x}$; **d)** 40 L

9. **a)** $SA = 2x^2 + \dfrac{2000}{x}$

b) 

c) as side length approaches 0, the surface area approaches $+\infty$; **d)** at least 10.2 cm

10. **a)** as $t \to 15^-$, $I \to +\infty$;

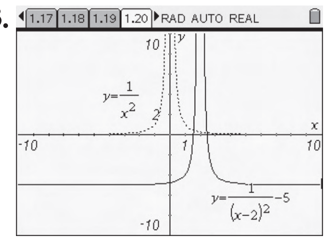
b) vertical asymptote: $t = 15$

11. **a)** Since the relationship is an inverse proportionality, $t = \dfrac{k}{r}$, for time, t, in minutes and rate, r, in L/min.

b) Solving $t = \dfrac{k}{r}$ for k we obtain $k = rt$. Using last year's information of $r = 1000$ L/min and $t = 45$ min, we get $k = (1000)(45)$ or $k = 45\,000$, so $t = \dfrac{k}{r}$ becomes $t = \dfrac{45\,000}{r}$. This year, the pump has $r = 900$ L/min as a rate so $t = \dfrac{45\,000}{r}$ becomes $t = \dfrac{45\,000}{900}$ which means a total time of 50 minutes this year will be needed.

12.
$$
\begin{array}{r}
x^2 - 5 \\
x + 2\overline{)x^3 - 2x^2 - 5x + 10} \\
\underline{x^3 - 2x^2} \qquad\qquad \\
0 - 5x + 10 \\
\underline{-5x - 10} \\
20
\end{array}
$$

Therefore, there is a parabolic asymptote at $y = x^2 - 5$.

13. 

vertical asymptote: $x = 2$; horizontal asymptote: $y = -5$; $y = \dfrac{1}{(x-2)^2} - 5$

14. $f^{-1}(x) = \dfrac{2}{x-3}$; the inverse is a function; similarities: same shape differences:

	$f(x) = \dfrac{2}{x} + 3$	$f^{-1}(x) = \dfrac{2}{x} - 3$
Domain	$\{x \in R, x \neq 0\}$	$\{x \in R, x \neq 3\}$
Range	$\{y \in R, y \neq 3\}$	$\{y \in R, y \neq 0\}$
Vertical asymptotes	$x = 0$	$x = 3$
Horizontal asymptotes	$y = 3$	$y = 0$
x-intercepts	$-\dfrac{2}{3}$	none
y-intercepts	none	$-\dfrac{2}{3}$

15. **a)** 4 **b)** 64

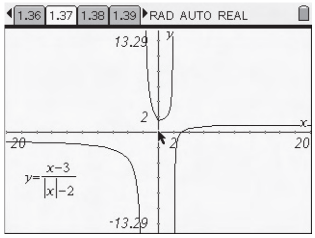
Chapter 3 Challenge Questions

C1. a) $C(x) = \dfrac{330\,240 + 258x^2}{320x}$

b) $x = 368.62$ km/h or $x = 3.47$ km/h

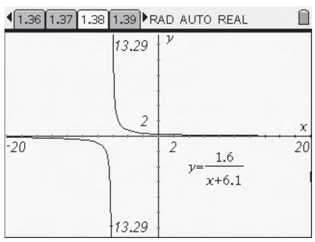
c) 35.77 km/h

C2. a) $f(x) = \dfrac{x-3}{x-2}$ **b)** $f(x) = \dfrac{x-3}{-x-2}$ **c)** $f(x)$ has two vertical asymptotes because x in $|x|$ can be positive or negative.



C3. h = 4.24 cm, d ≅ 4.24 cm

C4. **a)** $I(x) = \dfrac{1.6}{x + 6.1}$ **b)** $\{x \in R, x \geq 0\}$
c) approximately 0.26 A
d)



e) I approaches 0

C5. **a)** $d_i = \dfrac{f d_o}{d_o - f}$ **b)** approximately 17.1 cm

C6. $y = \dfrac{2x^2 + x + 36}{x^2 - 9}$ **C7.** $v \cong 23.57$ m/s

C8. $x = 19\,500$ items to maximize profit. Manufacturer breaks even at $x \cong 53$ items or $x \cong 18\,947$ items.

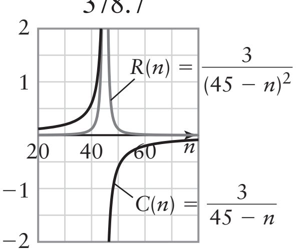
C9. 2.5 m/s

C10. **a)** reflected in the x-axis **b)** q_1 and q_2 are at maximum and of equal sign, and the distance r is at minimum

C11. $N(u) = \dfrac{nv}{u + v}$

$$N = \frac{343.7(50)}{378.7} = 45 \text{ Hz}$$

C12. **a)**



b) domain of $C(n)$: $n > 0, n \neq 45$, range of $C(n)$: $C(n) > 0$; domain of $R(n)$: $n > 0, n \neq 45$, range of $R(n)$: $R(n) > 0$ **c)** slope = 0.0075 **d)** $R(25) = 0.0075$

C13. **a)** $y = n$ **b)** vertical asymptotes $n = \pm 2$, x-intercept (0, 0), y-intercept (0, 0)
c) local minimum (3.46, 5.19) **d)** local maximum (−3.46, −5.19)

C14. The slope of the secant is $y = -2.5x$. The points on $g(x)$ that has a tangent with a slope of −2.5 is (0, 0) and (−4, −10).

Chapter 4

4.1 Radian Measure

1. **a)** $\dfrac{11\pi}{6}$ **b)** $\dfrac{\pi}{3}$ **c)** $\dfrac{5\pi}{12}$ **d)** $\dfrac{3\pi}{4}$ **e)** $\dfrac{5\pi}{4}$ **f)** $\dfrac{125\pi}{18}$

2. **a)** 180° **b)** 45° **c)** 150° **d)** 270° **e)** 315° **f)** 90° **g)** 213.75° **h)** 40°

3. **a)** 58.4° **b)** 100.3° **c)** 217.2° **d)** 343.8°

4. **a)** $\dfrac{\pi}{18}$ **b)** $\dfrac{16\pi}{45}$ **c)** $\dfrac{27\pi}{24}$ **d)** $\dfrac{19\pi}{12}$

5. 13.4 cm

6. $a = 14.6$ cm

$\theta = \dfrac{\pi}{4}$

$\theta = \dfrac{a}{r}$

$\dfrac{\pi}{4} = \dfrac{14.6}{r}$

$r = \dfrac{(4)(14.6)}{\pi}$ $\quad r \cong 18.6$ cm

7. 163.4 cm^2

8. **a)** 90°, $\dfrac{\pi}{2}$ **b)** 15°, $\dfrac{\pi}{12}$ **c)** 100°, $\dfrac{5\pi}{9}$ **d)** 25°, $\dfrac{5\pi}{36}$

9. **a)** $\dfrac{4\pi}{3}$ **b)** 60.4π **c)** $\dfrac{2\pi}{3}$ **d)** 2π

10. The radius of Earth r is 6336 km. The latitude of Philadelphia is 39° and the latitude of Ottawa is 45°.

$\theta = 45° - 39° = 6°$

Convert θ into radians

$\theta = 6\left(\dfrac{\pi}{180}\right) = \dfrac{\pi}{30}$

The distance between the two cities is the length of the arc a that subtends θ

$\theta = \dfrac{a}{r}$

$a = \theta r \quad a = \left(\dfrac{\pi}{30}\right)6336 \quad a \cong 663.5\ km$

11. $\dfrac{25\pi}{36}$ **12.** 0.3375 rad

13. The length of the arc doubles and the area of the sector quadruples.

14. **a)** $r = 2.2$ m

$\theta = 180° - 45° = 135°$

The area of a sector is $A = \dfrac{\theta}{360}\pi r^2$, where θ is in degrees.

$A = \dfrac{\theta}{360}\pi r^2$

$A = \dfrac{135}{360}\pi (2.2)^2$

$A = 1.815\pi \text{ m}^2 \quad A \cong 5.7 \text{ m}^2$

The total area of the sector is 5.7 m^2.

b) The radius r will increase by 0.5 m.

$r = 2.2 + 0.5 = 2.7$ m

$\theta = 135°$

$A = \frac{\theta}{360}\pi r^2$

$A = \frac{135}{360}\pi(2.7)^2 \quad A \cong 8.6 \text{ m}^2$

The area of the yard in which the dog can move will increase from 5.7 m^2 to 8.6 m^2, which is about 50% more space.

15. a) 14062.5 rad **b)** 5π rad/s

16. a) $\frac{8\pi}{15}$ rad/s **b)** 1792π m

17. a) 4188.8 s or 69.8 min **b)** 11 km/s

18. a) $\frac{7\pi}{4}$ **b)** $\frac{7\pi}{6}$ **c)** $\frac{4\pi}{3}$ **d)** $\frac{\pi}{5}$ **e)** $\frac{\pi}{3}$ **f)** $\frac{16\pi}{15}$

19. 3.37 m/s^2

20. a) $\frac{20\pi}{3}$ rad/s **b)** $\frac{80\pi^2}{3}$ m/s^2 **c)** 4π m

21. a) An airplane propeller rotates 20 times/s; therefore, the angular velocity is 20 rotations/s. Angular velocity = 20 rotations/s × 60 s = 1200 rotations/min. The propeller rotates through an angle of $20 \times 2\pi$, or 40π rad in 1 second. The angular velocity of an airplane propeller is 40π rad/s or 1200 rotations/min.

b) The propeller has diameter of 2 m. Therefore, the radius r is 1 m.

$v = r\frac{\theta}{t}$

$v = (1\text{m})(40\pi \text{ rad/s})$

$v = 40\pi \text{ m/s} \quad v \cong 125.7 \text{ m/s}$

The linear velocity of the propeller is 40π m/s.

22. a) 16 rotations/day; 32π rad/day

b) $1\,322\,752\pi$ km/day

23. a) 4.05 rad/h

b) Angular velocity of 3.8 rad/h

$v = r\left(\frac{\theta}{t}\right)$

$2700 = (x + 6336)(3.8)$

$\frac{2700}{3.8} = x + 6336$

$x = \frac{2700}{3.8} - 6336$

$x = 769.3$ km

4.2 Trigonometric Ratios and Special Angles

1. a) i) 0.7071 **ii)** 5.6713 **iii)** 0.3256 **iv)** 0.5299

b) i) 0.7074 **ii)** 5.6626 **iii)** 0.3255 **iv)** 0.5300

c) The answers are almost the same because the angles in parts a) and b) are the same.

2. a) −0.7071 **b)** 0.9239 **c)** 0.5774 **d)** −0.5

3. a) 1.0125 **b)** 0.2867 **c)** −2.9238 **d)** 1.0642

4. a) 1.7878 **b)** 1.6507 **c)** −1.1676 **d)** −3.4364

5. a) −1.1547 **b)** 1.3764 **c)** 1.3054 **d)** 1.0353

6.

θ	sin	cos	tan
(a) $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
(b) $\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
(c) $\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
(d) π	0	−1	0
(e) $\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
(f) $\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	−1

7. Use the unit circle from Section 4.2 to determine the exact values of the trigonometric rations for each angle. The coordinates at the unit circle determine the exact values of the cosine and sine functions for the given angle of rotation.

a) The terminal arm of an angle $\frac{\pi}{6}$ is in the first quadrant; the coordinates of the point of intersection are $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin\frac{\pi}{6} = y, \sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{\pi}{6} = x, \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\frac{\pi}{6} = \frac{y}{x}, \tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}, \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\csc\frac{\pi}{6} = \frac{1}{y}, \csc\frac{\pi}{6} = 2$

$\sec\frac{\pi}{6} = \frac{1}{x}, \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$\cot\frac{\pi}{6} = \frac{x}{y}, \cot\frac{\pi}{6} = \sqrt{3}$

b) The terminal arm of an angle $\frac{5\pi}{3}$ is in the fourth quadrant; the coordinates of the point of intersection are $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\frac{5\pi}{3} = y, \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{3} = x, \cos\frac{5\pi}{3} = \frac{1}{2}$

$\tan\frac{5\pi}{3} = \frac{y}{x}, \tan\frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}},$

$\tan\frac{5\pi}{3} = -\sqrt{3}$

$\csc \frac{5\pi}{3} = \frac{1}{y}, \csc \frac{5\pi}{3} = -\frac{2}{\sqrt{3}}$

$\sec \frac{5\pi}{3} = \frac{1}{x}, \sec \frac{5\pi}{3} = 2$

$\cot \frac{5\pi}{3} = \frac{x}{y}, \cot \frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$

c) The terminal arm of an angle $\frac{3\pi}{4}$ is in the second quadrant; the coordinates of the point of intersection are $P\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$\sin \frac{3\pi}{4} = y, \sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos \frac{3\pi}{4} = x, \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$\tan \frac{3\pi}{4} = \frac{y}{x}, \tan \frac{3\pi}{4} = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}}, \tan \frac{3\pi}{4} = -1$

$\csc \frac{3\pi}{4} = \frac{1}{y}, \csc \frac{3\pi}{4} = \sqrt{2}$

$\sec \frac{3\pi}{4} = \frac{1}{x}, \sec \frac{3\pi}{4} = -\sqrt{2}$

$\cot \frac{3\pi}{4} = \frac{x}{y}, \cot \frac{3\pi}{4} = -1$

d) The terminal arm of an angle $\frac{5\pi}{6}$ is in the second quadrant; the coordinates of the point of intersection are $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin \frac{5\pi}{6} = y, \sin \frac{5\pi}{6} = \frac{1}{2}$

$\cos \frac{5\pi}{6} = x, \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan \frac{5\pi}{6} = \frac{y}{x}, \tan \frac{5\pi}{6} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}},$

$\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

$\csc \frac{5\pi}{6} = \frac{1}{y}, \csc \frac{5\pi}{6} = 2$

$\sec \frac{5\pi}{6} = \frac{1}{x}, \sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$

$\cot \frac{5\pi}{6} = \frac{x}{y}, \cot \frac{5\pi}{6} = -\sqrt{3}$

e) The terminal arm of an angle $\frac{11\pi}{6}$ is in the fourth quadrant; the coordinates of the point of intersection are $P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin \frac{11\pi}{6} = y, \sin \frac{11\pi}{6} = -\frac{1}{2}$

$\cos \frac{11\pi}{6} = x, \cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan \frac{11\pi}{6} = \frac{y}{x}, \tan \frac{11\pi}{6} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}},$

$\tan \frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$

$\csc \frac{11\pi}{6} = \frac{1}{y}, \csc \frac{11\pi}{6} = -2$

$\sec \frac{11\pi}{6} = \frac{1}{x}, \sec \frac{11\pi}{6} = \frac{2}{\sqrt{3}}$

$\cot \frac{11\pi}{6} = \frac{x}{y}, \cot \frac{11\pi}{6} = -\sqrt{3}$

f) The terminal arm of an angle $\frac{2\pi}{3}$ is in the second quadrant; the coordinates of the point of intersection are $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin \frac{2\pi}{3} = y, \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos \frac{2\pi}{3} = x, \cos \frac{2\pi}{3} = -\frac{1}{2}$

$\tan \frac{2\pi}{3} = \frac{y}{x}, \tan \frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}, \tan \frac{2\pi}{3} = -\sqrt{3}$

$\csc \frac{2\pi}{3} = \frac{1}{y}, \csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$

$\sec \frac{2\pi}{3} = \frac{1}{x}, \sec \frac{2\pi}{3} = -2$

$\cot \frac{2\pi}{3} = \frac{x}{y}, \cot \frac{2\pi}{3} = -\frac{1}{\sqrt{3}}$

8. a) $\sin \theta = \frac{y}{r} = -\frac{1}{5}$

$y = -1, r = 5$

$x = \sqrt{r^2 - y^2}$

$x = \sqrt{5^2 - (-1)^2}$

$x = \sqrt{24}$

$\cos \theta = \frac{x}{r}, \cos \theta = \frac{\sqrt{24}}{5}$

$\tan \theta = \frac{y}{x}, \tan \theta = \frac{-1}{\sqrt{24}}$

$\csc \theta = \frac{r}{y}, \csc \theta = \frac{5}{-1}, \csc \theta = -5$

$\sec \theta = \frac{r}{x}, \sec \theta = \frac{5}{\sqrt{24}}$

$\cot \theta = \frac{x}{y}, \cot \theta = \frac{\sqrt{24}}{-1}, \cot \theta = -\sqrt{24}$

b) $\tan \theta = \frac{y}{x} = \frac{-2}{-1} = 2$

$y = -2, x = -1$

$r = \sqrt{x^2 + y^2}$

$y = \sqrt{(-1)^2 + (-2)}^2$

$r = \sqrt{5}$

$\sin \theta = \frac{y}{r}, \sin \theta = \frac{-2}{\sqrt{5}}$

$\cos\theta = \frac{x}{r}, \cos\theta = \frac{-1}{\sqrt{5}}$

$\csc\theta = \frac{r}{y}, \csc\theta = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{r}{x}, \sec\theta = \frac{\sqrt{5}}{-1}, \sec\theta = -\sqrt{5}$

$\cot\theta = \frac{x}{y}, \cot\theta = \frac{-1}{-2}, \cot\theta = \frac{1}{2}$

c) $\csc\theta = \frac{r}{y} = \frac{2}{-1} = -2$

$r = 2, y = -1$

In the third quadrant x is negative.

$x = -\sqrt{r^2 - y^2}$

$x = -\sqrt{2^2 + (-1)^2}$

$x = -\sqrt{3}$

$\sin\theta = \frac{y}{r}, \sin\theta = \frac{-1}{2}$

$\cos\theta = \frac{x}{r}, \cos\theta = -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{y}{x}, \tan\theta = \frac{-1}{-\sqrt{3}}, \tan\theta = \frac{1}{\sqrt{3}}$

$\sec\theta = \frac{r}{x}, \sec\theta = -\frac{2}{\sqrt{3}}$

$\cot\theta = \frac{x}{y}, \cot\theta = \frac{-\sqrt{3}}{-1}, \cot\theta = \sqrt{3}$

d) $\sec\theta = \frac{r}{x} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$x = 1, r = \sqrt{3}$

$y = \sqrt{r^2 - x^2}$

$y = \sqrt{(\sqrt{3})^2 - 1^2}$

$y = \sqrt{2}$

$\sin\theta = \frac{y}{r}, \sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$

$\cos\theta = \frac{x}{r}, \cos\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{y}{x}, \tan\theta = \frac{\sqrt{2}}{1}, \tan\theta = \sqrt{2}$

$\csc\theta = \frac{r}{y}, \csc\theta = \frac{\sqrt{3}}{\sqrt{2}}$

$\cot\theta = \frac{x}{y}, \cot\theta = \frac{1}{\sqrt{2}}$

9. **a)** $\tan\frac{\pi}{6}, \cot\frac{\pi}{3}, \tan\frac{7\pi}{6}, \cot\frac{4\pi}{3}$

b) $\cos\frac{5\pi}{6}, \cos\frac{7\pi}{6}, \sin\frac{4\pi}{3}, \sin\frac{5\pi}{3}$

c) $\sec\frac{\pi}{6}, \csc\frac{\pi}{3}, \csc\frac{2\pi}{3}, \sec\frac{11\pi}{6}$

d) $\sec\frac{3\pi}{4}, \sec\frac{5\pi}{4}, \csc\frac{5\pi}{4}, \csc\frac{7\pi}{4}$

10. 44.3 m

11. Let the vertical side of the triangle to the left be a_1, the horizontal side of the triangle to the left be b_1, the vertical side of the triangle to the right be a_2, the horizontal side of the triangle to the right be b_2.

$\sin(0.4\pi) = \frac{a_1}{10.4}, a_1 = 10.4\sin(0.4\pi),$

$a_1 \cong 9.9$ m

$\cos(0.4\pi) = \frac{b_1}{10.4}, b_1 = 10.4\cos(0.4\pi),$

$b_1 \cong 3.2$ m

$\sin(0.15\pi) = \frac{a_2}{10.4}, a_2 = 10.4\sin(0.15\pi),$

$a_2 \cong 4.7$ m

$\cos(0.15\pi) = \frac{b_2}{10.4}, b_2 = 10.4\cos(0.15\pi),$

$b_2 \cong 9.3$ m

Horizontal displacement is 6.1 m to the left.

Vertical displacement is 5.2 m up.

12. **a)** **i)** $\frac{\sqrt{3}}{2}$ **ii)** -1 **iii)** $-\sqrt{3}$

b) **i)** 0.866 and $\frac{\sqrt{3}}{2} = -0.866$ **ii)** -1

iii) -1.732 and $-\sqrt{3} = -1.732$

13. Answers will vary.

14. $\sin\theta = \frac{3}{\sqrt{13}}; \cos\theta = -\frac{2}{\sqrt{13}}; \tan\theta = -\frac{3}{2}$

$\csc\theta = \frac{\sqrt{13}}{3}; \sec\theta = -\frac{\sqrt{13}}{2}; \cot\theta = -\frac{2}{3}$

15. $146\left(\sin\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{6}\right)\right) = 12.8$ million km down

16. **a)** **i)** $$\frac{\sec\frac{\pi}{4}\cos\frac{2\pi}{3}}{\tan\frac{\pi}{6}\csc\frac{3\pi}{4}} = \frac{\left(\frac{1}{\cos\frac{\pi}{4}}\right)\left(\cos\frac{2\pi}{3}\right)}{\left(\tan\frac{\pi}{6}\right)\left(\frac{1}{\sin\frac{3\pi}{4}}\right)}$$

$$= \frac{\left(\cos\frac{2\pi}{3}\right)\left(\sin\frac{3\pi}{4}\right)}{\left(\cos\frac{\pi}{4}\right)\left(\tan\frac{\pi}{6}\right)}$$

$$= \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\left(-\frac{1}{2\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}\sqrt{3}}\right)}$$

$$= -\frac{\sqrt{3}}{2} \approx -0.866$$

ii) $\sin \frac{5\pi}{4} - \cos \frac{11\pi}{6} \cot \frac{\pi}{3}$

$= \sin \frac{5\pi}{4} - \left(\cos \frac{11\pi}{6}\right)\left(\frac{1}{\tan \frac{\pi}{3}}\right)$

$= \left(-\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= -\frac{1}{\sqrt{2}} - \frac{1}{2}$

$= -\frac{\sqrt{2}}{2} - \frac{1}{2}$

$= \frac{-\sqrt{2} - 1}{2} \approx -1.207$

b) i) $\frac{\sec \frac{\pi}{4} \cos \frac{2\pi}{3}}{\tan \frac{\pi}{6} \csc \frac{3\pi}{4}} = \frac{(1.414)(-0.5)}{(0.577)(1.414)}$

$\cong -0.866$

ii) $\sin \frac{5\pi}{4} - \cos \frac{11\pi}{6} \cot \frac{\pi}{3}$

$= (-0.707) - (0.866)(0.577)$

$\cong -1.207$

17. 2.63 m^2

4.3 Equivalent Trigonometric Expressions

1. $\sin \frac{\pi}{4} = \cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

2. $\sec \frac{\pi}{6} = \csc\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \csc\left(\frac{2\pi}{6}\right)$

$= \csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$

3. $\tan \frac{\pi}{3} = \cot\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \cot\left(\frac{3\pi - 2\pi}{6}\right)$

$= \cot \frac{\pi}{6} = \sqrt{3}$

4. $\cos \frac{2\pi}{9} = 0.766 = \sin\left(\frac{\pi}{2} + \frac{2\pi}{9}\right)$

$= \sin\left(\frac{9\pi + 4\pi}{9}\right) = \sin \frac{13\pi}{9} = 0.766$

5. $\frac{3\pi}{4}$; This answer must be added to $\frac{\pi}{2}$ to give $\frac{5\pi}{4}$, but this is also the measure of angle x.

6. Express $\frac{2\pi}{3}$ as a sum of $\frac{\pi}{2}$ and an angle.

$\frac{2\pi}{3} = \frac{\pi}{2} + \alpha$

$\alpha = \frac{2\pi}{3} - \frac{\pi}{2}$

$\alpha = \frac{4\pi - 3\pi}{6}$

$\alpha = \frac{\pi}{6}$

$\frac{2\pi}{3} = \frac{\pi}{2} + \frac{\pi}{6}$

Apply a trigonometric identity to determine y.

$\sin \frac{2\pi}{3} = \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos \frac{\pi}{6}$

$\therefore y = \frac{\pi}{6}$

7. $\frac{3\pi}{8}$ **8.** $\frac{\pi}{10}$ **9.** $\frac{\pi}{5}$

10. a) Since an angle of $\frac{5\pi}{12}$ lies in the first quadrant, it can be expressed as a difference between $\frac{\pi}{2}$ and an angle α.

$\frac{5\pi}{12} = \frac{\pi}{2} - \alpha$

$\alpha = \frac{\pi}{2} - \frac{5\pi}{12}$

$\alpha = \frac{6\pi - 5\pi}{12}$

$\alpha = \frac{\pi}{12}$

Apply a co-function identity:

$\cos \frac{5\pi}{12} = \cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$

$= \sin \frac{\pi}{12} \cong 0.2588$

b) Since an angle of $\frac{7\pi}{12}$ lies in the second quadrant, it can be expressed as a sum of $\frac{\pi}{2}$ and an angle α.

$\frac{7\pi}{12} = \frac{\pi}{2} + \alpha$

$\alpha = \frac{7\pi}{12} - \frac{\pi}{2}$

$\alpha = \frac{7\pi - 6\pi}{12}$

$\alpha = \frac{\pi}{12}$

Apply a trigonometric identity:

$\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{2} + \frac{\pi}{12}\right)$

$= -\sin \frac{\pi}{12} = -0.2588$

11. a) 1.21 **b)** 1.21 **12.** 2.92 rad

13. $b = \frac{\pi}{10}$, $b \cong 0.31$

14. $\sec x = -\csc 0.57$ rad

$\frac{1}{\cos x} = -\frac{1}{\sin 0.57}$

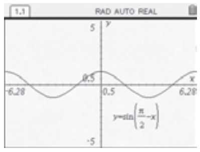
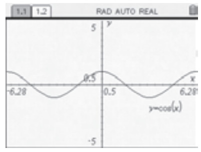
$\cos x = -\sin 0.57$

$\cos x = -0.5396$

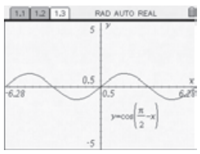
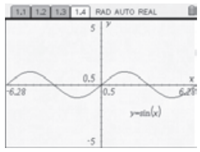
$x = 2.14$ rad

15. 0.05 rad **16.** 0.73 rad

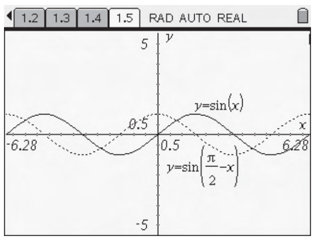
17. a) i)

ii)

b)

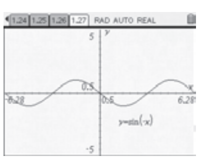
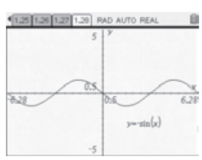


The difference is in the phase shift. Apply a phase shift of $\frac{\pi}{2}$ to the left on $y = \sin x$ and the result is $y = \left(\sin \frac{\pi}{2} - x\right)$; $\cos x$ is also a result of applying a phase shift of $\frac{\pi}{2}$ to the left on $y = \sin x$.

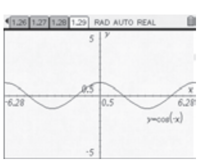
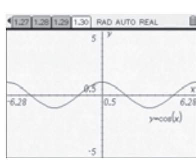
Therefore, $\sin\left(\frac{\pi}{2} - x\right) = \cos x$.

18. a) -0.9749 **b)** -0.9749

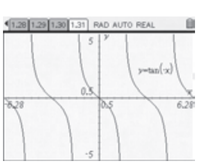
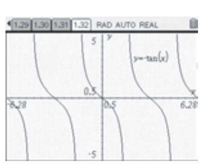
19. a)

b)

c)

The same relationship would be expected for the reciprocal trigonometric functions. This means that we would expect:
$\csc(-x) = -\csc x$; $\sec(-x) = \sec x$; $\cot(-x) = -\cot x$
They represent reflection in $\theta = 0$

20. Answers will vary.

21. Use compound-angle formulas, quotient identity, and reciprocal identities:

$$\sin\left(\alpha - \frac{\pi}{2}\right) = \sin \alpha \cos \frac{\pi}{2} - \cos \alpha \sin \frac{\pi}{2}$$
$$= \sin \alpha(0) - \cos \alpha(1) = -\cos \alpha$$
$$\cos\left(\alpha - \frac{\pi}{2}\right) = \cos \alpha \cos \frac{\pi}{2} + \sin \alpha \sin \frac{\pi}{2}$$
$$= \cos \alpha(0) + \sin \alpha(1) = \sin \alpha$$
$$\tan\left(\alpha - \frac{\pi}{2}\right) = \frac{\sin\left(\alpha - \frac{\pi}{2}\right)}{\cos\left(\alpha - \frac{\pi}{2}\right)}$$
$$= \frac{-\cos \alpha}{\sin \alpha} = -\cot \alpha$$
$$\csc\left(\alpha - \frac{\pi}{2}\right) = \frac{1}{\sin\left(\alpha - \frac{\pi}{2}\right)}$$
$$= \frac{1}{-\cos \alpha} = -\sec \alpha$$
$$\sec\left(\alpha - \frac{\pi}{2}\right) = \frac{1}{\cos\left(\alpha - \frac{\pi}{2}\right)} = \frac{1}{\sin \alpha} = \csc \alpha$$
$$\cot\left(\alpha - \frac{\pi}{2}\right) = \frac{1}{\tan\left(\alpha - \frac{\pi}{2}\right)} = \frac{1}{-\cot \alpha}$$
$$= -\tan \alpha$$
$$\sin(\alpha - \pi) = \sin \alpha \cos \pi - \cos \alpha \sin \pi$$
$$= \sin \alpha(-1) - \cos \alpha(0) = -\sin \alpha$$
$$\cos(\alpha - \pi) = \cos \alpha \cos \pi + \sin \alpha \sin \pi$$
$$= \cos \alpha(-1) + \sin \alpha(0) = -\cos \alpha$$
$$\tan(\alpha - \pi) = \frac{\sin(\alpha - \pi)}{\cos(\alpha - \pi)} = \frac{-\sin \alpha}{-\cos \alpha}$$
$$= \tan \alpha$$
$$\csc(\alpha - \pi) = \frac{1}{\sin(\alpha - \pi)} = \frac{1}{-\sin \alpha}$$
$$= -\csc \alpha$$
$$\sec(\alpha - \pi) = \frac{1}{\cos(\alpha - \pi)} = \frac{1}{-\cos \alpha}$$
$$= -\sec \alpha$$
$$\cot(\alpha - \pi) = \frac{1}{\tan(\alpha - \pi)} = \frac{1}{\tan \alpha} = \cot \alpha$$
$$\sin\left(\alpha - \frac{3\pi}{2}\right) = \sin \alpha \cos \frac{3\pi}{2} - \cos \alpha \sin \frac{3\pi}{2}$$
$$= \sin \alpha(0) - \cos \alpha(-1) = \cos \alpha$$
$$\cos\left(\alpha - \frac{3\pi}{2}\right) = \cos \alpha \cos \frac{3\pi}{2} + \sin \alpha \sin \frac{3\pi}{2}$$
$$= \cos \alpha(0) + \sin \alpha(-1) = -\sin \alpha$$
$$\tan\left(\alpha - \frac{3\pi}{2}\right) = \frac{\sin\left(\alpha - \frac{3\pi}{2}\right)}{\cos\left(\alpha - \frac{3\pi}{2}\right)} = \frac{\cos \alpha}{-\sin \alpha}$$
$$= -\cot \alpha$$

$$\csc\left(\alpha - \frac{3\pi}{2}\right) = \frac{1}{\sin\left(\alpha - \frac{3\pi}{2}\right)} = \frac{1}{\cos\alpha}$$
$$= \sec\alpha$$
$$\sec\left(\alpha - \frac{3\pi}{2}\right) = \frac{1}{\cos\left(\alpha - \frac{3\pi}{2}\right)} = \frac{1}{-\sin\alpha}$$
$$= -\csc\alpha$$
$$\cot\left(\alpha - \frac{3\pi}{2}\right) = \frac{1}{\tan\left(\alpha - \frac{3\pi}{2}\right)} = \frac{1}{-\cot\alpha}$$
$$= -\tan\alpha$$

a) $\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right) = -\sin(\alpha - \pi)$
$= -\cos\left(\alpha - \frac{3\pi}{2}\right)$

b) $\cos\alpha = -\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos(\alpha - \pi)$
$= \sin\left(\alpha - \frac{3\pi}{2}\right)$

c) $\tan\alpha = -\cot\left(\alpha - \frac{\pi}{2}\right) = \tan(\alpha - \pi)$
$= -\cot\left(\alpha - \frac{3\pi}{2}\right)$

d) $\csc\alpha = \sec\left(\alpha - \frac{\pi}{2}\right) = -\csc(\alpha - \pi)$
$= -\sec\left(\alpha - \frac{3\pi}{2}\right)$

e) $\sec\alpha = -\csc\left(\alpha - \frac{\pi}{2}\right) = -\sec(\alpha - \pi)$
$= \csc\left(\alpha - \frac{3\pi}{2}\right)$

f) $\cot\alpha = -\tan\left(\alpha - \frac{\pi}{2}\right) = \cot(\alpha - \pi)$
$= -\tan\left(\alpha - \frac{3\pi}{2}\right)$

22. a) $\sin\alpha = -\cos\left(\alpha + \frac{\pi}{2}\right) = -\sin(\alpha + \pi)$
$= \cos\left(\alpha + \frac{3\pi}{2}\right)$

$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right) = -\cos(\alpha + \pi)$
$= -\sin\left(\alpha + \frac{3\pi}{2}\right)$

$\tan\alpha = -\cot\left(\alpha + \frac{\pi}{2}\right) = \tan(\alpha + \pi)$
$= -\cot\left(\alpha + \frac{3\pi}{2}\right)$

$\csc\alpha = -\sec\left(\alpha + \frac{\pi}{2}\right) = -\csc(\alpha + \pi)$
$= \sec\left(\alpha + \frac{3\pi}{2}\right)$

$\sec\alpha = \csc\left(\alpha + \frac{\pi}{2}\right) = -\sec(\alpha + \pi)$
$= -\csc\left(\alpha + \frac{3\pi}{2}\right)$

$\cot\alpha = -\tan\left(\alpha + \frac{\pi}{2}\right) = \cot(\alpha + \pi)$
$= -\tan\left(\alpha + \frac{3\pi}{2}\right)$

b) They also represent phase shifts of the original function.

23. a) 0.8391 **b)** −0.8391

24. a) $-\frac{\pi}{2}$ **b)** Answers will vary.

25. a) $\sin x$ **b)** $-\sin x - \cos^2 x$

26. $\text{L.S.} = \dfrac{\sin\left(x + \frac{\pi}{2}\right)}{\sin x}$
$$= \frac{\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}}{\sin x}$$
$$= \frac{\sin x(0) + \cos x(1)}{\sin x} = \frac{\cos x}{\sin x} = \cot x$$
R.S. = $\tan x$
L.S. ≠ R.S.

4.4 Compound-Angle Formulas

1. a) $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ **b)** $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$
c) $\cos\frac{11\pi}{12} = \frac{-\sqrt{6} - \sqrt{2}}{4}$ **d)** $\cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}$

2. $\frac{\sqrt{2} + \sqrt{6}}{4}$

3. a) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$ **b)** $\frac{1 + \sqrt{3}}{2\sqrt{2}}$ **c)** $\frac{-1 - \sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{1 - \sqrt{3}}{2\sqrt{2}}$

4. Answers may vary. For example:
$$\sin\frac{13\pi}{36} = \sin\left(\frac{\pi}{4} + \frac{\pi}{9}\right)$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{9} + \cos\frac{\pi}{4}\sin\frac{\pi}{9}$$
$$= \left(\frac{1}{\sqrt{2}}\right)\cos\frac{\pi}{9} + \left(\frac{1}{\sqrt{2}}\right)\sin\frac{\pi}{9}$$
$$= \frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{9} + \sin\frac{\pi}{9}\right)$$

5. Answers may vary. For example:
$$x2\,\frac{-\sqrt{3} + \tan\frac{\pi}{8}}{1 + \sqrt{3}\tan\frac{\pi}{8}}$$

6. a) $\frac{\sqrt{2} - \sqrt{6}}{4}$ **b)** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **c)** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **d)** $\frac{\sqrt{2} + \sqrt{6}}{4}$

7. **a)** $\sin \frac{23\pi}{12} = \sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \sin \frac{\pi}{4} \cos \frac{\pi}{3} - \cos \frac{\pi}{4} \sin \frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1 - \sqrt{3}}{2\sqrt{2}}$

b) $\cos \frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$= \cos \frac{3\pi}{4} \cos \frac{\pi}{3} - \sin \frac{3\pi}{4} \sin \frac{\pi}{3}$

$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-1 - \sqrt{3}}{2\sqrt{2}}$

c) $\tan \frac{23\pi}{12} = \tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \dfrac{\tan \frac{\pi}{4} - \tan \frac{\pi}{3}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{3}}$

$= \frac{1 - \sqrt{3}}{1 + (1)(\sqrt{3})} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

d) $\csc \frac{5\pi}{12} = \csc\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) = \dfrac{1}{\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)}$

$= \dfrac{1}{\sin \frac{3\pi}{4} \cos \frac{\pi}{3} - \cos \frac{3\pi}{4} \sin \frac{\pi}{3}}$

$= \dfrac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{1}{\frac{1 + \sqrt{3}}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}}{1 + \sqrt{3}}$

8. **a)** $\cos x = \frac{\sqrt{3}}{2}$; $\cos y = \frac{\sqrt{15}}{4}$

b) i) $\frac{\sqrt{15} + \sqrt{3}}{8}$ **ii)** $\frac{\sqrt{15} - \sqrt{3}}{8}$

iii) $\frac{3\sqrt{5} - 1}{8}$ **iv)** $\frac{3\sqrt{5} + 1}{8}$

9. **a)** $\cos x = -\frac{\sqrt{77}}{9}$; $\cos y = \frac{3}{5}$

b) i) $\frac{-8 - 3\sqrt{77}}{45}$ **ii)** $\frac{-8 + 3\sqrt{77}}{45}$

iii) $\frac{4\sqrt{77} - 6}{45}$ **iv)** $\frac{4\sqrt{77} + 6}{45}$

10. **a)** $\cos x = \frac{3}{5}$ and x is in the first quadrant

$\sin x = \frac{\sqrt{5^2 - 3^2}}{5} = \frac{\sqrt{16}}{5} = \frac{4}{5}$

$\sin y = \frac{24}{25}$ and y is in the first quadrant

$\cos y = \frac{\sqrt{25^2 - 24^2}}{25} = \frac{\sqrt{49}}{25} = \frac{7}{25}$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= \left(\frac{4}{5}\right)\left(\frac{7}{25}\right) - \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = \frac{28}{125} - \frac{72}{125} = -\frac{44}{125}$

b) $\sin x = \frac{5}{13}$ and x is in the first quadrant

$\cos x = \frac{\sqrt{13^2 - 5^2}}{13} = \frac{\sqrt{144}}{13} = \frac{12}{13}$

$\sin y = \frac{12}{13}$ and y is in the first quadrant

$\cos y = \frac{\sqrt{13^2 - 12^2}}{13} = \frac{\sqrt{25}}{13} = \frac{5}{13}$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{5}{13}\right)\left(\frac{5}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{12}{13}\right) = \frac{25}{169} + \frac{144}{169}$

$= \frac{169}{169} = 1$

c) $\cos x = \frac{2}{3}$ and x is in the first quadrant

$\sin x = \frac{\sqrt{3^2 - 2^2}}{3} = \frac{\sqrt{5}}{3}$

$\sin y = \frac{10}{13}$ and y is in the first quadrant

$\cos y = \frac{\sqrt{13^2 - 10^2}}{13} = \frac{\sqrt{69}}{13}$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{2}{3}\right)\left(\frac{\sqrt{69}}{13}\right) - \left(\frac{\sqrt{5}}{3}\right)\left(\frac{10}{13}\right) = \frac{2\sqrt{69}}{39} - \frac{10\sqrt{5}}{39}$

$= \frac{2\sqrt{69} - 10\sqrt{5}}{39}$

d) $\cos x = \frac{3}{5}$ and x is in the first quadrant

$\sin x = \frac{\sqrt{5^2 - 3^2}}{5} = \frac{\sqrt{16}}{5} = \frac{4}{5}$

$\sin y = \frac{4}{5}$ and y is in the first quadrant

$\cos y = \frac{\sqrt{5^2 - 4^2}}{5} = \frac{\sqrt{9}}{5} = \frac{3}{5}$

$\cos (x - y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{12}{25} + \frac{12}{25} = \frac{24}{25}$

e) $\cot x = \frac{6}{5}$ and x is in the first quadrant

$\tan x = \frac{1}{\cot x} = \frac{5}{6}$

$\sec y = \frac{4}{5}$ and y is in the first quadrant

$\tan y = \frac{\sqrt{3^2 - 2^2}}{2} = \frac{\sqrt{5}}{2}$

$\tan (x + y) = \dfrac{\tan x + \tan y}{1 + \tan x \tan y}$

$$= \frac{\left(\frac{5}{6}\right) + \left(\frac{\sqrt{5}}{2}\right)}{1 + \left(\frac{5}{6}\right)\left(\frac{\sqrt{5}}{2}\right)} = \frac{\frac{5 + 3\sqrt{5}}{6}}{1 + \frac{5\sqrt{5}}{12}} = \frac{\frac{5 + 3\sqrt{5}}{6}}{\frac{12 - 5\sqrt{5}}{12}}$$

$$= \frac{12(5 + 3\sqrt{5})}{6(12 - 5\sqrt{5})} = \frac{2(5 + 3\sqrt{5})}{12 - 5\sqrt{5}}$$

f) $\sec x = \frac{4}{3}$ and x is in the first quadrant

$$\cos x = \frac{1}{\sec x} = \frac{3}{4}$$

$$\sin x = \frac{\sqrt{4^2 - 3^2}}{4} = \frac{\sqrt{7}}{4}$$

$\tan y = \frac{13}{5}$ and y is in the first quadrant

$$\sin y = \frac{13}{\sqrt{13^2 + 5^2}} = \frac{13}{\sqrt{194}}$$

$$\cos y = \frac{5}{\sqrt{13^2 + 5^2}} = \frac{5}{\sqrt{194}}$$

$$\csc(x - y) = \frac{1}{\sin(x - y)}$$

$$= \frac{1}{\sin x \cos y - \cos x \sin y}$$

$$= \frac{1}{\left(\frac{\sqrt{7}}{4}\right)\left(\frac{5}{\sqrt{194}}\right) - \left(\frac{3}{4}\right)\left(\frac{13}{\sqrt{194}}\right)}$$

$$= \frac{4\sqrt{194}}{5\sqrt{7} - 39}$$

11. $\frac{\sqrt{3}\cos\alpha - \sin\alpha}{2}$

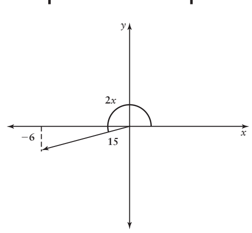
12. $$\csc\left(\frac{3\pi}{2} + \theta\right) = \frac{1}{\sin\frac{3\pi}{2}\cos\theta + \cos\frac{3\pi}{2}\sin\theta}$$

$$= \frac{1}{(-1)\cos\theta + (0)\sin\theta}$$

$$= \frac{1}{-\cos\theta + 0} = -\sec\theta$$

13. $\cos\frac{13\pi}{4} = -\cos\frac{\pi}{4}$

14. a)



b) second quadrant **c)** $-\sqrt{\frac{3}{10}}$ **d)** 2.15

e) -0.548 and $-\sqrt{\frac{3}{10}} = -0.5477225575$

15. a) $\sin b = \frac{4}{5}$, $\tan b = -\frac{4}{3}$ **b)** $-\frac{7}{25}$ **c)** $-\frac{24}{25}$ **d)** $\frac{24}{7}$ **e)** 2.21 rad **f)** The angle $2b$ lies in the third quadrant, as indicated by the signs of the three primary ratios.

16. a) $\frac{\sqrt{2 - \sqrt{3}}}{2}$ **b)** Answers will vary.

17. $\sin 2x = -\frac{24}{25}$; $\cos 2x = -\frac{7}{25}$

18. a) $\sin 2x = \frac{24}{25}$; $\cos 2x = \frac{7}{25}$ **b)** $\sin 2x = -1$; $\cos 2x = 0$ **c)** $\sin 2x = \frac{\sqrt{15}}{8}$; $\cos 2x = \frac{7}{8}$ **d)** $\sin 2x = -\frac{12}{13}$; $\cos 2x = -\frac{5}{13}$

19. $\sin 3x = 3\sin x - 4\sin^3 x$

20. $\cos 3x = 4\cos^3 x - 3\cos x$

21. a) $\sqrt{\frac{2 + \sqrt{3}}{2}}$ **b)** $\sqrt{\frac{2 + \sqrt{2}}{2}}$

22. $\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$

$$\begin{aligned} RS &= \frac{1}{2}[\cos x \cos x + \sin x \sin y \\ &\quad + \cos x \cos y - \sin x \sin y] \\ &= \frac{1}{2}[2\cos x \cos y] \\ &= \cos x \cos y \\ &= LS \end{aligned}$$

23. $R = \frac{2v^2\cos\theta\sin(\theta - \beta)}{g\cos^2\beta}$

$\beta = \frac{\pi}{6}$

$$R = \frac{2v^2\cos\theta\sin\left(\theta - \frac{\pi}{6}\right)}{g\cos^2\frac{\pi}{6}}$$

$$= \frac{2v^2\cos\theta\left(\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6}\right)}{g\cos^2\frac{\pi}{6}}$$

$$= \frac{2v^2\cos\theta\left(\sin\theta\left(\frac{\sqrt{3}}{2}\right) - \cos\theta\left(\frac{1}{2}\right)\right)}{g\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{2v^2\cos\theta\left(\frac{1}{2}\right)(\sqrt{3}\sin\theta - \cos\theta)}{g\left(\frac{3}{4}\right)}$$

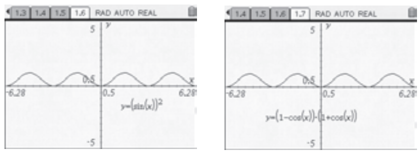
$$= \frac{4v^2\cos\theta(\sqrt{3}\sin\theta - \cos\theta)}{3g}$$

The range of the rocket is

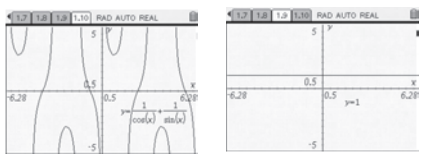
$$R = \frac{4v^2\cos\theta(\sqrt{3}\sin\theta - \cos\theta)}{3g}$$

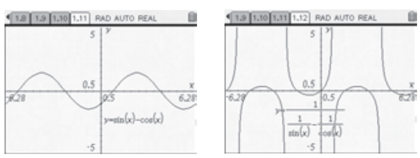
4.5 Prove Trigonometric Identities

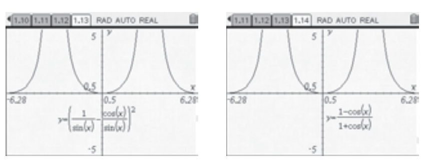
1. **a)** 1 **b)** $\csc x$ **c)** $\csc x$ **d)** 2
2. **a)** yes **b)** yes **c)** no **d)** no **e)** yes **f)** yes

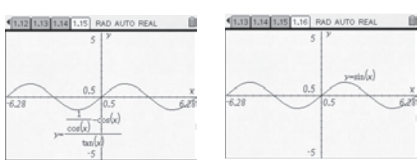
3. a) 

b) 

c) 

d) 

e) 

f) 

4. Answers may vary. Sample answer:

c) $\sec x + \csc x = \dfrac{\sin x + \cos x}{\sin x \cos x}$

d) $\sin x - \cos x = \dfrac{\sec x - \csc x}{\sec x \csc x}$

5. a) This is not an identity, i.e.,

$\tan^2 x + \sec^2 x \neq 1$

Counter-example: $x = \dfrac{\pi}{4}$

L.S. $= \tan^2 \dfrac{\pi}{4} + \sec^2 \dfrac{\pi}{4} = (1)^2 + \left(\sqrt{2}\right)^2$

$= 1 + 2 = 3$

R.S. $= 1$

L.S. $\neq$ R.S.

Therefore, $\tan^2 x + \sec^2 x \neq 1$.

b) This is an identity, i.e.,

$\cos x \tan^2 x = \sin x \tan x$

L.S. $= \cos x \tan^2 x$ Use quotient identity.

$= \cos x \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\sin^2 x}{\cos x}$

$= \sin x \dfrac{\sin x}{\cos x}$ Use quotient identity.

$= \sin x \tan x$

$=$ R.S.

L.S. $=$ R.S.

Therefore, $\cos x \tan^2 x = \sin x \tan x$ is an identity.

c) This is an identity, i.e.,

$\tan 2x - \sin 2x = 2 \tan 2x \sin^2 x$

L.S. $= \tan 2x - \sin 2x$ Use quotient identity.

$= \dfrac{\sin 2x}{\cos 2x} - \sin 2x$

$= \dfrac{\sin 2x - \sin 2x \cos 2x}{\cos 2x}$

$= \dfrac{\sin 2x(1 - \cos 2x)}{\cos 2x}$

$= \dfrac{\sin 2x}{\cos 2x}(1 - \cos 2x)$ Use quotient identity and double-angle formula.

$= \tan 2x\left(1 - (1 - 2\sin^2 x)\right)$

$= \tan 2x(1 - 1 + 2\sin^2 x)$

$= \tan 2x(2\sin^2 x)$

$= 2 \tan 2x \sin^2 x$

$=$ R.S.

L.S. $=$ R.S.

Therefore, $\tan 2x - \sin 2x = 2 \tan 2x \sin^2 x$ is an identity.

d) This is an identity, i.e.,

$\sin 2x = \tan x(1 + \cos 2x)$

R.S. $= \tan x(1 + \cos 2x)$ Use double-angle formula.

$= \tan x(2\cos^2 x)$ Use quotient identity.

$= \dfrac{\sin x}{\cos x}(2\cos^2 x)$

$= 2 \sin x \cos x$ Use double-angle formula.

$= \sin 2x$

$=$ L.S.

L.S. $=$ R.S.

Therefore, $\sin 2x = \tan x(1 + \cos 2x)$ is an identity.

e) This is not an identity, i.e.,

$\sec 2x \neq \dfrac{\sec x \csc x}{2}$

Counter-example: $x = \dfrac{\pi}{6}$

$\text{L.S.} = \sec\left(2\frac{\pi}{6}\right) = \sec\frac{\pi}{3} = 2$

$\text{R.S.} = \dfrac{\sec\left(\frac{\pi}{6}\right)\csc\left(\frac{\pi}{6}\right)}{2} = \dfrac{\left(\frac{2}{\sqrt{3}}\right)(2)}{2} = \dfrac{2}{\sqrt{3}}$

L.S. ≠ R.S.

Therefore, $\sec 2x \neq \dfrac{\sec x \csc x}{2}$.

f) This is an identity, i.e., $\sin^2 x + \cos^2 x + \tan^2 x = \sec^2 x$

$\text{L.S.} = \sin^2 x + \cos^2 x + \tan^2 x$

$= 1 + \tan^2 x$ Use quotient identity.

$= 1 + \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$ Use Pythagorean identity.

$= \dfrac{1}{\cos^2 x}$ Use reciprocal identity.

$= \sec^2 x$

= R.S.

L.S. = R.S.

Therefore, $\sin^2 x + \cos^2 x + \tan^2 x = \sec^2 x$ is an identity.

g) $\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

$\text{L.S.} = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$

$= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$

$= \dfrac{\cos x - \sin x}{\cos x}$

$= 1 - \dfrac{\sin x}{\cos x}$ Use quotient identity.

$= 1 - \tan x$

= R.S.

L.S. = R.S.

Therefore, $\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$ is an identity.

6. Answers may vary. For example:

a) counter-example: $x = \dfrac{\pi}{6}$

b) counter-example: $x = \dfrac{\pi}{3}$

c) counter-example: $x = \dfrac{2\pi}{3}$

d) counter-example: $x = \dfrac{\pi}{4}$

7. $\sin\left(\frac{\pi}{2} + \theta\right) = \sin\left(\frac{\pi}{2}\right)\cos\theta + \cos\left(\frac{\pi}{2}\right)\sin\theta$

$= (1)\cos\theta + (0)\sin\theta = \cos\theta$

8. **a)** $-\sin x$ **b)** $-\cos x$ **c)** $-\cos y$ **d)** $\cos y$

9. $\tan\dfrac{3\pi}{2}$ is undefined

Use quotient identity and compound-angle formula instead.

10. $\sin\left(x - \frac{4\pi}{3}\right) = \dfrac{\sqrt{3}\cos x - \sin x}{2}$

$\text{L.S.} = \sin\left(x - \frac{4\pi}{3}\right)$ Use double angle formula.

$= 2\sin\left[\frac{1}{2}\left(x - \frac{4\pi}{3}\right)\right]\cos\left[\frac{1}{2}\left(x - \frac{4\pi}{3}\right)\right]$

$= 2\sin\left(\frac{x}{2} - \frac{2\pi}{3}\right)\cos\left(\frac{x}{2} - \frac{2\pi}{3}\right)$

Use compound-angle formula.

$= 2\left[\sin\frac{x}{2}\cos\frac{2\pi}{3} - \cos\frac{x}{2}\sin\frac{2\pi}{3}\right]\left[\cos\frac{x}{2}\cos\frac{2\pi}{3} + \sin\frac{x}{2}\sin\frac{2\pi}{3}\right]$

$= 2\left[\sin\frac{x}{2}\left(-\frac{1}{2}\right) - \cos\frac{x}{2}\left(\frac{\sqrt{3}}{2}\right)\right]\left[\cos\frac{x}{2}\left(-\frac{1}{2}\right) + \sin\frac{x}{2}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(-\sin\frac{x}{2} - \sqrt{3}\cos\frac{x}{2}\right)\left(-\cos\frac{x}{2} + \sqrt{3}\sin\frac{x}{2}\right)$

$= \left(\frac{1}{2}\right)\left(\sin\frac{x}{2}\cos\frac{x}{2} - \sqrt{3}\sin^2\frac{x}{2} + \sqrt{3}\cos^2\frac{x}{2} - 3\sin\frac{x}{2}\cos\frac{x}{2}\right)$

$= \left(\frac{1}{2}\right)\left[-2\sin\frac{x}{2}\cos\frac{x}{2} + \sqrt{3}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)\right]$ Use double-angle formula.

$= \left(\frac{1}{2}\right)\left[-\sin x + \sqrt{3}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)\right]$

Use double-angle formula.

$= \left(\frac{1}{2}\right)(\sqrt{3}\cos x - \sin x)$

$= \dfrac{\sqrt{3}\cos x - \sin x}{2}$

= R.S.

L.S. = R.S.

Therefore, $\sin\left(x - \frac{4\pi}{3}\right) = \dfrac{\sqrt{3}\cos x - \sin x}{2}$ is an identity.

11. $\cos\left(2x + \frac{\pi}{2}\right) = -\sin 2x$

$$\begin{aligned}
\text{L.S.} &= \cos\left(2x + \frac{\pi}{2}\right) \\
&= \cos\left[2\left(x + \frac{\pi}{4}\right)\right] && \text{Use double-angle formula} \\
&= 1 - 2\sin^2\left(x + \frac{\pi}{4}\right) && \text{Use compound-angle formula} \\
&= 1 - 2\left(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}\right)^2 \\
&= 1 - 2\left[\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right]^2 \\
&= 1 - 2\left[\left(\frac{1}{\sqrt{2}}\right)(\sin x + \cos x)\right]^2 \\
&= 1 - 2\left(\frac{1}{2}\right)[(\sin x + \cos x)]^2 \\
&= 1 - (\sin x + \cos x)^2 \\
&= 1 - (\sin^2 x + 2\sin x \cos x + \cos^2 x) \\
& \qquad \text{Use Pythagorean identity} \\
&= 1 - (1 + 2\sin x \cos x) \\
&= -2\sin x \cos x && \text{Use double angle formula} \\
&= -\sin 2x \\
&= \text{R.S.}
\end{aligned}$$

12. Answers may vary.

13. a) $\sin 4x = \sin(2(2x)) = 2\sin 2x \cos 2x$
b) $\sin 6x = \sin(2(3x)) = 2\sin 3x \cos 3x$
$\sin 8x = \sin(2(4x)) = 2\sin 4x \cos 4x$
These identities will hold for all $\sin kx$, where k is a positive integer because the angle kx can be expressed as $2\left(\frac{kx}{2}\right)$; therefore, $\sin kx = \sin\left(2\left(\frac{kx}{2}\right)\right) = 2\sin\frac{kx}{2}\cos\frac{kx}{2}$.

14. a) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$
c) $0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$ **d)** $\frac{3\pi}{4}, \frac{7\pi}{4}$
e) $0 \le x \le 2\pi$ **f)** $\frac{\pi}{3}, \frac{5\pi}{3}$

15. a–b) Answers will vary.

16. Answers will vary.

17. a–b) Answers will vary.

18. Answers will vary.

19. a) No, the graphs are not the same for all values. **b)** Answers may vary. For example: Let $x = 0$; L.S. $\ne$ R.S.

20. $B = \frac{F\csc\theta}{I\ell}$
Use reciprocal identity $\csc\theta = \frac{1}{\sin\theta}$.
$B = \frac{F\left(\frac{1}{\sin\theta}\right)}{I\ell}$ $\quad B = \frac{F}{I\ell\sin\theta}$

21. a) $I = I_0\cos^2\theta$ **b)** half the original intensity

22. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

$$\begin{aligned}
&(\sin x \cos y + \cos x \sin y) \\
&\quad (\sin x \cos y - \cos x \sin y) = \text{LS} \\
&\sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \text{LS} \\
&\sin^2 x(1 - \sin^2 y) - \\
&\quad (1 - \sin^2 x)\sin^2 y = \text{LS} \\
&\sin^2 x - \sin^2 x \sin^2 y \\
&\quad - \sin^2 y + \sin^2 x \sin^2 y = \text{LS} \\
&\sin^2 x - \sin^2 y = \text{LS} \\
&= \text{RS}
\end{aligned}$$

23. a)
$$\begin{aligned}
\text{R.S.} &= \frac{1}{2}[\cos(x - y) - \cos(x + y)] \\
&= \frac{1}{2}[(\cos x \cos y + \sin x \sin y) \\
&\qquad - (\cos x \cos y - \sin x \sin y)] \\
&= \frac{1}{2}[2\sin x \sin y] \\
&= \sin x \sin y \\
&= \text{L.S.}
\end{aligned}$$

b) i) $\frac{1}{2}[\cos(x + y) + \cos(x - y)]$
$= \cos x \cos y$

ii) $\frac{1}{2}[\sin(x + y) - \sin(x - y)]$
$= \cos x \sin y$

iii) $\frac{1}{2}[\sin(x + y) + \sin(x - y)]$
$= \sin x \cos y$

24. a) $\sin\frac{x + y}{2}\sin\frac{x - y}{2} = \frac{1}{2}[\cos y - \cos x]$

b) $\sin\frac{x + y}{2}\cos\frac{x - y}{2} = \frac{1}{2}[\sin x + \sin y]$

c) $\cos\frac{x + y}{2}\sin\frac{x - y}{2} = \frac{1}{2}[\sin x - \sin y]$

d) $\cos\frac{x + y}{2}\cos\frac{x - y}{2} = \frac{1}{2}[\cos x + \cos y]$

25. a–c) Answers may vary.

Chapter 4 Challenge Questions

C1. 94.6 m

C2. Any values for the radius and the radian measure that satisfy the relationship $52 = r^2\theta$ are possible solutions.

C3. 1.7425 m/s

C4. As you move from the equator (where the circumference is a maximum) to the poles, the circumference decreases by a factor of cos L. At the poles, the circumference is 0 (because of the fact that at the poles the angle is 90º and cos 90º is zero).

C5. 3.43°. As the angle increases, the speed must also increase, as these two variables are in direct proportionality to each other in this formula.

C6. $\frac{\pi}{4}$ radians or 45°

C7. a) $\frac{\cos(x + h) - \cos x}{h}$

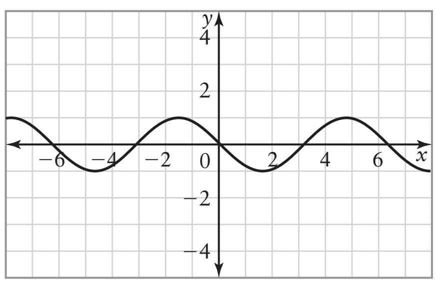
b) $\frac{\cos(x + 0.1) - \cos x}{0.1}$

$= \frac{\cos x \cos(0.1) - \sin x \sin(0.1) - \cos x}{0.1}$

$= \cos x\left(\frac{\cos(0.1) - 1}{0.1}\right) - \sin x\left(\frac{\sin(0.1)}{0.1}\right)$

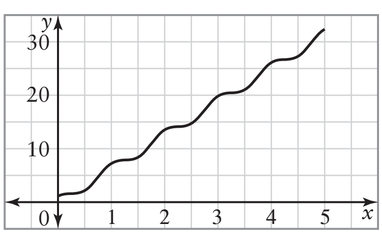
$= -0.050 \cos x - 0.998 \sin x$

c)



The expression after the cos x is approximately equal to zero [and gets closer to zero if h is smaller than 0.1 (0.001, for example)], and the expression after the sin x is approximately equal to 1 [and gets closer to 1 if h is smaller than 0.1 (0.001, for example)]. Therefore, the expression is approximately equal to $-\sin x$.

C8. a)



b) moving left: (0.5, 1), (1.5, 2), (2.5, 3), (3.5, 4), (4.5, 5)
moving right: (0, 0.5), (1, 1.5), (2, 2.5), (3, 3.5), (4, 4.5)

c) Answers may vary. **d)** 2π

e) When $t = 0, 1, 2, 3, 4$, and 5, the velocity has the same value of approximately 6.3.

C9. 60°

C10. max = 129.9 V and min = −129.9 V

Chapter 5

5.1 Graphs of Sine, Cosine, and Tangent Functions

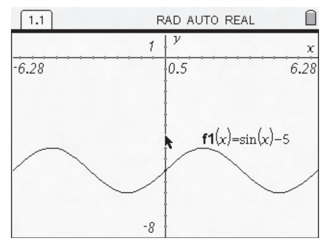
1. a) max −4 at $x = -\frac{3\pi}{2}, \frac{\pi}{2}$
min −6 at $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

b) max 8 at $x = -2\pi, 0, \pi$
min 6 at $x = -\pi, \pi$

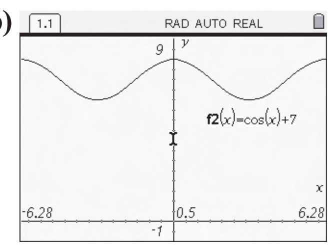
c) max 2 at $x = -\frac{3\pi}{2}, \frac{\pi}{2}$
min 0 at $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

d) max −2 at $x = -2\pi, 0, \pi$
min −4 at $x = -\pi, \pi$

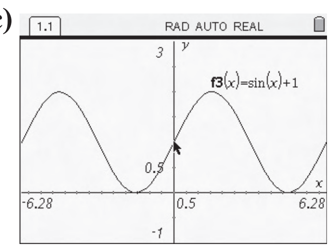
2. a)



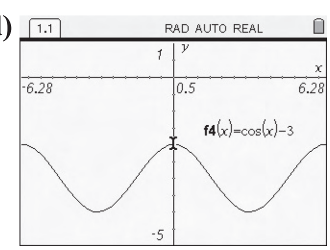
b)



c)



d)



3. a) $y = \frac{1}{2}\sin x$ **b)** $y = 3 \cos x$

c) $y = -7\sin x$ **d)** $y = -\frac{1}{4}\cos x$

4. a) $y = \sin\left(x - \frac{2\pi}{3}\right)$ **b)** $y = \cos\left(x - \frac{4\pi}{5}\right)$

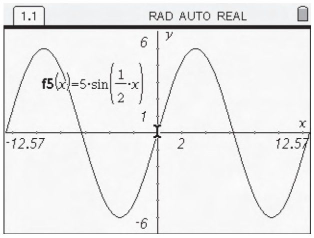
c) $y = \sin\left(x + \frac{5\pi}{7}\right)$ **d)** $y = \cos\left(x + \frac{11\pi}{7}\right)$

5. a) $y = \sin \frac{1}{2}x$ **b)** $y = \cos 4x$

c) $y = \sin \frac{4}{3}x$ **d)** $y = \cos \frac{8}{5}x$

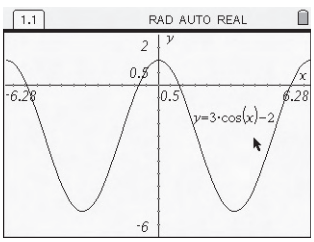
6. a) $y = 5 \sin \frac{1}{2}x$

b)



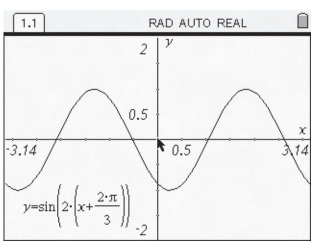
7. a) 3 **b)** 2 down **c)** $y = 3 \cos x - 2$

d)



8. a) π **b)** $\frac{2\pi}{3}$ left **c)** $y = \sin\left[2\left(x + \frac{2\pi}{3}\right)\right]$

d)



9. a) To determine the amplitude:

$\frac{\max - \min}{2} = \frac{0.82 - 0.08}{2} = 0.370$

The period is 4 seconds, so

$\frac{2\pi}{k} = 4$

$4k = 2\pi; k = \frac{\pi}{2}$

Since the minimum amount of air is in the lungs at $t = 0$, the cosine function needs to be reflected in the t-axis. The point (0, 0.08) must be on the function, so

$V = -0.370 \cos\left(\frac{\pi}{2}t\right) + d$ becomes

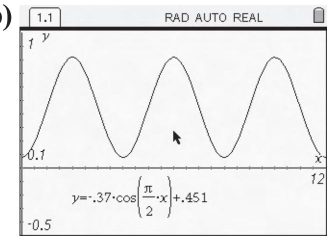
$0.08 = -0.370 \cos\left(\frac{\pi}{2}(0)\right) + d$

$0.08 = -0.370 + d$

$d = 0.45$

Therefore, $V = -0.370 \cos\left(\frac{\pi}{2}t\right) + 0.45$ L

b)



c) $V = -0.370 \cos\left(\frac{\pi}{2}(5.5)\right) + 0.45$

$= 0.71$ L

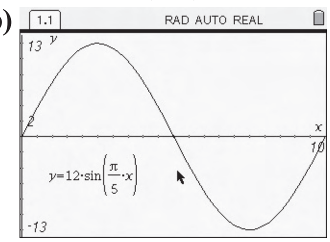
10. $y = 2 \cos\left(\frac{\pi}{15}x\right)$

11. a) $y = 13.1 \sin\left[\frac{\pi}{6}(x - 4)\right] + 8.6$

b) 2.05° **c)** 19.9°

12. a) $y = 12 \sin\left(\frac{\pi}{5}x\right)$

b)

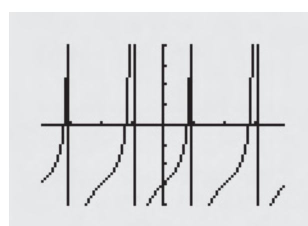


c) 11.4 cm below equilibrium position

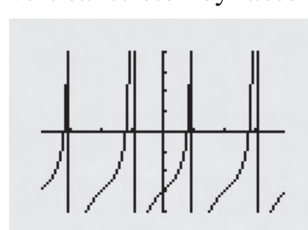
d) 6.3 s, 8.7 s

13. a) 25 **b)** 30 up **c)** 90 **d)** $\frac{\pi}{45}$

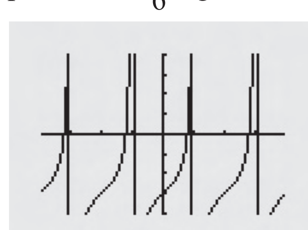
14. a) vertical translation 3 down



b) vertical stretch by factor of 2

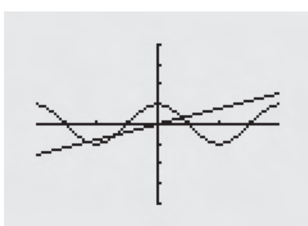


c) phase shift $\frac{\pi}{6}$ right

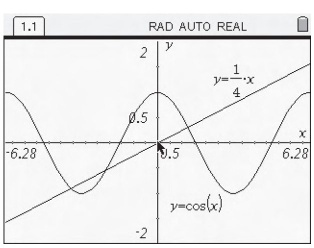


d) change of period to $\frac{\pi}{4}$

15. a) The solutions must be from the interval $-4 \le x \le 4$, since beyond this, $y = \frac{1}{4}x$ exceeds the amplitude of cos x (which is 1). For $0 < x < 4$, $y = \frac{1}{4}x$ is positive. According to the CAST rule, for $0 < x < 4$, cos x is only positive once (cos x is positive in the first quadrant, but negative in the second and third [which is where $x = 4$ stops]). For $-4 < x < 0$, $y = \frac{1}{4}x$ is negative. According to the CAST rule, for $-4 < x < 0$, cos x is negative twice (cos x is positive in the fourth quadrant, but negative in the third and second [which is where $x = -4$ stops]). Therefore, there will be three points of intersection in total.



b)



5.2 Graphs of Reciprocal Trigonometric Functions

1. 3.28, 6.14 **2.** 1.37, 4.91 **3.** 3.06, 6.20

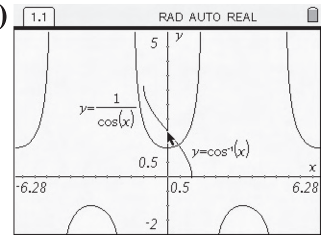
4. a) $y = \sec\left(x - \frac{\pi}{2}\right)$

b) Yes. The secant function is periodic, with period 2π; therefore adding a multiple of 2π to the phase shift will not change the graph. The following transformations are all equivalent to $y = \csc x$:

$y = \sec\left(x - \left(\frac{\pi}{2} + 2\pi n\right)\right)$, n is an integer

$y = \sec\left(x + \left(\frac{3\pi}{2} + 2\pi n\right)\right)$, n is an integer

5. a)



b) approximately 0.446 rad

6. 2215.4 m

7. a) According to the diagram,

$\cos x = \frac{1200}{d}$

So $d = \frac{1200}{\cos x}$

Since the function d is in terms of x and $\frac{1}{\cos x} = \sec x$, this function can be written as $d(x) = 1200 \sec x$

b) $d\left(\frac{7\pi}{12}\right) = 1200 \sec\left(\frac{7\pi}{12}\right)$

$= \frac{1200}{\cos\left(\frac{7\pi}{12}\right)}$ *or* $1200\left(\frac{1}{\cos\left(\frac{7\pi}{12}\right)}\right)$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

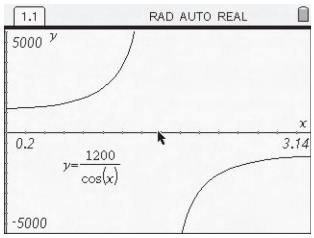
$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$

So $\frac{1}{\cos\left(\frac{7\pi}{12}\right)} = \frac{2\sqrt{2}}{1 - \sqrt{3}}$

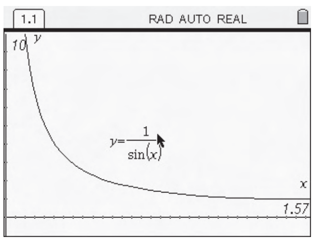
$d\left(\frac{7\pi}{12}\right)$ becomes

$d\left(\frac{7\pi}{12}\right) = 1200\left(\frac{2\sqrt{2}}{1 - \sqrt{3}}\right)$

$= \frac{2400\sqrt{2}}{1 - \sqrt{3}}$

c)


8. **a)** $a(\theta) = \csc \theta$

b)


9. $\tan \theta = \frac{d}{5}$ or $d = 5 \tan \theta$

One complete revolution every 3 seconds means that the period is 3.

$\frac{2\pi}{k} = 3$

So $3k = 2\pi$

$k = \frac{2\pi}{3}$

Therefore, the function $d = 5 \tan \theta$ can be written as a function of time, using the period of rotation

$d(t) = 5 \tan\left(\frac{2\pi}{3}t\right)$

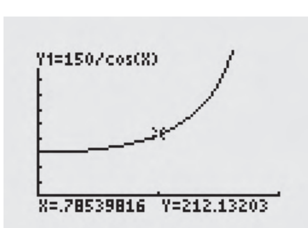
10. **a)** The cotangent function is the reciprocal of the tangent function and $\tan^{-1}$ is the opposite operation of tangent.

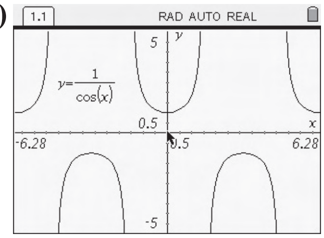
b) $\cot \frac{1}{\sqrt{3}} = 1.5352$, $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

11. No. Answers may vary. Sample answer:

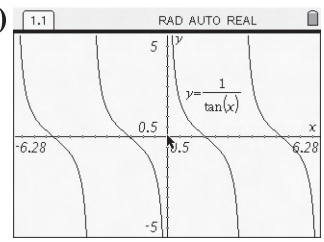
$\sec^2 x = \csc^2\left(x - \frac{\pi}{2}\right)$

12. **a)** $d = 150 \sec x$ **b)** $100\sqrt{3}$ m **c)** 173.2 m

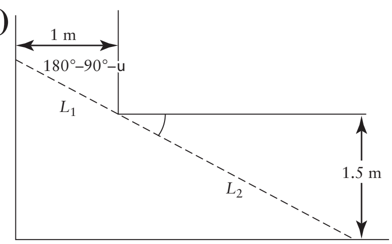
d)


13. **a)**


b) i) vertical stretch by factor of 2
ii) change of period to $\frac{2\pi}{3}$
iii) vertical translation 4 down
iv) phase shift 2 left

14. **a)**


b) i) vertical stretch by factor of 4
ii) change of period to $\frac{\pi}{3}$ **iii)** vertical translation 5 up **iv)** phase shift 2 right

15. **a)**


$L = L_1 + L_2$

$L_1 = \frac{1}{\sin(180° - 90° - \theta)}$

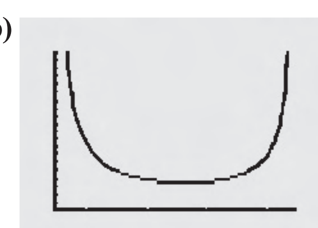
$L_1 = \frac{1}{\sin(90° - \theta)}$ $\qquad L_2 = \frac{1.5}{\sin \theta}$

$L_1 = \frac{1}{\cos \theta}$ $\qquad L_2 = 1.5 \csc \theta$

$L_1 = \sec \theta$

$L = L_1 + L_2$

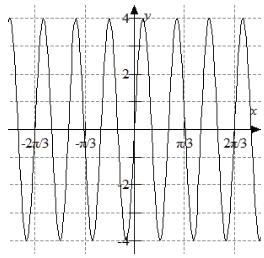
$L = \sec\theta + 1.5 \csc \theta$

b)


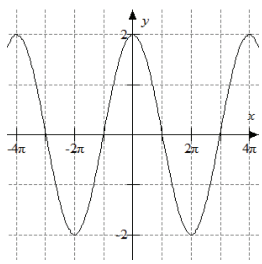
c) 48.86° **d)** 3.5 m

5.3 Sinusoidal Functions of the Form $f(x) = a \sin[k(x - d)] + c$ and $f(x) = \cos[k(x - d)] + c$

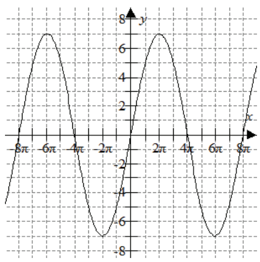
1. a) 4; $\frac{2\pi}{9}$



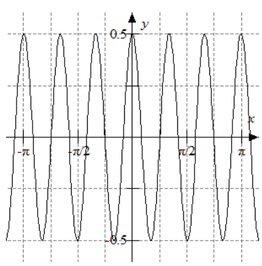
b) 2; 4π



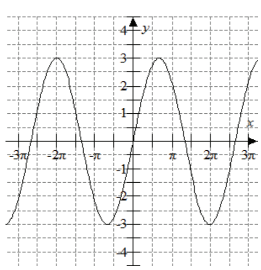
c) 7; 8π



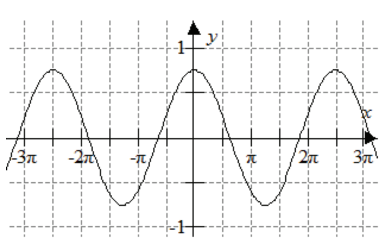
d) $\frac{1}{2}$; $\frac{\pi}{3}$



e) 3; $\frac{8\pi}{3}$



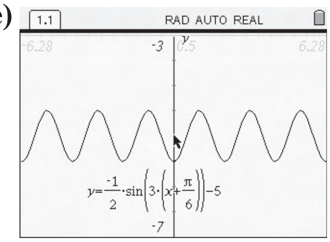
f) $\frac{3}{4}$; $\frac{5\pi}{2}$



2. a) $y = 5 \sin(3x)$ **b)** $y = 2 \cos\left(\frac{1}{2}x\right)$

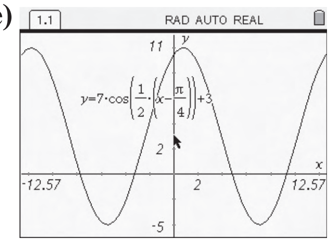
3. a) $\frac{1}{2}$ **b)** $\frac{2\pi}{3}$ **c)** $\frac{\pi}{6}$ left **d)** 5 down

e)



4. a) 7 **b)** 4π **c)** $\frac{\pi}{4}$ right **d)** 3 up

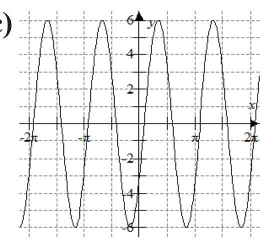
e)



5. a) amplitude 6; period π; phase shift $\frac{\pi}{12}$ right; vertical tra nslation 0

b) $y = 6 \sin\left[2\left(x - \frac{\pi}{12}\right)\right]$

c)

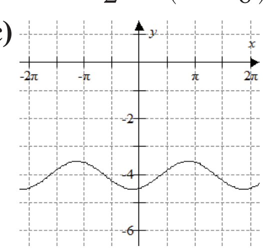


d) same

6. a) amplitude $\frac{1}{2}$; period 2π; phase shift $\frac{\pi}{8}$ left; vertical translation 4 down

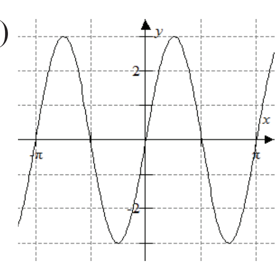
b) $y = -\frac{1}{2}\cos\left(x + \frac{\pi}{8}\right) - 4$

c)

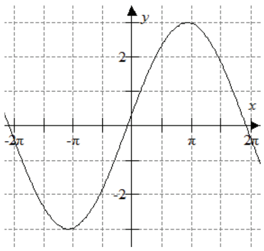


d) same

7. a) $y = 3 \sin(2x)$

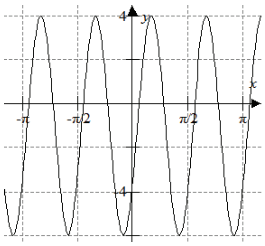


b) $y = 3\sin\left[\frac{1}{2}\left(x + \frac{\pi}{12}\right)\right]$

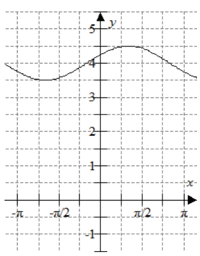


8. a) $y = 5\cos\left[4\left(x - \frac{\pi}{6}\right)\right] - 1$

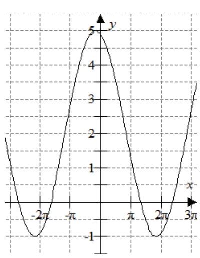
b)



9. a) $y = \frac{1}{2}\cos\left(x - \frac{\pi}{3}\right) + 4$



b) $y = 3\cos\left(\frac{1}{2}\left(x + \frac{\pi}{6}\right)\right) + 2$



10. a) $y = 2\sin\left[2\left(x + \frac{\pi}{2}\right)\right]$

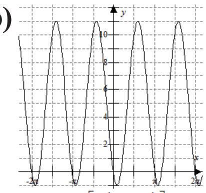
b) $y = 3\sin\left[4\left(x - \frac{\pi}{3}\right)\right]$

11. a) $y = \frac{1}{2}\cos\left[\frac{1}{2}\left(x + \frac{\pi}{6}\right)\right]$

b) $y = 4\cos[2(x - \pi)] - 3$

12. a) $y = 6\sin\left[2\left(x + \frac{2\pi}{3}\right)\right] + 5$

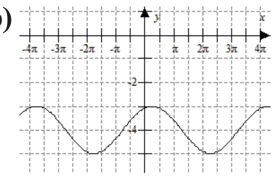
b)



$y = 6\sin\left[2\left(x + \frac{2\pi}{3}\right)\right] + 5$

13. a) $y = \cos\left[\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right] - 4$

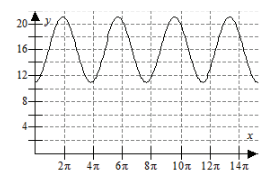
b)



$y = \cos\left[\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right] - 4$

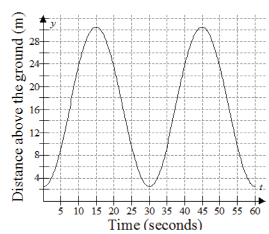
14. a) $y = -5\cos\left(\frac{\pi}{6}x\right) + 16$

b)



$y = -5\cos\left(\frac{\pi}{6}x\right) + 16$

15. a)



b) A period of 30 seconds means

$\frac{2\pi}{k} = 30$

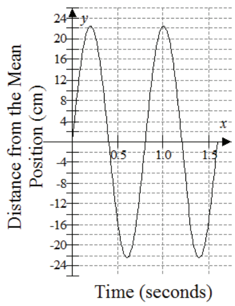
$30k = 2\pi; k = \frac{\pi}{15}$

The amplitude of the function is the radius of the wheel, so amplitude = 14. The centre of the wheel is 14 + 2.5 = 16.5 m off the ground, so $d = 16.5$. As well, if the wheel position starts at the lowest position at $t = 0$, then the cosine function must be reflected in the x-axis to have a minimum value at $t = 0$. All of this together means that

$y = -14\cos\left(\frac{\pi}{15}t\right) + 16.5$

c) If the wheel speeds up, the period of the function gets shorter, so k increases.

16. a)

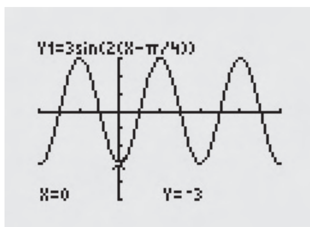


b) $y = 22.5 \sin\left(\frac{5\pi}{2}x\right)$

17. a) amplitude 3, period π, phase shift $\frac{\pi}{4}$ rad to the right, no vertical translation

b) $y = 3 \sin\left[2\left(x - \frac{\pi}{4}\right)\right]$

c)

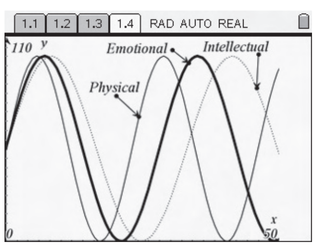


d) Answers may vary. Sample answer:

$y = 3 \cos\left[2\left(x - \frac{\pi}{2}\right)\right]$

18. a) $\frac{2\pi}{23}, \frac{\pi}{14}, \frac{2\pi}{33}$

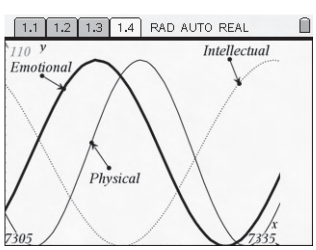
b)



c) yes, at approximately 10 171.5 days

d) at $t = 7305$ days; intellectual: 87.79%; emotional: 18.83%; physical: 18.45%

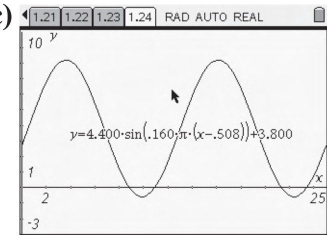
Potential for the next 30 days:



19. a) approximately 4:08 p.m. (16.133 3 hours)

b) $h(t) = 4.4 \sin[0.16\pi(x - 0.508\ 3)] + 3.8$

c)



d) 8.2 m

20. a) The amplitude can be found as follows: $\frac{\max - \min}{2} = \frac{350 - 50}{2} = 150$

The function has a period of 12 months so

$\frac{2\pi}{k} = 12$

$12k = 2\pi;\ k = \frac{\pi}{6}$

If February is the starting month of the data at $t = 1$, then the phase shift is 1 unit to the right. The point (1, 50) is on the function, which means that:

$s(t)$ = a minimum point

$150 \cos\left[\frac{\pi}{6}(t - 1)\right] + d$

Becomes $s(1) = 150 \cos\left[\frac{\pi}{6}(1 - 1)\right] + d$

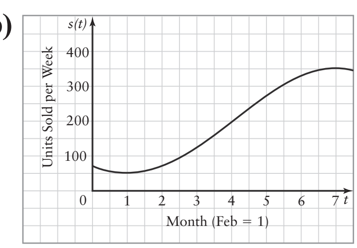
$50 = -150 + d$

$d = 200$

All of this information produces the function

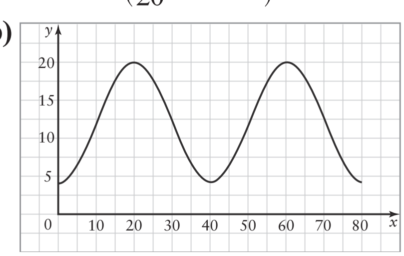
$s(t) = -150 \cos\left[\frac{\pi}{6}(t - 1)\right] + 200.$

b)



21. a) $y = 8 \sin\left(\frac{\pi}{20}(x - 10)\right) + 12$

b)



5.4 Solving Trigonometric Equations

1. **a)** 0.52, 2.62 **b)** 2.69, 3.59 **c)** 0.79, 3.93 **d)** 1.77, 4.51 **e)** 2.09, 5.24 **f)** 3.48, 5.94

2. **a)** $\frac{5\pi}{4}, \frac{7\pi}{4}$ **b)** $\frac{5\pi}{6}, \frac{7\pi}{6}$ **c)** $\frac{\pi}{3}, \frac{4\pi}{3}$ **d)** 2.94, 6.09

3. **a)** 0.78, 2.37, 3.92, 5.51 **b)** 0.84, 2.30, 3.98, 5.44 **c)** 1.48, 1.66, 4.62, 4.80 **d)** 1.10, 2.04, 4.24, 5.18 **e)** 0.49, 2.66, 3.63, 5.80

4. **a)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
   c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

5. **a)** 1.98. 3.84 **b)** 0, 2.47 **c)** 1.26 **d)** 1.02

6. **a)** $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ **b)** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$
   c) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **d)** $\frac{2\pi}{3}, \frac{4\pi}{3}$, 1.23, 5.05
   e) 1.82, 4.46 **f)** no solution **g)** $\frac{\pi}{3}, \frac{5\pi}{3}$, 1.82, 4.46 **h)** $\frac{\pi}{3}, \frac{5\pi}{3}$ **i)** $\frac{\pi}{2}$, 3.31, 6.12

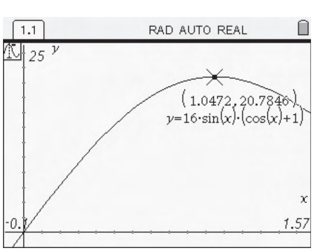
7. **a)** $2\cos^2\theta + \cos\theta - 1 = 0$
   $(\cos\theta + 1)(2\cos\theta - 1 = 0)$
   $\cos\theta + 1 = 0$ or $2\cos\theta - 1 = 0$
   $\cos\theta = -1 \qquad \cos\theta = \frac{1}{2}$
   $\theta = 180° \qquad \theta = 60°$

   We eliminate the solution 180° due to the restriction on the variable given in the question, so the solution is 60°.

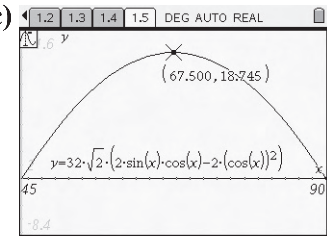
   b) $A = 16\sin 60°(\cos 60° + 1)$
   $= 16\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} + 1\right)$
   $= 16\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{2}\right)$
   $= \frac{48\sqrt{3}}{4}$
   $= 12\sqrt{3} = 20.8$

   Therefore the maximum area is 20.8 cm^2

   c)



8. **a)** 67.5°
   b) 18.75 *m*
   c)



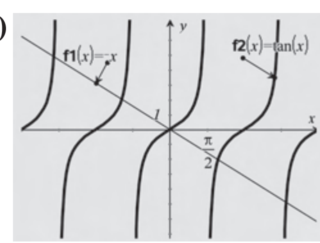
   max R = 18.745 m at θ = 67.5°

9. **a)** 30°, 60° **b)** 123.6 m

10. $-\frac{5\pi}{3}, -\frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

11.
$$\frac{1 + \sec x}{\sec x} = \frac{\sin^2 x}{1 - \cos x}$$
$$\frac{1 + \frac{1}{\cos x}}{\frac{1}{\cos x}} = \frac{1 - \cos^2 x}{1 - \cos x}$$
$$\frac{\frac{\cos x}{\cos x} + \frac{1}{\cos x}}{\frac{1}{\cos x}} = \frac{(1 - \cos x)(1 + \cos x)}{(1 - \cos x)}$$
$$\frac{\frac{\cos x + 1}{\cos x}}{\frac{1}{\cos x}} = 1 + \cos x$$
$$\frac{\cos x + 1}{\cos x} \times \frac{\cos x}{1} = 1 + \cos x$$
$$\cos x + 1 = 1 + \cos x$$
$$0 = 0$$
Since this is always true, all values are solutions to the original expression, as a result the solution is $x \in [-2\pi, 2\pi]$.

12. **a)**



   b) 2.02, 4.91

5.5 Making Connections and Instantaneous Rates of Change

1. **a) i)**
$$\frac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{5\pi}{90}\right) + 15\right]}{10 - 5}$$
$$= \frac{19.104\,241\,72 - 17.083\,778\,13}{5}$$
$$= 0.404$$

ii) $\dfrac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{9\pi}{90}\right) + 15\right]}{10-9}$

$= \dfrac{19.104\,241\,72 - 18.708\,203\,93}{1}$

$= 0.396$

iii) $\dfrac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{9.9\pi}{90}\right) + 15\right]}{10 - 9.9}$

$= \dfrac{19.104\,241\,72 - 19.064\,855\,04}{0.1}$

$= 0.394$

iv) $\dfrac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{9.99\pi}{90}\right) + 15\right]}{10-9.99}$

$= \dfrac{19.104\,241\,72 - 19.100\,305\,29}{0.01}$

$= 0.394$

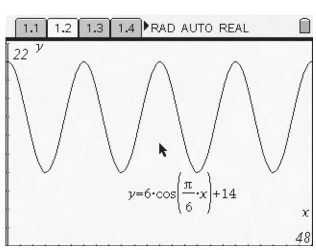
b) As the two points from part a) get closer and closer together at the value of $x = 10$, the behaviour of the secant slopes becomes the value of the tangent slope at $x = 10$. As a result, the tangent slope is approximately 0.394 m/s.

c) The instantaneous rate of change represents the speed of the particle at $t = 10$ *s*.

d) The slope of points on a sinusoidal function that are in phase to each other should have the same slope. Therefore, with this function having a period of 180 seconds $\frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{90}}$ or 180, we would expect the speed to be the same at $t = 194$ s, $t = 374$ s, and so on.

2. a) $d(t) = 6\cos\left(\frac{\pi}{6}t\right) + 14$

b)



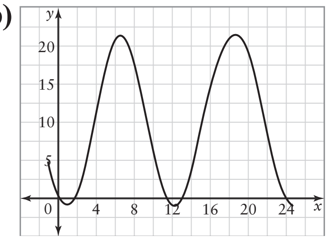
c) i) −0.804 **ii)** −0.409 **iii)** −0.273 **iv)** −0.137 **v)** −0.069 **vi)** −0.014

d) approximately −0.016 **e)** speed by which water depth changes at time $t = 9{:}00$ a.m.

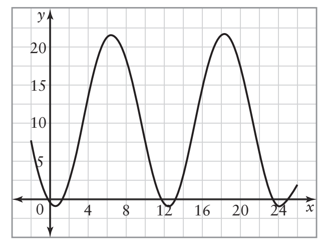
3. a) 120; 40π **b)** $E = 120\sin(40\pi t)$

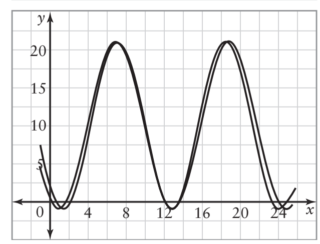
4. a) $y = 12.2\sin\left[\frac{\pi}{6}(x - 4)\right] + 10.9$

b)



c) $y = 11.76\sin(0.55x - 2.35) + 10.66$

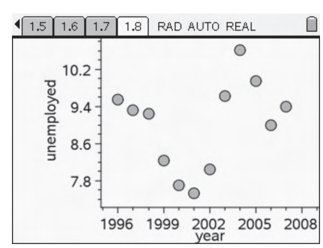




The two graphs are very close to each other.

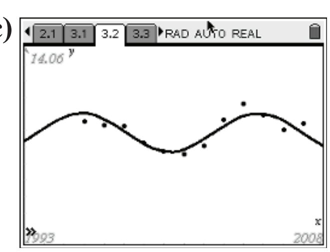
d) Answers may vary. For example: $(x, y) = (4.27, 10.9)$ **e)** 6.49 °C/month **f)** the rate by which average monthly temperature changes at month x

5. a)

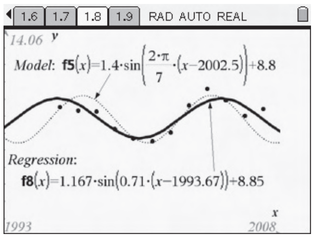


b) $y = -1.543\sin\left[\frac{\pi}{4}(x - 6.5)\right] + 9.071$

c)



does not fit really well

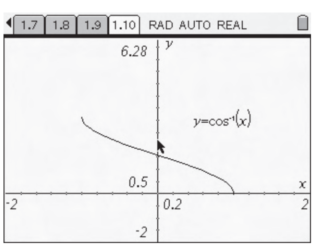
d) 

Answers may vary. For example: The regression equation and the model are pretty close.

e) 1998 **f)** −0.8 **g)** the rate by which the number of unemployed people changes at year x

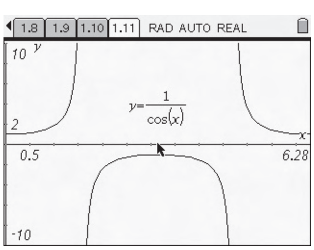
6. a) Answers may vary. For example: At $t = 5$, the weight is at its minimum position, and would have a slope of zero. The speed would be a max at a time of $t = 1.5$ seconds, half way from a min (at $t = 1$) and a max ($t = 2$). **b)** 9.5 cm/s **c)** speed of the weight (going up) at time $t = 1.5$ s

7. a) i) 0.8363 m/s **ii)** 0.7766 m/s **iii)** 0.7639 m/s **iv)** 0.7626 m/s
b) 0.763 m/s
c) No, the graph of the sine function changes its slope continually and would not likely yield the same value at a different value of t.

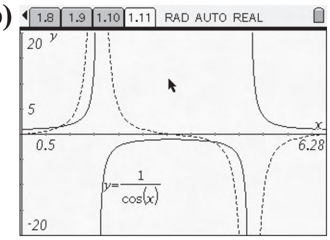
8. a) 

No, fails the vertical line test. Restrict range to $[0, \pi]$.

b) $x = 0$ **c)** -1

9. a) 

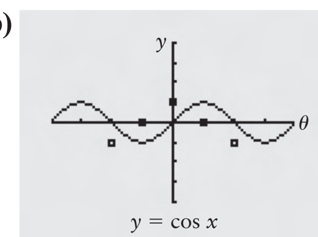
$0, \pi, 2\pi; x \to \frac{\pi}{2}; x \to \frac{3\pi}{2}$

b) 

c) Answers may vary.

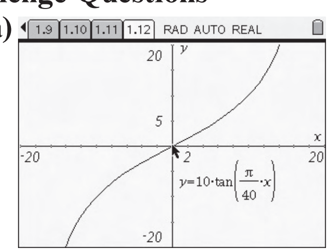
10. a)

Value of θ	Instantaneous Rate of Change
$-\pi$	-1
$\frac{-\pi}{2}$	0
0	1
$\frac{\pi}{2}$	0
π	-1

b) 

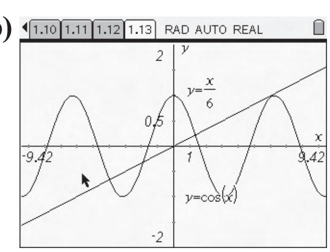
c) The instantaneous rate of change of the sine function is the cosine function.

Challenge Questions

C1. a) 

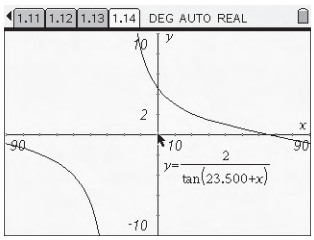
b) 2.4 m **c)** 24.1 m

C2. a) 1.72 **b)** 2.27 **c)** 12.27
d) $d(t) = 1.72 \cos[0.163\pi (t - 0.05)] + 2.27$
e) 3.98 m

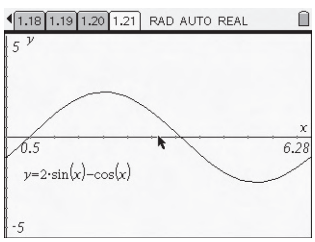
C3. a) 3
b) 

C4. $\frac{\pi}{4}$

C5. a) From the diagram, in the summer:
$\tan\theta = \frac{2}{d}$ so $d = \frac{2}{\tan\theta}$, and since the angle is a function of the latitude, given by $23.5 + l$, we have $d(l) = \frac{2}{\tan(23.5 + l)}$

b)


c) Answers may vary.

C6. a)


b) 2.034 s, 5.176 s; 3.605 s

C7. 0.001335 s **C8.** April, October

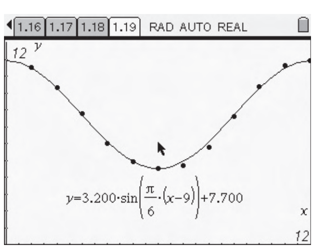
C9. $-\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

C10. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

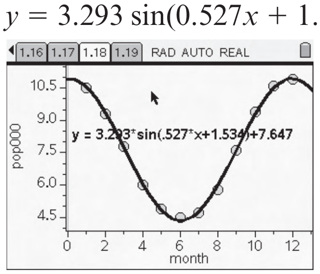
C11. 26 **C12.** $\frac{\sqrt{2}}{2}$

C13. a) 2.6° **b)** 190 km/h

C14. a) $y = 3.2\sin\left[\frac{\pi}{6}(x - 9)\right] + 7.7$

b)


Excellent fit.

c) $y = 3.293\sin(0.527x + 1.534) + 7.647$


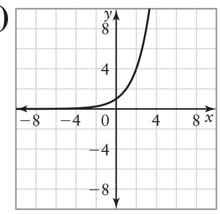
Very close.

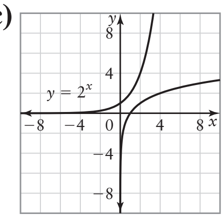
d) months 0, 6, 12; month 9

Chapter 6

6.1 The Exponential Function and Its Inverse

1. a) i) domain $\{x \in \mathbb{R}\}$ and range $\{y > 0, y \in \mathbb{R}\}$,
ii) no x-intercept
iii) y-intercept at (0, 1)
iv) the function is always positive
v) the function is always increasing
vi) $y = 0$

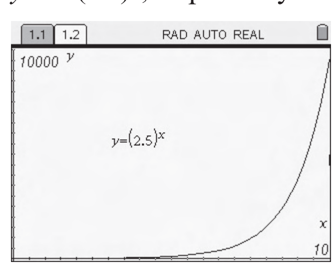
b)


c)


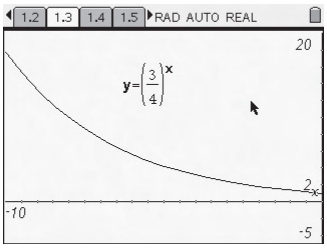
d) i) domain $\{x > 0, x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}\}$
ii) x-intercept at (0, 1)
iii) no y-intercept
iv) the function is positive for $x > 1$ and is negative for $0 < x < 1$
v) the function is always increasing
vi) $x = 0$

2. a) Data set (ii) is exponential. Successive terms have constant ratios. **b)** $y = 5^x$

3. Tables b) and d) represent exponential functions. Functions are $y = 1.5(3)^x$ and $y = 4(0.8)^x$, respectively.

4.


a) i) 3.75 **ii)** 9.38 **iii)** 23.44 **iv)** 58.59
b) i) 5.73 **ii)** 14.32 **iii)** 35.79 **iv)** 89.48
c) positive and increasing

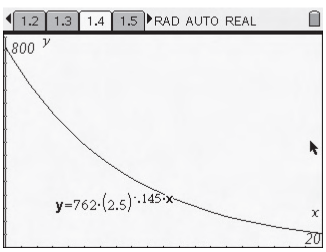
5. 

a) i) -1.05 **ii)** -0.79 **iii)** -0.59 **iv)** -0.44 **v)** -0.33

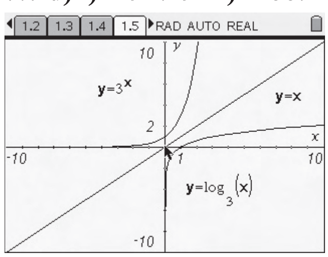
b) i) -0.91 **ii)** -0.68 **iii)** -0.51 **iv)** -0.38 **v)** -0.29

c) negative and increasing

6. **a) i)** 760 **ii)** 711.15 **iii)** 582.65 **iv)** 391.12

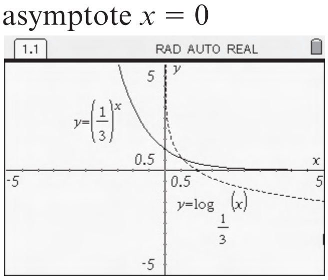
b) 

c) -97.7 **d) i)** -94.49 **ii)** -88.41

7. **a)–c)** 

d) $f(x) = 3^x$ **i)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y > 0\}$
ii) no x-intercept
iii) y-intercept 1
iv) positive for all x in the domain
v) increasing for all x in the domain
vi) horizontal asymptote $y = 0$

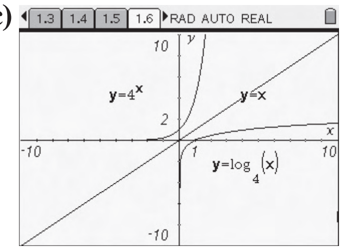
$f^{-1}(x) = \log_3 x$ **i)** $\{x \in \mathbb{R}, x > 0\}, \{y \in \mathbb{R}\}$ **ii)** x-intercept 1 **iii)** no y-intercept **iv)** positive: $x > 1$, negative: $0 < x < 1$ **v)** increasing for all x in the domain **vi)** vertical asymptote $x = 0$

8. **a)–c)** 

i) $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y > 0\}$
ii) no x-intercepts **iii)** y-intercept 1
iv) positive for all x in the domain
v) decreasing for all x in the domain
vi) horizontal asymptote at $y = 0$

d) $g^{-1}(x) = \log_{1/3} x$

i) $\{x \in \mathbb{R}, x > 0\}, \{y \in \mathbb{R}\}$
ii) x-intercept 1 **iii)** no y-intercept
iv) positive: $0 < x < 1$, negative: $x > 1$
v) decreasing for all x in the domain
vi) vertical asymptote at $x = 0$

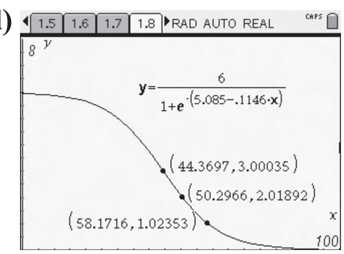
9. **a)–c)** 

d) $y = 4^x$

10. **a)** 21.46 **b)** 0.18

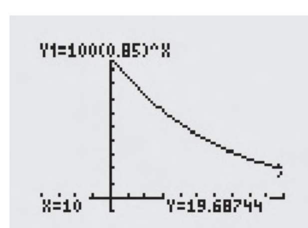
11. **a)** 46.75 watts **b)** 22.09 watts **c)** -0.101, -0.051 **d)** -0.105, -0.049

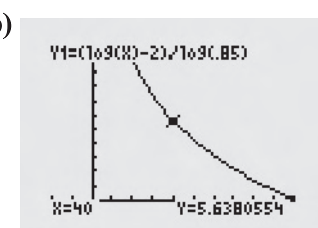
12. **a)** 0.01 **b)** 0.86 **c)** 5.03

d) 

approximately 58.2 F, 50.3 F, 44.4 F

e) -0.021, -0.139

13. **a)** 

b) 

c) 4.3 m **d)** −11.3%/m **e)** −7.21%/m; greater, magnitude of slope is increasing as d increases

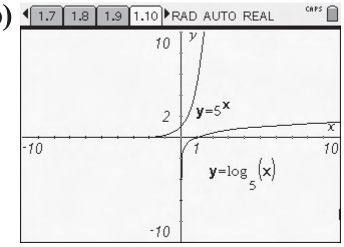
14.

	$f(x) = \left(\frac{1}{2}\right)^x$	Inverse of f
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}, x > 0\}$
Range	$\{y \in \mathbb{R}, y > 0\}$	$\{y \in \mathbb{R}\}$
X-intercept	none	1
Y-intercept	1	none
Intervals for which $f(x)$ is positive	$(-\infty, +\infty)$	(0,1)
Intervals for which $f(x)$ is increasing	none	none
Equation of asymptote	$y = 0$	$x = 0$

6.2 Logarithms

1. **a)** $\log_3 243 = 5$ **b)** $\log_6 \frac{1}{216} = -3$
2. **a)** 2 **b)** −6 **c)** 2 **d)** −2.6
3. **a)** $4^3 = 64$ **b)** $10^y = 30$
4. $x = \log_5 125$ **5.** −3
6. $5^4 = 625$
7. **a)** $\log_3 9 = 2$ **b)** $\log_4 16 = 2$
   c) $\log_6\left(\frac{1}{36}\right) = -2$ **d)** $\log_{\frac{1}{3}}\left(\frac{1}{27}\right) = 3$
   e) $\log_7 y = x$ **f)** $\log_{10} 10000 = 4$
   g) $\log_5\left(\frac{1}{125}\right) = -3$ **h)** $\log_b a = x$
8. **a)** 3 **b)** 5 **c)** −2 **d)** −4 **e)** −4 **f)** 4 **g)** 5 **h)** 7 **i)** 8 **j)** 9
9. **a)** 5 **b)** −3 **c)** −3 **d)** −9 **e)** 2 **f)** −7
10. **a)** $2^3 = 8$ **b)** $3^{-2} = \frac{1}{9}$ **c)** $a^6 = 3$ **d)** $3^x = 2$
   e) $2^x = 6$ **f)** $2^{1.3} = x$ **g)** $\sqrt{2}^x = \pi$ **h)** $5^{7y} = 3x$
11. **a)**

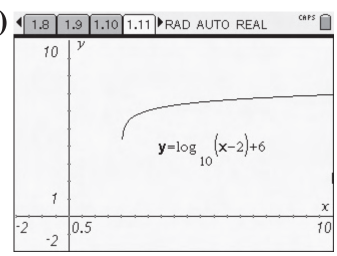
b)



12. a) 2.1 **b)** −0.9 **c)** 1.1 **d)** 3.3 **e)** −1.9 **f)** 3.9
13. a) 6.3 **b)** 12.6 **14. a)** 11.56 **b)** 13.78
15. a) 218.7 km **b)** 76 158 859.21 s (or approximately 21 155 hours)
16. a) 2.36×10^{25} **b)** 10.5 h **c)** 14.9 h
17. a) 4.3 **b)** 10^{-11} mol/L
18. a) Sirius is 23 times more luminous than the Sun. Vega is 53.8 times more luminous than the Sun. The North Star is 5875 times more luminous than the Sun. Deneb is 63 715 times more luminous than the Sun. **b)** Answers may vary.
19. a) 2374 years **b)** 19 000 years **c)** $R = 10^{\frac{A}{19000}}$
d) 99%

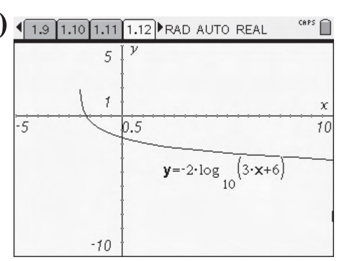
6.3 Transformations of Logarithmic Functions

1. **a)**



b) i) $\{x \in \mathbb{R}, x > 2\}$, $\{y \in \mathbb{R}\}$ **ii)** x-intercept 2.000001 **iii)** no y-intercept **iv)** vertical asymptote $x = 2$

2. **a)**

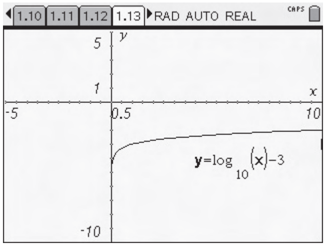


b) i) $\{x \in \mathbb{R}, x > -2\}$, $\{y \in \mathbb{R}\}$
ii) x-intercept $-\frac{5}{3}$ **iii)** y-intercept −1.56
iv) vertical asymptote $x = -2$

3. a) and iv); b) and iii); c) and ii); d) and i)
4. **a)** translation 3 units to the left
   b) translation 2 units to the right and 4 up

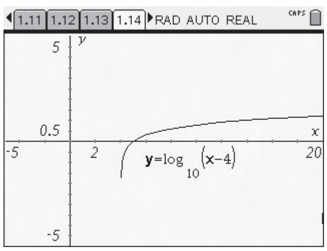
c) translation 3 units to the right and 2 up
d) translation 2 units to the left and 3 up

5. a)



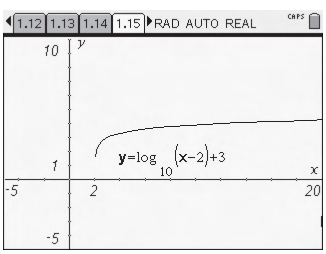
domain $\{x \in \mathbb{R}, x > 0\}$, range $\{y \in \mathbb{R}\}$; no y-intercept; x-intercept 1000; vertical asymptote $x = 0$

b)



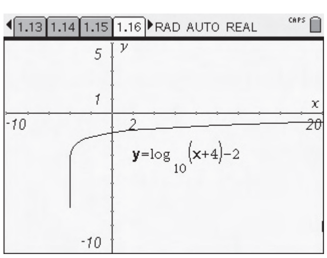
domain $\{x \in \mathbb{R}, x > 4\}$, range $\{y \in \mathbb{R}\}$; no y-intercept; x-intercept 5; vertical asymptote $x = 4$

c)



domain $\{x \in \mathbb{R}, x > 2\}$, range $\{y \in \mathbb{R}\}$; no y-intercept; x-intercept 2.001; vertical asymptote $x = 2$

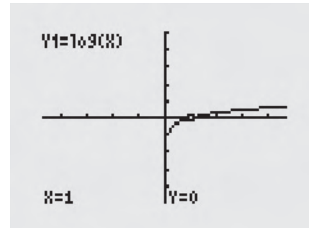
d)



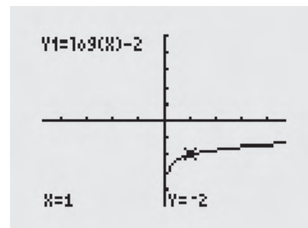
domain $\{x \in \mathbb{R}, x > -4\}$, range $\{y \in \mathbb{R}\}$; y-intercept -1.4; x-intercept 96; vertical asymptote $x = -4$

6. a) $y = 2 \log x$ **b)** $y = \frac{1}{4} \log x$ **c)** $y = 5 \log x$
d) $y = \frac{1}{2} \log x$

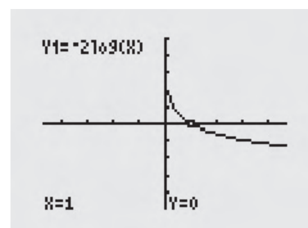
7. a) $\{x \in \mathbb{R}, x > 0\}$, $\{y \in \mathbb{R}\}$



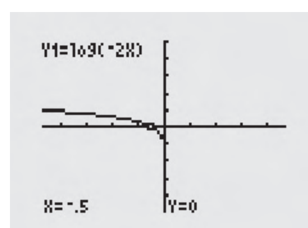
b) $\{x \in \mathbb{R}, x > 0\}$, $\{y \in \mathbb{R}\}$



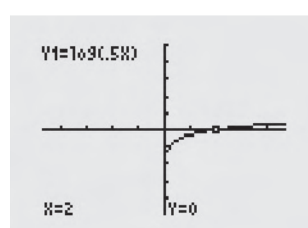
c) $\{x \in \mathbb{R}, x > 0\}$, $\{y \in \mathbb{R}\}$



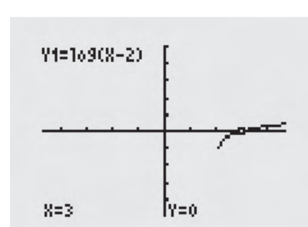
d) $\{x \in \mathbb{R}, x < 0\}$, $\{y \in \mathbb{R}\}$

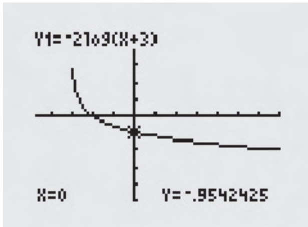


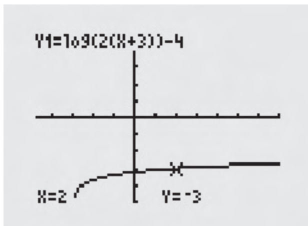
e) $\{x \in \mathbb{R}, x > 0\}$, $\{y \in \mathbb{R}\}$

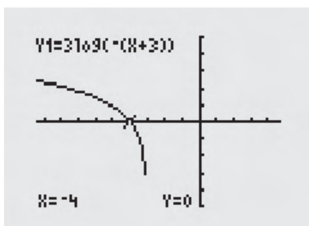


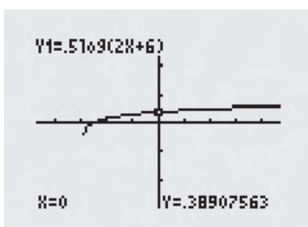
f) $\{x \in \mathbb{R}, x > 2\}$, $\{y \in \mathbb{R}\}$



8. a)


b)


c)


d)


9. a) $y = -\log\left[\frac{1}{2}(x + 5)\right] - 3$

b) $y = -\log\left[\frac{1}{2}x + 5\right] - 3$

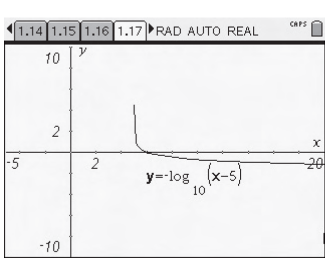
10. $y = 5\log(x + 1)$

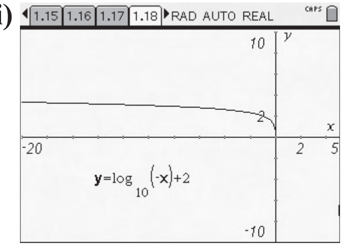
11. a) compress vertically by factor of $\frac{1}{3}$

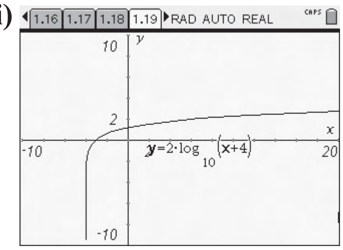
b) stretch horizontally by factor of 4

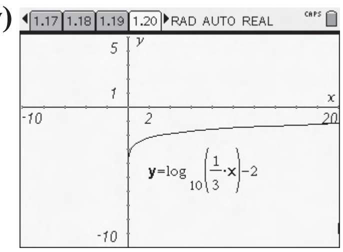
c) compress horizontally by factor of $\frac{1}{2}$

d) stretch horizontally by factor of 3

12. a) i)


ii)


iii)


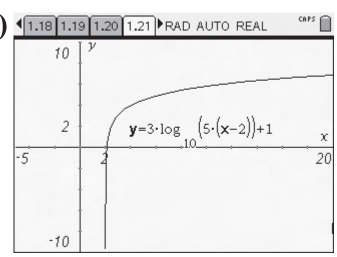
iv)


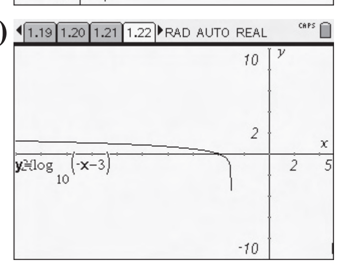
b) i) domain $\{x \in \mathbb{R}, x > 5\}$; range $\{y \in \mathbb{R}\}$; vertical asymptote $x = 5$;

ii) domain $\{x \in \mathbb{R}, x < 0\}$; range $\{y \in \mathbb{R}\}$; vertical asymptote $x = 0$

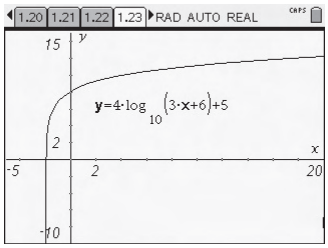
iii) domain $\{x \in \mathbb{R}, x > -4\}$; range $\{y \in \mathbb{R}\}$; vertical asymptote $x = -4$

iv) domain $\{x \in \mathbb{R}, x > 0\}$; range $\{y \in \mathbb{R}\}$; vertical asymptote $x = 0$

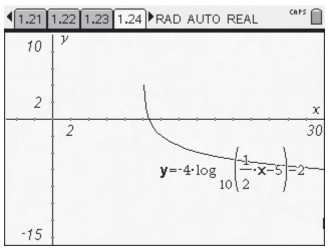
13. a)


b)


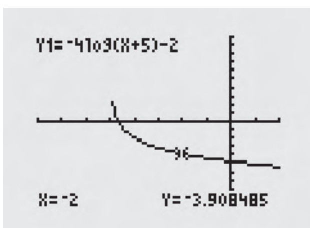
c)



d)

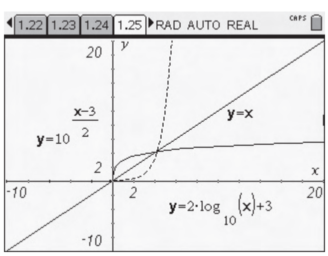


14.



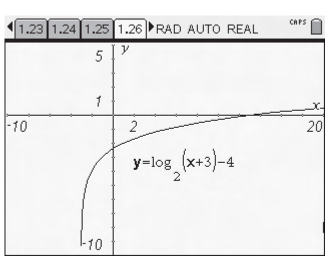
15. 127.155 h

16. a–c)

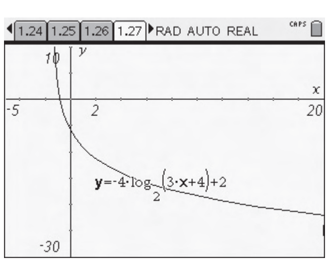


d) i) domain $\{x \in \mathbb{R}\}$ **ii)** range $\{y \in \mathbb{R}, y > 0\}$ **iii)** horizontal asymptote $y = 0$

17. a)

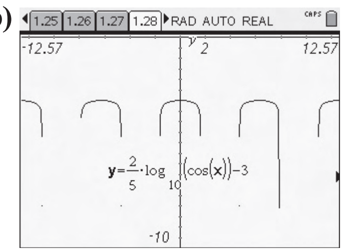


b)



18. a) domain $\left\{x \in ..., \left(-\frac{5\pi}{2}, \frac{3\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(-\frac{3\pi}{2}, \frac{5\pi}{2}\right), ...\right\}$ The function S(x) is defined only when cos x is positive. The cos x function is positive in the intervals: $..., \left(-\frac{5\pi}{2}, \frac{3\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(-\frac{3\pi}{2}, \frac{5\pi}{2}\right), ...$ range $\{y \in \mathbb{R}, y < -3\}$, due to the fact that cos x will always be positive and less than one, and the log of this will be negative, causing the value of the function to always be less than -3.

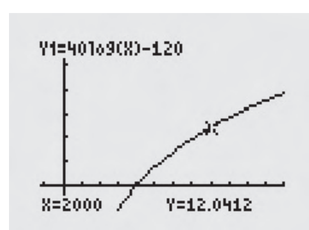
b)



c) Answers may vary. Sample answer: No, because pattern is too cyclic, most likely created by a naturally occurring, repeating event such as the rotation of a pulsar.

19. a) i) 12 years **ii)** 19 years

b) approximately \$1585



c) $A \geq 1000$ because $n \geq 0$

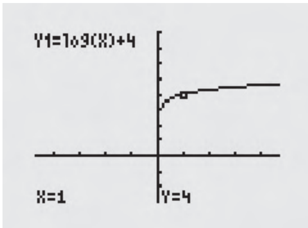
20. a) compress horizontally by a factor of $\frac{1}{2}$, translate left 3 units, reflect in the x-axis

b) reflect in the y-axis, stretch vertically by a factor of 3, translate up 4 units

21. The function $y = -\log(-x)$ can be obtained by reflecting $y = \log x$ in the y-axis and reflecting in the x-axis, so that a point (a, b) is transformed to (−a, −b). Reflection of the function $y = \log x$ in the line $y = x$ has the exact effect, so the two functions are inverses.

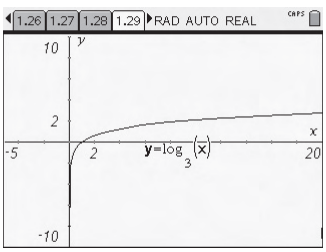
22. a) Each pair of graphs is identical.

b) translated up 4 units



6.4 Power Laws of Logarithms

1. 9
2. 1.76
3. 1.75
4.



5. a) i) \$1500 **ii)** \$1881.60 **iii)** \$2360.28
b) i) 6.1 years **ii)** 12.2 years
6. a) 15 **b)** 3.37 **c)** -6 **d)** -1
7. a) $\frac{2}{3}$ **b)** 4 **c)** 18 **d)** -9
8. a) 1.29 **b)** 0.43 **c)** 2 **d)** 1.60
9. a) i) \$1500 **ii)** \$1694.59 **iii)** \$1914.42
b) i) 14.2 years **ii)** 28.4 years
10. a) 2.613 **b)** 3.210 **c)** 0.732 **d)** -0.594
e) -2.727 **f)** -6.530
11. a) $\log_4 9$ **b)** $\log_5 12$ **c)** $\log_{\frac{5}{7}} \frac{1}{3}$
d) $\log_{(x-5)}(x+3)$
12. a) 4.292 **b)** 100 **c)** 1.250 **d)** 4.096
13. a) Initially, the investment is made at $t = 0$, so

$$A(t) = 1200(1.06)^{2t}$$
$$A(0) = 1200(1.06)^0$$
$$= 1200$$

Therefore, the initial investment is \$1200

b) To triple means that the investment needs to grow to \$3600, so

$$A(t) = 1200(1.06)^{2t}$$
$$3600 = 1200(1.06)^0$$
$$3 = (1.06)^{2t}$$
$$\ell n\, 3 = 2\, t\, \ell n(1.06)$$
$$\frac{\ell n\, 3}{2\, \ell n(1.06)} = t$$
$$t = 9.4$$

Therefore, it will take approximately 9.4 years for the investment to grow to \$3600.

14. a) 21 **b)** 21 **c)** Answers may vary. For example: Power law of logarithms is easier.
15. a) $t = \log_{0.5}\left(\frac{d}{250}\right)$ becomes

$$t = \log_{0.5}\left(\frac{1}{250}\right)$$
$$= 7.97$$

Therefore the docking procedures can begin approximately 8 hours after the controlled breaking starts.

b) Due to the answer for a), we have a domain $\{d \in \mathbb{R}, 1 \le x \le 250\}$, range $\{t \in \mathbb{R}, 0 \le t \le 8\}$
16. a) $A(t) = I(1.04)^t$, where I is the initial investment
b) i) 28 years **ii)** 35.3 years
c) i) 26.5% **ii)** 60.1%
17. a) $\frac{\log_3 1024}{\log_3 4}$ **b)** $\frac{\log_3 37}{\log_3 10}$
18. a) $A(t) = I(1.0275)^{2t}$, where I is the initial investment
b) i) 12.8 years **ii)** 20.2 years

6.5 Making Connections: Logarithmic Scales in the Physical Science

1. a) -2 **b)** 8.3 **c)** 0.001 mol/L
d) tomato juice has 10 times higher concentration of hydronium ions than soft drinking water
2. approximately 4 times
3. approximately 8 times
4. a) maximum stereo output is 100 times as intense as a shout **b)** 100 dB
5. a) 15 849 times **b)** 4.7
6. a) 3 **b)** 3.1 **c)** 5 **d)** 7.4
7. a) 10^{-15} mol/L **b)** 10^{-4} mol/L
c) 5.012×10^{-12} mol/L
d) 6.31×10^{-7} mol/L
8. Substitute $I = 1.15 \times 10^{-10}$ in $L = 10 \log$

$$L = 10 \log$$
$$= 10 \log 115$$
$$= 20.6 \text{ dB}$$

9. 39.5 dB **10.** 100 times
11. a) 10^7 times **b)** 10^{15} times
12. 3981 times **13.** 6

14. **a)** 5 011 872 times **b)** 2.5×10^{-6} times

15. 3162 times

16. **a)** The factor of the increase is given by $\log\left(\frac{L}{L_O}\right)$, so when the brightness increased by 200 times, the factor can be found by evaluating log(200) which is 2.301. Therefore, the magnitude increased by a factor of 2.3

b) The factor of the increase is given by log $\left(\frac{L}{L_O}\right)$, so when the brightness increased by 1000 times, the factor can be found by evaluating log(1000) which is 3 Therefore, the magnitude increased by a factor of 3

17. **a)** 10^{-6} W/m^2 **b)** 0.5 W/m^2

Chapter 6 Challenge Questions

C1. 3.35 mg, 0.45 mg

C2. $N = P(1 - e^{-0.15d})$ becomes

$$N = 1000(1 - e^{-0.15(3)})$$
$$= 1000(0.36237)$$
$$= 362.37$$

Therefore, 362 students would have heard the rumour after 3 days.

C3. a) 38.2 W **b)** 0.84 W **c)** −0.0827 W/day, −0.0149 W/day **d)** −0.0856 W/day, −0.0019 W/day

C4. 6.8 **C5. a)** 50 119 **b)** star B

C6. a)
$$h = \frac{-\ln\left(\frac{P}{760}\right)}{0.145}$$
$$= \frac{-\ln\left(\frac{271.5}{760}\right)}{0.145}$$
$$= \frac{1.029\,356\,294}{0.145}$$
$$= 7.099 \text{ km}$$

b)
$$8.85 = \frac{-\ln\left(\frac{P}{760}\right)}{0.145}$$
$$8.85 \times 0.145 = -\ln\left(\frac{P}{760}\right)$$
$$-1.283\,25 = \ln\left(\frac{P}{760}\right)$$
$$e^{-1.283\,25} = \frac{P}{760}$$
$$P = 760 \times e^{-1.283\,25}$$
$$P = 210.6 \text{ or approx 211 mm of mercury}$$

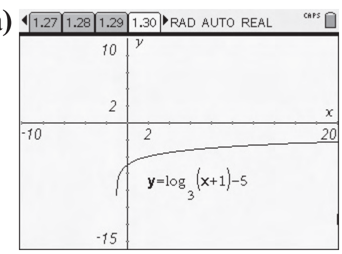
c)
$$2.1 = \frac{-\ln\left(\frac{P}{760}\right)}{0.145}$$
$$2.1 \times 0.145 = -\ln\left(\frac{P}{760}\right)$$
$$-0.3045 = \ln\left(\frac{P}{760}\right)$$
$$e^{-0.304\,5} = \frac{P}{760}$$
$$P = 760 \times e^{-0304\,5}$$
$$P = 560.49 \text{ or approx 560.5 mm or mercury}$$

The pressure difference can be found by subtraction: (760 − 560.5) = 199.5 mm of mercury

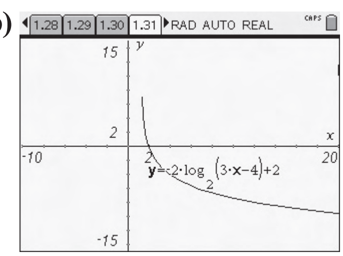
C7. a) 3541.5 m **b)** 83 mm or mercury

C8. a) 9.8 years **b)** 4.3 years

C9. a)



b)



C10. a) $0 < a < 1$

b) point of intersection (x, y) is always in quadrant 1, where $0 < x < 1$ and $0 < y < 1$

C11. a) $A(t) = I(1.002\,307\,692)^{52t}$

b) i) 9.2 years **ii)** 11.6 years

c) i) 604% **ii)** 1099%

C12. 4.04 years **C13.** approximately 10%

C14. 1 584 893 times **C15.** 4

C16. 7 **C17.** 6.31×10^{-5} mol/L

C18. 20 times **C19.** $x(t) = \frac{\left(ab2^{\frac{t}{c}} - 1\right)}{a2^{\frac{t}{c}} - b}$

Chapter 7

7.1 Equivalent Forms of Exponential Equations

1. **a)** 2^{16} **b)** 2^{14} **c)** 2^{-9} **d)** $2^{\log_2 11}$

2. **a)** 3^8 **b)** 3^{21} **c)** 3^{-6} **d)** $3^{\log_3 \frac{1}{5}}$

3. a) 4^9 b) 2^{-10} c) $4^{\frac{3\log 5}{\log 4}}$

4. a) $3^{\frac{4}{3}}$ b) 3^1

5. a) $x = -10$ b) $t = 28$ c) $x = \frac{17}{9}$

6. a) 27^3 b) $9^{\frac{9}{2}}$ c) $3^9 = 27^3 = 9^{\frac{9}{2}} = 19\,683$

7. a) 4^{10} b) $4^{\frac{3}{5}}$ c) $4^{\frac{49}{10}}$ d) 4^3

8. a) $x = \frac{13}{3}$ b) $w = 4$ c) $x = 12$ d) $w = \frac{5}{6}$

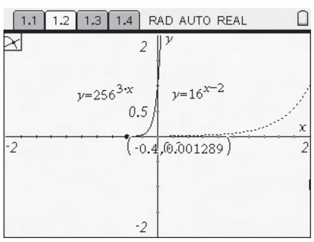
9. a) $x = -3$ b) $x = -\frac{23}{14}$ c) $y = -\frac{8}{7}$ d) $k = \frac{39}{8}$

10. a) $x = 7$ b) $x = 7$ c) Answers may vary.

11. a) $x = -\frac{7}{2}$ b) $k = -\frac{3}{2}$

12. a) $x = \frac{1}{\log 3}$ b–c) Answers may vary.

13. a) $x = -\frac{2}{5}$ b) $x = -\frac{2}{5}$

c) 

$x = -\frac{2}{5} = -0.4$ d) Answers may vary.

14. a) $x = -\frac{1}{3}$ b) $x = -\frac{1}{3}$

15. a) i) $x = \log 5$ ii) $x = \log 3$ iii) $x = \log 7$
b) $x = \log b$

16. a) $x = \frac{11}{15}$ b) $x = \frac{11}{15}$ c) $x = \frac{11}{15}$
d) Answers may vary.

17. a) i) no solution ii) $x < 5$ b) Answers may vary. For example: Graph both side of the inequality. Identify where the graphs intersect. Identify the interval(s) where the inequality is true.

7.2 Techniques for Solving Exponential Equations

1. Graph 1: $y = 200\left(\frac{1}{3}\right)^{\frac{x}{3}}$; Graph 4: $y = 70\left(\frac{1}{3}\right)^{x}$
Graph 3: $y = 70\left(\frac{1}{3}\right)^{\frac{x}{3}}$; Graph 2: $y = 200\left(\frac{1}{3}\right)^{x}$;

2. a) 6.19 b) 6.18 c) 2.71 d) 8.78 e) −4.10
f) −1.46 g) −12.01 h) −5

3. a) approximately 76.7 mg b) approximately 6.2 min c) no

4. a) $x = \frac{2\log 4}{\log 4 - \log 3}$ b) $x = \frac{3\log 7}{\log 5 - \log 7}$
c) $x = \frac{2\log 4 + 4\log 9}{\log 9 - \log 4}$
d) $x = \frac{2\log 5 - 5\log 8}{3\log 8 - \log 5}$

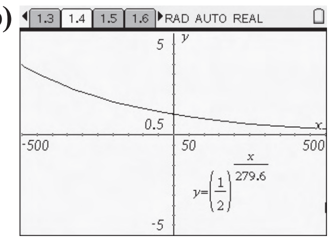
5. a) $x = 9.638$ b) $x = -17.350$
c) $x = 14.257$ d) $x = -1.551$

6. a) $z^2 + z - 12 = 0, a = 1, b = 1, c = -12$
b) $z = 3$ or $z = -4$ c) $z = -4$

7. a) $z^2 - 2z - 3 = 0, a = 1, b = -2, c = -3$
b) $z = 3$ or $z = -1$ c) $z = -1$ 8. $\frac{3}{4}$ h

9. a) 10.7 min b) 35.5 min

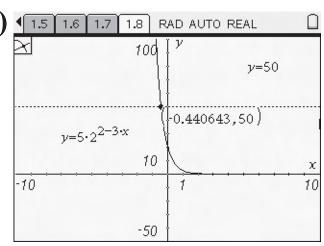
10. a) approximately 279.6 days

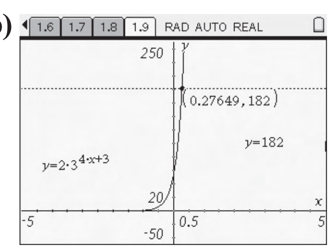
b) 

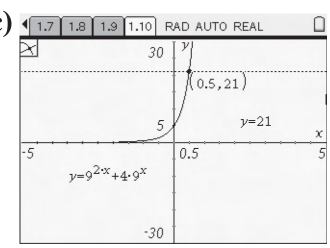
c) i) graph would decrease faster
ii) graph would decrease slower
d) i) graph would stretch vertically
ii) graph would compress vertically

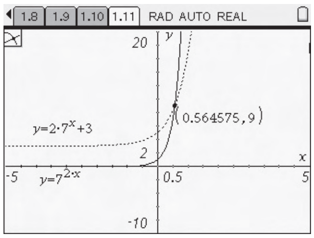
11. $x = \log_3 6$

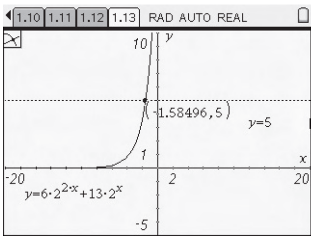
12. a) $x \cong -0.44$ b) $x \cong 0.28$ c) $x = 0.5$
d) $x \cong 0.56$ e) $x \cong -1.58$ f) $x = -1$

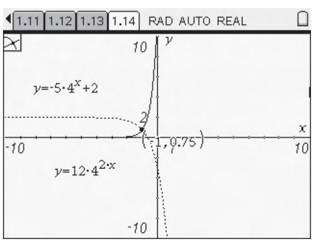
13. a) 

b) 

c) 

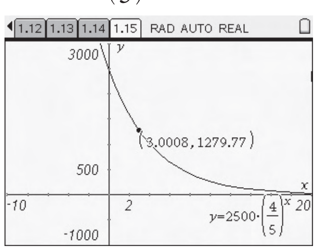
d) 

e) 

f) 

14. a) 3 years **b)** 8.2 years

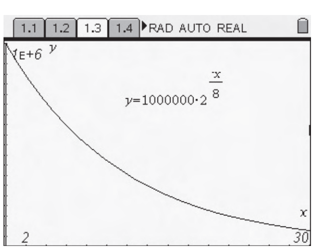
15. a) $V(t) = 2500\left(\frac{4}{5}\right)^t$

b)–c) 

d) 1280

16. 24.6 days

17. a) i) 8 days **ii)** approximately 26.6 days

b) 

c) i) half-life of 8 will be replaced by a larger number

ii) graph decreases slower

18. a) $P(t) = 5000(2)^t$ **b)** $P(t) = 3500(3)^t$

c) $t \cong 1.9$ days; approximately 2 days after each of the movie trailers was posted, they reached the same number of people.

19. a) 65 h **b)** 49 h **20. a)** 1.18 m **b)** 29

21. $P(t) = 4(2)^{\frac{t \log 1.4}{24 \log 2}}$ **22. a)** 16.5% **b)** 360.5%

23. a) $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{15}}$ **b)** $\{t \in R, t \geq 0\}$, $\{A \in R, 0 < A \leq A_0\}$

7.3 Product and Quotient Laws of Logarithms

1. a) log 40 **b)** log 8 **c)** $\log_2 24$ **d)** 1

2. a) 1.602 **b)** 0.903 **c)** 4.585 **d)** 1

3. a) $\log(6xyz)$, $x > 0, y > 0, z > 0$

b) $\log_2\left(\frac{3xy}{z^5}\right)$, $x > 0, y > 0, z > 0$

c) $\log\left(\frac{a^3y^4}{z^2}\right)$, $a > 0, y > 0, z > 0$

d) $\log\left(\frac{9x^2y^3}{2\sqrt{w}}\right)$, $x > 0, y > 0, w > 0$

4. a) 8 **b)** 3 **c)** 6.5 **d)** 3

5. a) 8 **b)** 4 **c)** 6 **d)** 2

6. a) 1 **b)** 2 **c)** 1 **d)** 4

7. a) 1 **b)** 2 **8.** $\log_3 5(x-2)^2$

9. $\log a + 4 \log b - \frac{1}{4} \log c$

10. a) $\log \frac{x+2}{3}$, $x > 6$

b) $\log(x^2 + 3x + 9)$, $x > 3$

11. a) $\log_5 x + \log_5 y$, $x > 0, y > 0$

b) $\log_8 a - \log_8 b$, $a > 0, b > 0$

c) $3 + \log_2 5$

d) $1 + \log_6 20$

12. a) $-\frac{3}{2} \log x$, $x > 0$ **b)** $\frac{38}{3} \log x$, $x > 0$

c) $\frac{29}{6} \log x$, $x > 0$ **d)** $\frac{11}{2} \log x$, $x > 0$

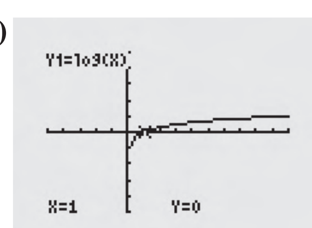
13. a) $\log(x-3)$, $x > 3$ **b)** $\log(x-4)$, $x > 7$

c) $\log\left(\frac{x+9}{3}\right)$, $x > 4$ **d)** $\log\left(\frac{2x+3}{x-2}\right)$, $x < -2, x > 2$

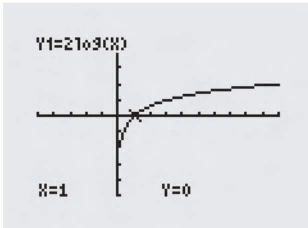
14. a) $4 \log m$, $m > 0$ **b)** $\log p$, $p > 0$

c) $\log \frac{x^2 - 5x - 6}{x - 3}$, $x > 6$

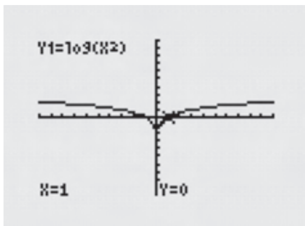
d) $\log(3x-2)$, $x > \frac{3}{2}$

15. a) 

b)



c)



d) same for $x > 0$; power law

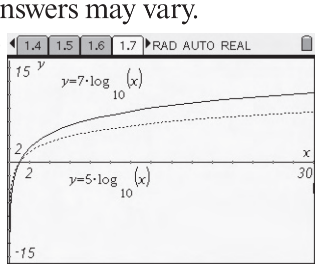
16. a) $y = \frac{(x-3)^2}{x}$, $\{x \in \mathbb{R}, x > 3\}$

b) $y = \frac{x}{1000 - x}$, $\{x \in \mathbb{R}, x > 0, x \neq 1000\}$

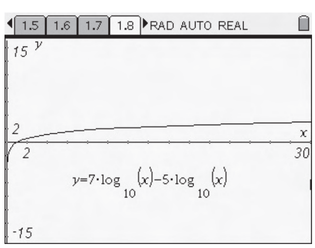
c) $y = \frac{x^2}{(x+5)^2}$, $\{x \in \mathbb{R}, x > 0\}$

17. Answers may vary.

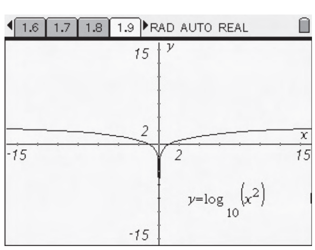
18. a)



b)



c)



d) $r(x)$ and $s(x)$ are the same for $x > 0$. This illustrates power law of logarithms.

19. a) approximately 8.7 years

b) i) Firm B **ii)** Firm B

c) Answers may vary. Sample answer: Other issues such as promotions, health insurance, vacation, maternity, travel, commuting time, etc., may influence Ifra's decision.

20. a) i) approximately −0.42

ii) approximately −0.67

b) E is negative; the cell is expending energy.

21. a) $A_v = 20\log\left(\frac{V_o}{V_i}\right)$ **b)** approximately 4.24

22. Answers may vary.

7.4 Techniques for Solving Logarithmic Equations

1. a) $x = 15$ **b)** $x = 135$ **c)** $x = 927$
d) $x = 6$ **e)** $x = 106$ **f)** $n = \frac{100}{3}$

2. a) $x = 7$ **b)** $x = \frac{76}{3}$ **c)** $x = 44$
d) $x = -25$ **e)** $t = 6$ **f)** $n = -3, n = 5$

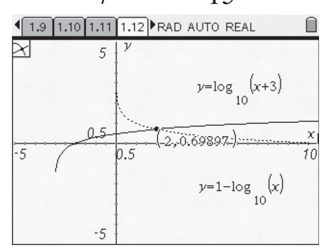
3. a) $x = 5$ **b)** $x = \pm 3$ **c)** no solution
d) no solution **e)** $x = 5$ **f)** $x = 6$

4. $x = 13$

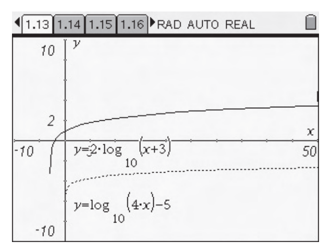
5. a) $x = 3 \pm 3\sqrt{2}$ **b)** $x = \frac{-21 \pm \sqrt{40\,441}}{2}$

6. a) $x = 2$ **b)** $x = \frac{6}{7}$ **c)** $x = \frac{7}{15}$

7. a) $x = 2$



b) no solution

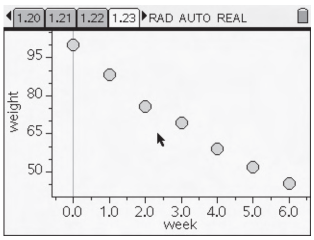


8. a) approximately 9139.6 m **b)** approximately 5539.0 m **c)** Answers may vary. For example: The difference in height will be calculated as follows:
$h = 18\,400\log\frac{P_0(original)}{P_0\,(new)}$, where
P_0 (*original*) is the original air pressure at ground level and P_0 (*new*) is the changed air pressure at ground level.

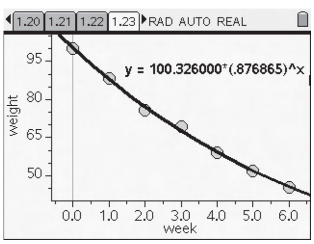
9. **a)** 3229.5 m **b)** 7.6 mm of mercury
c) 61.49 mm of mercury
10. $x = 16$ **11.** $x \cong 2.6$

7.5 Making Connections: Mathematical Modeling with Exponential and Logarithmic Functions

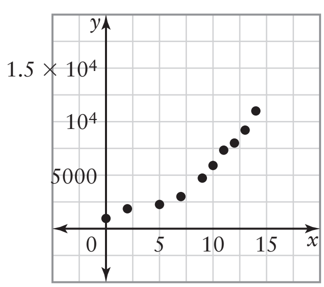
1. **a)**



b) $A(t) = 100e^{-0.13t}$
c) approximately −3.1 weeks (3.1 weeks ago) **d)** approximately 0.15 gm
e) $A(t) = 100.326(0.876\ 865)^t$



2. **a)**



b) From the graphing calculator:
$A = 1035(1.185\ 23)^t$, with $t = 0$ at 1983.
To convert this to an exponential function in base e:

$$(1.18523)^t = e^{kt}$$
$$\ln(1.18523)^t = e^{kt}$$
$$t\ln(1.18523) = kt$$
$$k = \ln(1.18523)$$
$$k = 0.170$$

Therefore $A = 1035e^{0.170t}$

c)

$$A = 1035e^{0.170t}$$
$$15\ 000 = 1035e^{0.170t}$$
$$\frac{15\ 000}{1035} = e^{0.170t}$$
$$\ln\left(\frac{15\ 000}{1035}\right) = 0.170t$$
$$t = \frac{\ln\left(\frac{15\ 000}{1035}\right)}{0.170}$$
$$t = 15.7 \text{ or approx } 16$$

Therefore, the population of home-schooled students reached 15 000 in approximately 1999 (1983 + 16).

d)

$$A = 1035e^{0.170t}$$
$$A = 1035e^{0.170(37)}$$
$$= 1035e^{6.29}$$
$$= 1035(539.15)$$
$$= 558\ 023.7$$

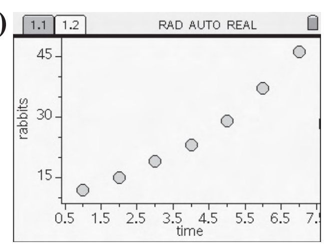
Therefore, in 2020, there will be 558 024 home-schooled students in Canada.

e) verified using a graphing calculator

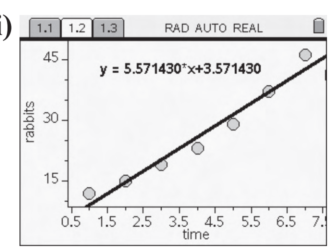
3. **a)** $A(t) = 2500(1.025)^{2t}$
b) approximately \$ 3200.21
c) approximately 14 years

4. **a)** $A(t) = 2500(1.025)^{2t} - 75$
b) translation 75 units down

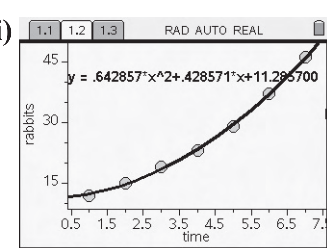
5. **a)**

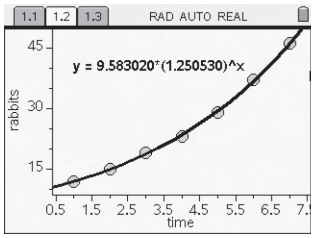


b) **i)**



ii)



iii) 

c) i) $y = 5.571\ 43x + 3.571\ 43$

ii) $y = 0.642\ 857x^2 + 0.428\ 571x + 11.285\ 7$

iii) $y = 9.583\ 02(1.250\ 53)^x$

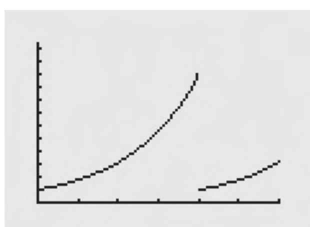
d) Exponential model is preferred because it best expresses steady growth for several months before the collection of data.

e) approximately 10.5 years

f) Answers may vary. For example: Population of predators, food supply, health issues, etc., may affect this trend of rabbit population.

6. a) $P = 200(1.122)^t$

b) $P = 200(1.122)^{t-20}$

c) 

7. a) approximately 17.2 min

b) approximately 18.7 °C; approximately 20.8 °C

8. a) 10^9 **b)** 100 dB

Chapter 7 Challenge Questions

C1. a) 6^3 **b)** 6^{-2} **c)** $6^{\frac{7}{5}}$ **C2. a)** $x = 2$

b) $x = -23$

C3. a) approximately 297.3 mg

b) approximately 185.8 days

C4. a) $x = \dfrac{2 \log 4}{\log 4 - \log 7}$

b) $x = \dfrac{7 \log 3 + 3 \log 2}{\log 2 - \log 3}$

C5. a) $b \neq 0$ **b)** $b > 0$ **c)** $b < 0$

C6. a) no solution **b)** $x = 1$

c) no solution **d)** $x = \log_7 2$

C7. a) approximately 722.5

b) approximately 124.6

C8. a) 5 **b)** 3 **c)** 2 **d)** 2

C9. a) $\log_8 21$ **b)** $\log_a\left(\dfrac{bd^c}{s^r}\right)$, $a > 0, a \neq 1, b > 0, d > 0, s > 0$

C10. a) $2 \log_4 a + \log_4 b - \dfrac{1}{2}\log_4 c$, $a > 0, b > 0, c > 0$

b) $\dfrac{1}{3}\log_9 y + \dfrac{1}{3}\log_9(y + 1)$, $y > 0$

C11. a) $\log\left(\dfrac{2}{x - 5}\right)$, $x > 5$

b) $\log\left(\dfrac{x + 2}{x - 5}\right)$, $x < -2, x > 5$

C12. a) $x = -22$ **b)** $x = \dfrac{9}{2}$ **C13.** $x = 3$

C14. a) 10 hours **b)** approximately 46.7%

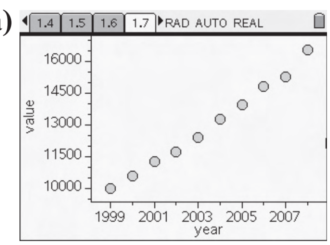
C15. Answers may vary.

C16. Answers may vary.

C17. a) $x = 4096$

b) $x = 0$, $x = \dfrac{\log_5\left(\frac{3}{2}\right)}{2} \cong 0.13$

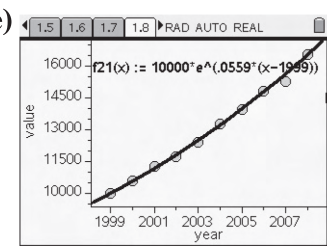
$x = \dfrac{\log_5 2}{2} \cong 0.22$

C18. a) 

b) $A(t) = 10\ 000e^{0.055\ 9(t-1999)}$

c) $A \cong \$\ 32\ 345.80$

d) $t \cong 2015.4$ (approximately first half of year 2015)

e) 

C19. a) $A(t) = 1500(1.044)^{2t}$

b) approximately \$ 4216.03

c) approximately 12.8 years

d) i) translation of 75 up

ii) vertical stretch by factor of 2

C20. a) approximately 8.5 days **b)** 45 items

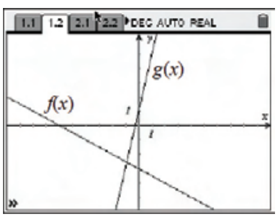
Chapter 8

8.1 Sum and Difference of Functions

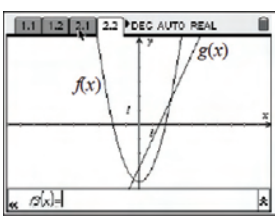
1. a) i) $y = 7x - 4$ **ii)** $y = 5x + 4$
iii) $y = -5x - 4$
b) i) $y = -5x + 10$ **ii)** $y = -x + 4$
iii) $y = x - 4$
c) i) $y = x^2 + 2$ **ii)** $y = x^2 - 6$
iii) $y = -x^2 + 6$
d) i) $y = 2x^2 + 8x - 9$ **ii)** $y = 2x^2 - 2x - 5$
iii) $y = -2x^2 + 2x + 5$

2. a) $h(x) = 8x + 2, h(-2) = -14$
b) $h(x) = 4x + 12, h(4) = 28$
c) $h(x) = -4x - 12, h(-3) = 0$

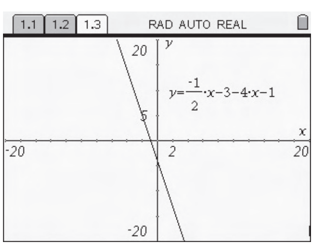
3. a) $h(x) = 2x^2 + 5x - 4, h(0) = -4$
b) $h(x) = 2x^2 - 5x + 10, h(-5) = 85$
c) $h(x) = -2x^2 + 5x - 10, h(6) = -52$

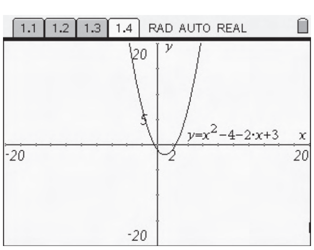
4. a)


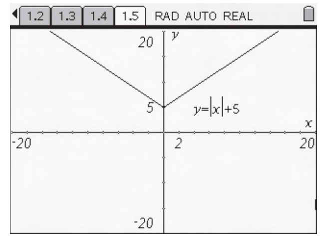
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

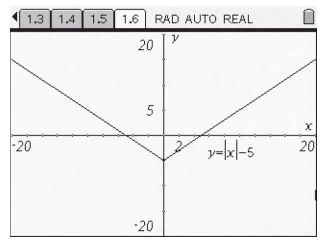
b)


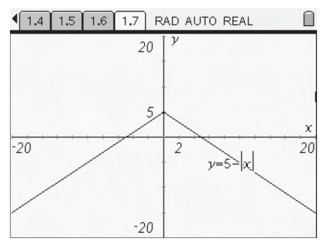
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} \mid y \geq -8\}$

5. a)


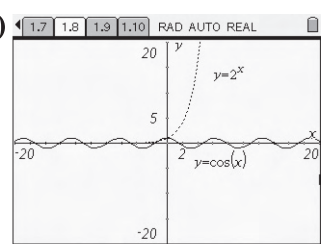
b)


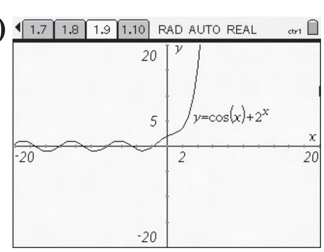
6. $f(x) = |x|, g(x) = 5$
a) i) $y = f(x) + g(x) = |x| + 5$


ii) $y = f(x) - g(x) = |x| - 5$


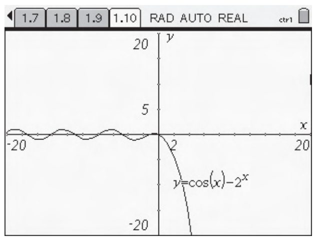
iii) $y = g(x) - f(x) = 5 - |x|$


b) i) Apply translation of 5 units up.
ii) Apply translation of 5 units down.
iii) Apply reflection in the x-axis first and then apply translation of 5 units up.
c) i) The domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R} \mid y \geq 5\}$. **ii)** The domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R} \mid y \geq -5\}$. **iii)** The domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R} \mid y \leq -5\}$.

7. a)


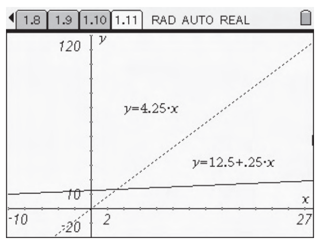
b)


c)



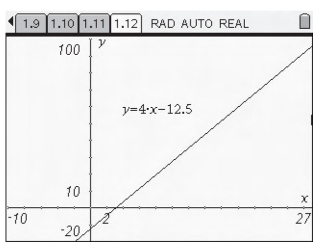
8. a) $C(p) = 12.50 + 0.25p$, $R(p) = 4.25p$

b)



c) Break-even point (3.125, 13.281 25) is where the cost equals the revenue and the courier starts to make profit.

d) $P(p) = 4p - 12.50$



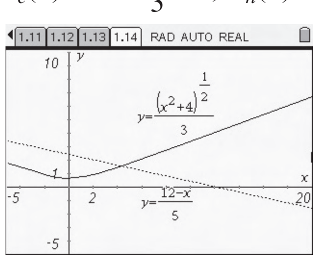
e) $C(p)$: $\{p \in \mathbb{R} \mid 0 \leq p \leq 27\}$, $\{C \in \mathbb{R} \mid 12.50 \leq C \leq 19.25\}$
$R(p)$: $\{p \in \mathbb{R} \mid 0 \leq p \leq 27\}$, $\{R \in \mathbb{R} \mid 0 \leq \mathbb{R} \leq 114.75\}$
$P(p)$: $\{p \in \mathbb{R} \mid 0 \leq p \leq 27\}$, $\{P \in \mathbb{R} \mid -12.50 \leq P \leq 95.50\}$

f) \$95.50

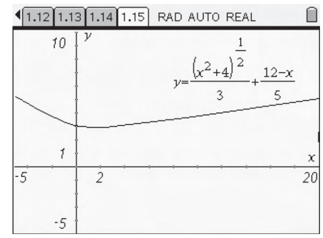
9. a) i) reduce fixed cost ii) reduce variable cost

b) Answers may vary. For example: If more than 23 (rounded up from 22.5) packages are delivered per day, then reducing variable cost is the better option because the daily profit is higher.

10. a) $T_c(x) = \dfrac{(x^2+4)^{\frac{1}{2}}}{3}$, $T_h(x) = \dfrac{12-x}{5}$

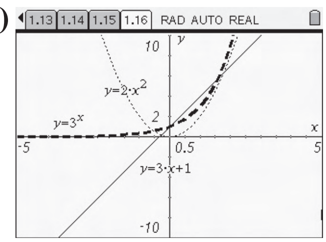


b) $T_c(x) + T_h(x) = \dfrac{(x^2+4)^{\frac{1}{2}}}{3} + \dfrac{12-x}{5}$

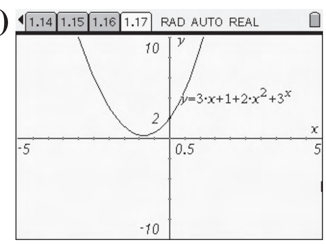


c) The domain is $\{x \in R \mid x \geq 0\}$ and the range is $\{T \in R \mid y > 2.9\}$.

11. a)



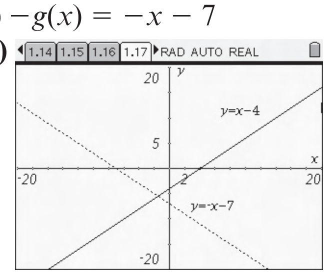
b)



c) $h(x) = 3^x$ d) As x increases, the rate of change of $f(x) = 3x + 1$ is positive and constant. As x increases, the rate of change of $g(x) = 2x^2$ is positive and increasing. As x increases, the rate of change of $h(x) = 3^x$ is positive and increasing faster than $g(x)$, which influences the rate of change of the combined function $T(x)$.

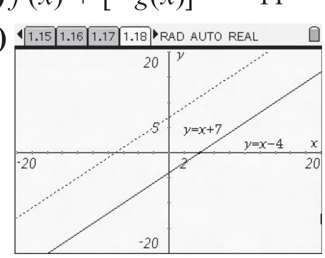
12. a) $-g(x) = -x - 7$

b)



c) $f(x) + [-g(x)] = -11$

d)



e) $f(x) - g(x) = -11$, same as in part c)

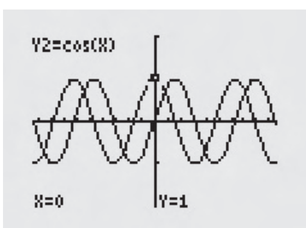
13. a) Let x represent the number of units sold: $C(x) = 175x + 750\,000$ **b)** $S(x) = 599x$ **c)** $P(x) = 424x - 750\,000$

14. $f(x) = 2x^2 + 7x + 8$ and $g(x) = 3x^2 - 5x + 3$

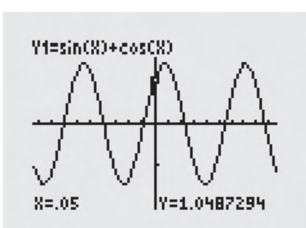
15. Answers may vary. Sample answer: If the domains of the two functions are not the same, a value of either $f(x)$ or of $g(x)$ will not have a corresponding value from the other function to combine with in the addition or subtraction. As a result, the two functions could not be combined.

16. Answers may vary. Sample answer: The degree of the addition of the functions $f(x)$ and $g(x)$ will be determined by the highest degree of the two individual functions. This highest degree will determine the type of function for $(f + g)(x)$.

17. a)

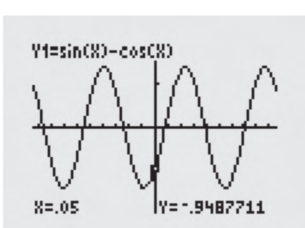


b)



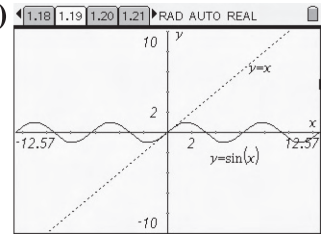
c) $y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$

d)



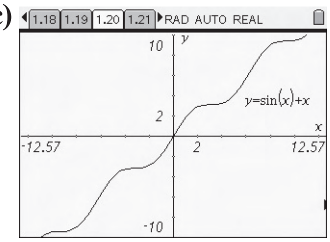
e) $y = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

18. a)

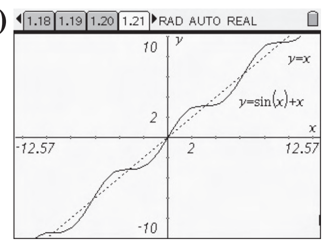


b) periodic (like the sine function), but follows the slope of the line $g(x) = x$

c)



d)



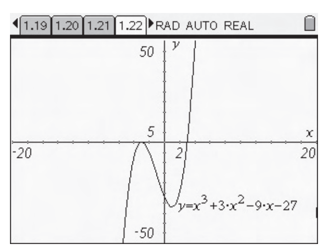
infinitely many intersection points

e) ..., -3π, -2π, $-\pi$, 0, π, 2π, 3π, ...; $g(x)$ intersects $h(x)$ at the zeros of $f(x)$

f) The curvature is up because $g(x)$ has positive slope.

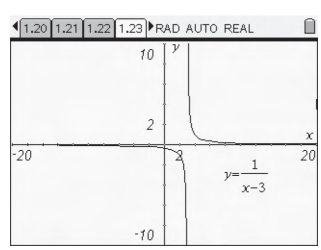
8.2 Products and Quotients of Functions

1. a) $y = x^3 + 3x^2 - 9x - 27$



$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

b) $y = \dfrac{1}{x - 3}$

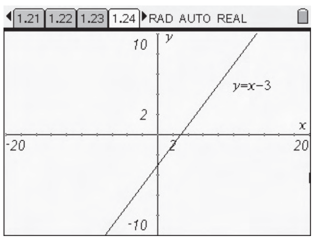


$\{x \in \mathbb{R} \mid x \neq 3, x \neq -3\}$,

$\left\{y \mid y \in \mathbb{R} \mid y \neq -\frac{1}{6}\right\}$; hole at $\left(-3, -\frac{1}{6}\right)$;

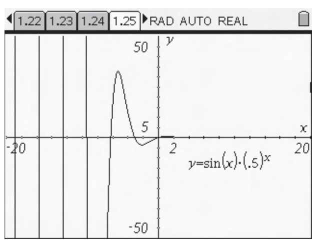
horizontal asymptote: $y = 0$ vertical asymptote: $x = 3$

c) $y = x - 3$



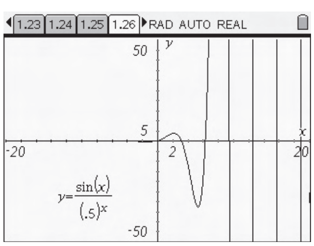
$\{x \in \mathbb{R} \mid x \neq -3\}$, $\{y \in \mathbb{R} \mid y \neq -6\}$; hole at $(-3, -6)$

2. a) $y = 0.5^x \sin x$



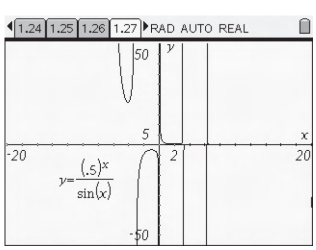
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

b) $y = \dfrac{\sin x}{0.5^x}$



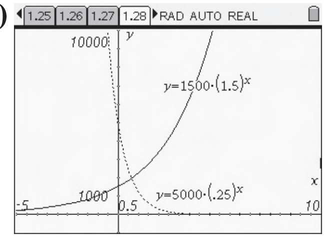
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$;

c) $y = \dfrac{0.5^x}{\sin x}$



$\{x \in \mathbb{R} \mid x \neq k\pi, k = 0, \pm 1, \pm 2, ...\}$, $\{y \in \mathbb{R} \mid y \neq 0\}$; vertical asymptotes: $x = k\pi, k = 0, \pm 1, \pm 2, ...$

3. a)

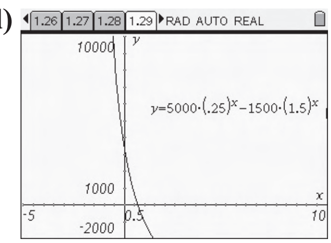


exponential functions

b) $P(t)$: $\{t \in \mathbb{R} \mid t \geq 0\}$, $\{P \in \mathbb{R} \mid P \geq 1500\}$; $F(t)$: $\{t \in \mathbb{R} \mid t \geq 0\}$, $\{F - R \mid 0 \leq F \leq 5000\}$

c) point of intersection: (0.67, 1969.77); the crisis point is when the food supply from surrounding farms is not enough for the increasing population of the town.

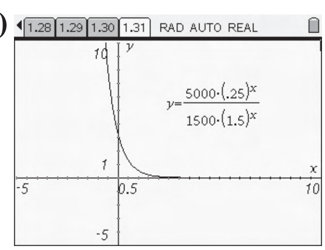
d)



It shows how much time the town has before the food supply from surrounding farms is not enough.

e) The t-intercept is approximately 0.67 and it represents the crisis point. **f)** The model for $P(t)$ is not valid for t values greater than the crisis point, because the surrounding farms are not the only source of food; the town can bring food from other locations.

4. a)



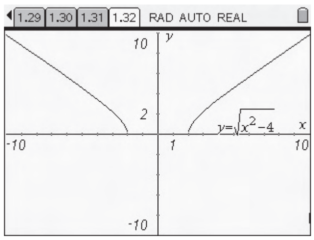
This function represents food available from surrounding farms per person.The function is an exponential function.

b) crisis point coordinates: (0.67, 1); at $t = 0.67$ years, the ratio of $\dfrac{F(t)}{P(t)}$ is 1, which means there is just enough food for each person in town from the surrounding farms.

c) Before the crisis point, the town had enough food supply from surrounding farms. After the crisis point, the food supply from surrounding farms is not enough, which means they would have to bring food from other locations.

5. $f(-x) = \sqrt{x^2 - 4}$, $g(x) = \cos x$

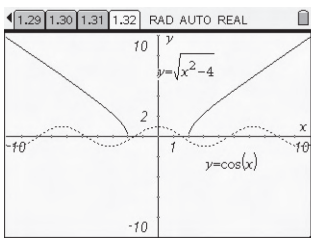
a)



$f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4} = f(x)$

The function is even.

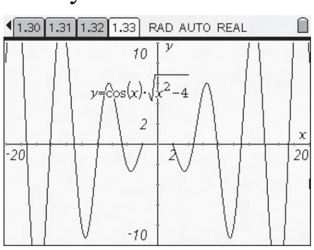
b)



$g(-x) = \cos(-x) = \cos x$

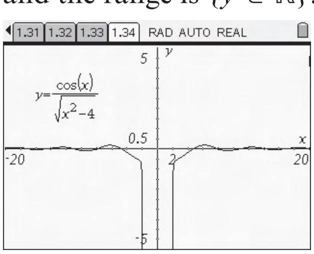
The function is even.

c) The function is periodic (period 2π). As $x \to \pm\infty$, the amplitude increases. It is symmetric about the y-axis.



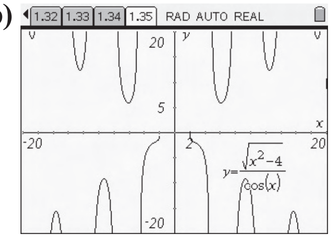
d) The domain is $\{x \in \mathbb{R} \mid x \geq 2, x \leq -2\}$ and the range is $\{y \in \mathbb{R}\}$.

6. a)



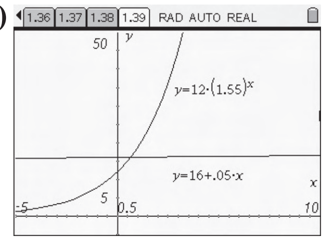
even $\{x \in \mathbb{R} \mid x > 2, x < -2\}$, $\{y \in \mathbb{R} \mid y < 0.170\,623\}$

b)



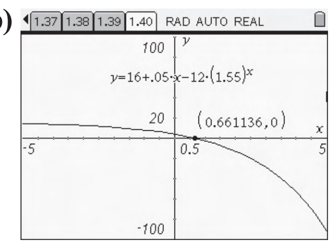
even $\{x \in \mathbb{R} \mid x > 2, x < -2, x \neq \frac{k\pi}{2},$ $k = \pm1, \pm3, \pm5, \pm7, \ldots\}$, $\{y \mid y \in \mathbb{R}, y \pm 5\}$

7. a)



Both functions increase over time; $P(t)$ has a slope that is positive and increasing and $F(t)$ has a slope that is positive and constant. $P(t)$ is increasing at faster rate than $F(t)$.

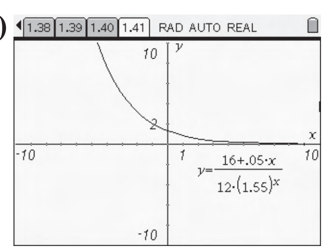
b)



The function is decreasing. Between $t = 0$ and $t \cong 0.66$, there is a food surplus. After $t \cong 0.66$, there is food shortage.

c) (0, 4); the maximum food surplus, 4, occurred at year $t = 0$

8. a)



The function is decreasing.

b) $t = 0$; same result as in question 7. b) Analyzing the difference function (as in question 7. b) and analyzing the quotient function are two different methods that provide the same answers.

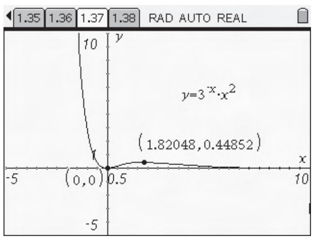
c) When $\frac{F(t)}{P(t)} > 1$, there is food surplus;

$\frac{F(t)}{P(t)} > 1$ occurs at the interval (0, 0.67)

When $\frac{F(t)}{P(t)} < 1$, there is food shortage;

$\frac{F(t)}{P(t)} < 1$ occurs at the interval (0.67, ∞)

9. $f(x) = 3^{-x}$, $g(x) = x^2$, $y = f(x)$
$g(x) = (3^{-x})x^2$

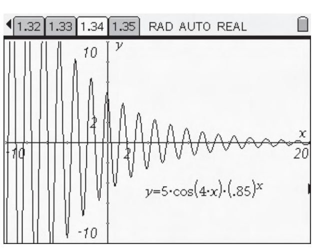


Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R} \mid y \geq 0\}$; x-intercept: 0; y-intercept: 0; Horizontal asymptote: $y = 0$; End behaviour: As $x \to +\infty$, $y \to 0$ from above; as $x \to -\infty$, $y \to +\infty$; Minimum: (0, 0); Local maximum: (1.82, 0.45)

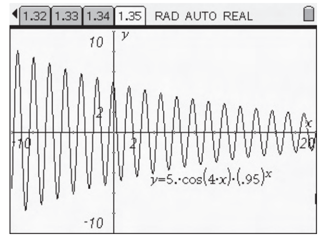
	$(-\infty, 0)$	$x = 0$	(0, 1.85)	$x = 1.85$	$(1.85, +\infty)$
Sign of Function	+	0	+	+	+
Sign of Slope	−	0	+	0	−

10. a) i) $y = x^3 - 7x^2 + 15x - 9$; **ii)** $y = \frac{1}{x - 1}$
b) $\{x \in \mathbb{R} \mid x \neq 1, x \neq 3\}$,
$\left\{y \in \mathbb{R} \mid y \neq 0, y \neq \frac{1}{2}\right\}$

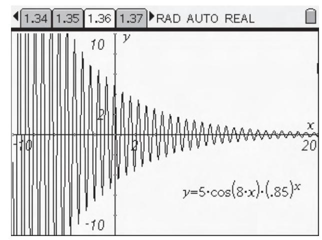
11. a) 5 m
b) Original function:



i) Air resistance reduced:



ii) Swing lengthened:



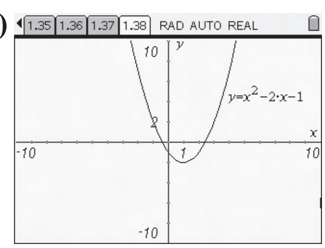
12. a)–c) Answers may vary. **d)** For functions, odd × odd = even, odd × even = odd, and even × even = even. This is exactly the same as the multiplication of numbers.

13. $f(x) = 3x^2 + 4x - 1$
$g(x) = 2x + 4$
or
$\{f(x) = 3x^2 + \frac{7}{2}x - 1$
$g(x) = \frac{1}{2}x^2 + 2x + 4$

8.3 Composite Functions

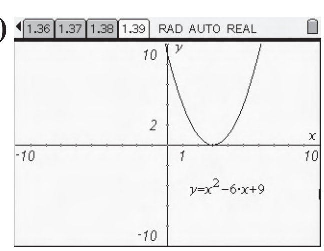
1. a) $0.6t^2 + 20t + 160$ **b)** 486.4
2. a) $y = x^2 - 2x - 1$ **b)** $y = x^2 - 6x + 9$
c) $y = x - 4$ **d)** $y = x^4 - 4x^3 + 4x^2$
e) $y = x$

3. a)

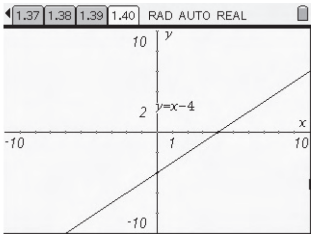


$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid y \geq -2\}$

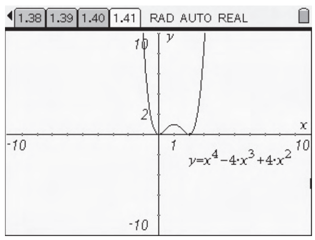
b)



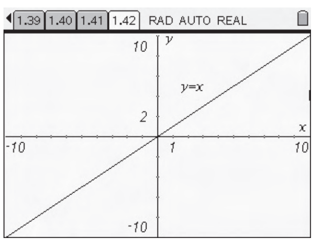
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid y \geq 0\}$

c) 

$\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

d) 

$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} \mid y \geq 0\}$

e) 

$\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

5. $f(x) = x^2 - 3x + 5,\ g(x) = \dfrac{1}{x - 2}$

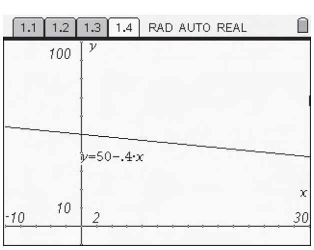
a) $f(1) = (1)^2 - 3(1) + 5 = 1 - 3 + 5 = 3$

$g(f(1)) = g(3) = \dfrac{1}{3 - 2} = \dfrac{1}{1} = 1$

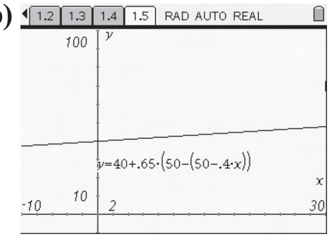
b) $g(4) = \dfrac{1}{4 - 2} = \dfrac{1}{2}$

$f(g(4)) = f\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2 - 3\left(\dfrac{1}{2}\right) + 5 =$

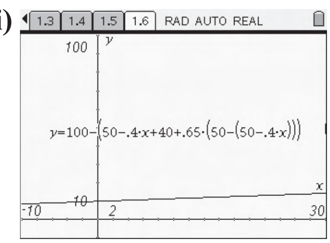
$\dfrac{1}{4} - \dfrac{3}{2} + 5 = \dfrac{1 - 6 + 20}{4} = \dfrac{15}{4}$

6. a) 

Popularity is decreasing
i) 50% **ii)** −0.4; popularity is decreasing at 0.4 % per day **iii)** There may be more than two candidates. At the beginning of the campaign Candidate A has 50% popularity; Candidate B has 40% popularity. The rest 10% could be popularity of other candidate(s) or could be undecided voters.

b) 

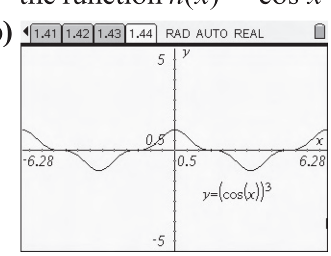
Popularity is increasing
i) 40% **ii)** 0.26; popularity is increasing at 0.26 % per day **iii)** The answer depends on how long the campaign is and how many undecided voters there are.

c) i) 

The graph represents the popularity of the rest of the candidate(s). Popularity is increasing **ii)** If the campaign lasts less than about 15 days, candidate A will win. If the campaign lasts more than about 15 days, candidate B will win. **iii)** The results will be the same as in part c). The last two candidates have no chance of winning.

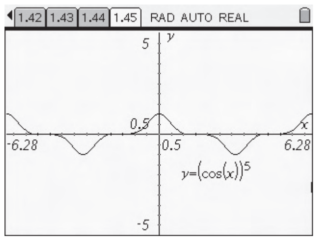
7. a) $\pm\sqrt[4]{x}$ **b)** x **c)** $\pm\sqrt[4]{x}$ **d)** The answer in part c) is an inverse of the answer in part b) **e)** 2; 4; −2. Notice that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

8. a) periodic function with amplitude of 1 and the curve changes its concavity from up to down or down to up at the zeros of the function $h(x) = \cos x$

b) 

c) The function is periodic; the period is 2π, same as $h(x) = \cos x$ **d)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid -1 \le y \le 1\}$

9. a) periodic function with amplitude of 1; the curve changes its concavity from up to down or down to up at the zeros of the function $h(x) = \cos x$

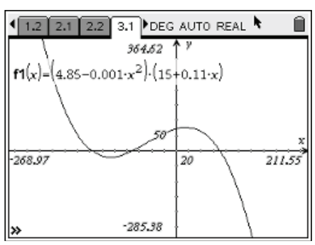


The function is periodic; the period is 2π, same as $h(x) = \cos x$; $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid -1 \le y \le 1\}$

b) same domain, range, period, amplitude, zeros; difference in the shape at the inflection points

10. a) $V(t) = -0.00011x^3 - 0.015t^2 + 0.5335x + 72.75$

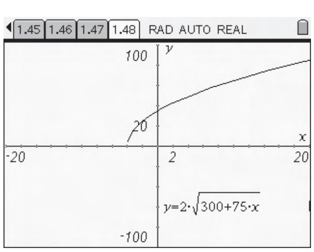
b)



c) 15 s

11. $N(t) = 300 + 75t$, $W(N) = 2\sqrt{N}$

a) $W(t) = W(N(t)) = W(300 + 75t) = 2\sqrt{300 + 75t}$ **b)** t is the time in years since 2007. $t = 0$ represents the year 2007 and therefore the domain of the new function is $\{t \in \mathbb{R} \mid t \ge 0\}$. At $t = 0$, the size of the company's workforce is $W(0) = 2\sqrt{300 + 75(0)} = 2\sqrt{300} = 20\sqrt{3} = 34.6$, which is rounded to 35. Therefore the range of the new function is $\{W \in \mathbb{R} \mid W \ge 20\sqrt{3}\}$.



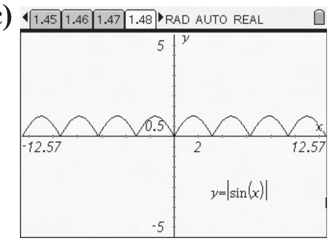
12. a) $C(h) = 2375 + 47.5h$

b) approximately 4.7 hours

13. a) i) $f(g(x)) = 2(4x - 7)^3$ **ii)** $h(g(x)) = \frac{-1}{4x - 7}$ **iii)** $g^{-1}(h(x)) = \frac{-1}{4x} + \frac{7}{4}$ **b)** 2

14. a) $\{x \in \mathbb{R}\}$ **b)** periodic function; looks like the sine function but every negative section of the sine function is reflected in the x-axis (scalloped shape)

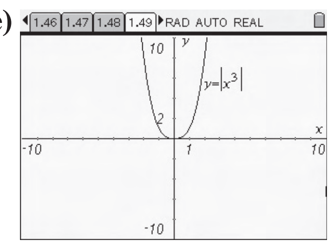
c)



$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid 0 \le y \le 1\}$

d) "parabolic" shape

e)

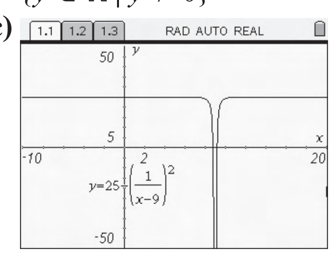


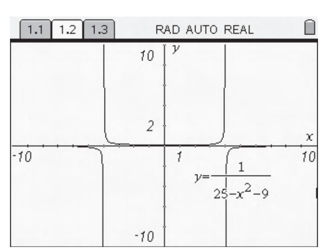
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R} \mid y \ge 0\}$

15. a) $\{x \in \mathbb{R} \mid x \ne 9\}$, $\{y \in \mathbb{R} \mid y < 25\}$

b) $\{x \in \mathbb{R} \mid x \ne -4, x \ne 4\}$, $\{y \in \mathbb{R} \mid y \ne 0\}$

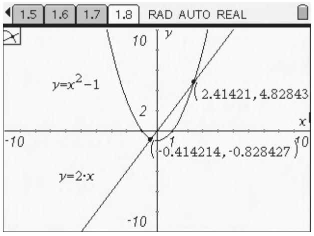
c)





8.4 Inequalities of Combined Functions

1. a)



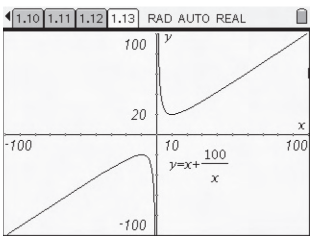
Points of intersection: $(1 + \sqrt{2}, 2 + 2\sqrt{2})$, $(1 - \sqrt{2}, 2 - 2\sqrt{2})$

b) i) $1 - \sqrt{2} < x < 1 + \sqrt{2}$

ii) $x < 1 - \sqrt{2}, x > 1 + \sqrt{2}$

2. $\ldots, \left(-\frac{9\pi}{2}, -\frac{7\pi}{2}\right), \left(-\frac{5\pi}{2}, -\frac{3\pi}{2}\right), (-1.076\,9, 0)$

3. a)



This function is a sum of a linear function and a reciprocal of linear function. It has two asymptotes $x = 0$ and $y = x$. The graph is made up of two branches with end behaviour as follows: As $x \to 0^+$, $y \to \infty$; As $x \to 0^-$, $y \to -\infty$; As $x \to \infty$, $y \to x$; As $x \to -\infty$, $y \to x$
Since n represents years, n cannot be negative, and also n cannot be zero. Therefore, the domain of interest for this problem is $\{n \in \mathbb{R} \mid n > 0\}$

b) $C(n) < 55$

$$n + \frac{100}{n} < 55$$

$$\frac{n^2 + 100}{n} < 55$$

$$n^2 + 100 < 55n$$

$$n^2 - 55n + 100 < 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

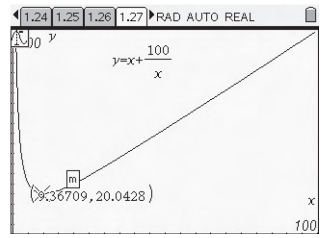
$$n = \frac{55 \pm \sqrt{55^2 - 4(1)(100)}}{2(1)}$$

$$n = \frac{55 \pm \sqrt{2625}}{2}$$

$n \cong 1.9$ or $n \cong 53.1$

$C(n) < 55$ in the interval $(1.9, 53.1)$
$C(n)$ is below 55 when inventory is between approximately 2 and 53 cell phones. Therefore, minimum number of cell phones that can be ordered is 2 and maximum is 53.

c) Use the graphing calculator to find local minimum.

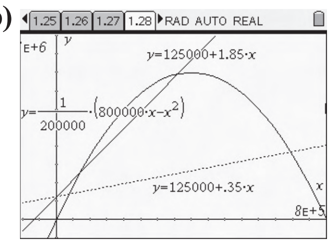


The optimum order size that will minimize storage costs is $n = 10$.

d) The best number to order is 10 cell phones because the storage costs are at minimum, $C = \$20$.

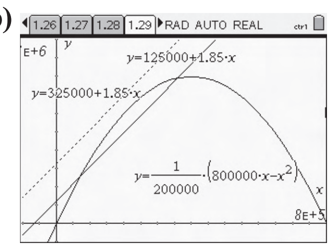
4. a) i) minimum number decreases, maximum number increases **ii)** maximum potential profit increases

b)



5. a) i) the restaurant chain could sell any number of hamburgers but at a loss
ii) profit turns to loss

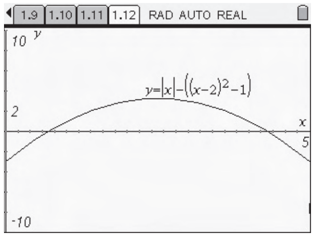
b)



6. a) i) $\frac{5 - \sqrt{13}}{2} < x < \frac{5 + \sqrt{13}}{2}$

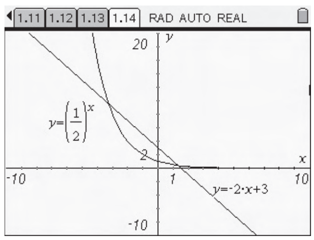
ii) $x < \frac{5 - \sqrt{13}}{2}, x > \frac{5 + \sqrt{13}}{2}$

b)

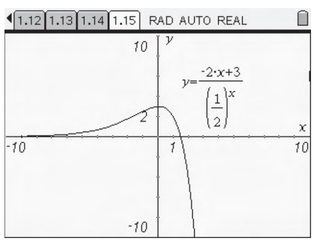


c) $\frac{5-\sqrt{13}}{2} < x < \frac{5+\sqrt{13}}{2}$; same as a) i)

7. a)



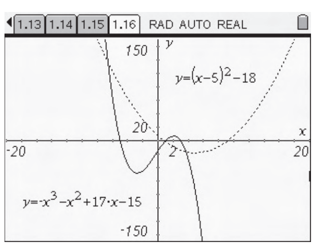
b)



c) i) If $\frac{u(x)}{v(x)} > 1$, then $u(x) > v(x)$

ii) If $\frac{u(x)}{v(x)} < 1$, then $u(x) < v(x)$

8. a)



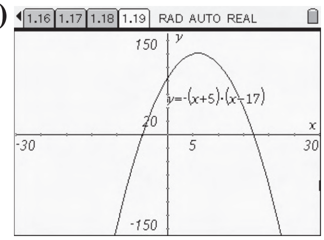
b) i) The graph of $f(x)$ is above the graph of $g(x)$ between $x \cong 0.9$ and $x \cong 3.7$, and also when x is less than approximately -6.6. Therefore, $f(x) > g(x)$ on the intervals $(-\infty, -6.6) \cup (0.9, 3.7)$

ii) The graph of $f(x)$ is below the graph of $g(x)$ between $x \cong -6.6$ and $x \cong 0.9$, and also when x is greater than approximately 3.7. Therefore, $f(x) < g(x)$ on the intervals $(-6.6, 0.9) \cup (3.7, \infty)$

9. Answers will be the same as question 8 when using the following methods:
Method 1: Analyzing the difference function $y = f(x) - g(x)$, and
Method 2: Analyzing the quotient function $y = \frac{f(x)}{g(x)}$

10. $-0.824 < x < 0.824$ **11.** $x > 0.718$

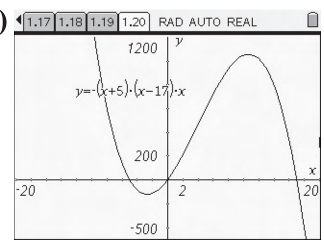
12. a)



b) $-5 < x < 17$; therefore, given that minimum ticket price is $12, maximum realistic ticket price is $12 < x < 17$.

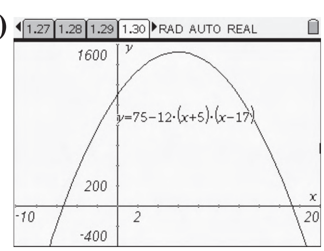
c) $\{p \in \mathbb{R} | 12 \le p < 17\}$, $\{N \in \mathbb{R} | 0 < N \le 85\}$

13. a)

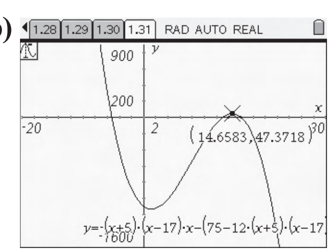


b) $0 < x < 17$; yes **c)** No, maxima of $N(p)$ occurs at $p = 6$ and maxima of $R(p)$ occurs at $p = 10.66$. In this case a combination of fewer visitors paying higher ticket price produces higher revenue than more people paying lower ticket price. **d)** $10.66

14. a)



b)

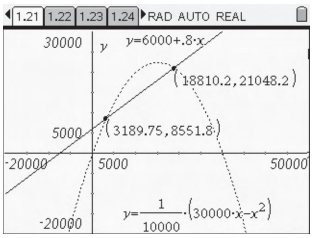


profit function

c) $13.052\,4 < p < 16.144\,1$, profit **d)** no, because profit depends on both revenue and cost

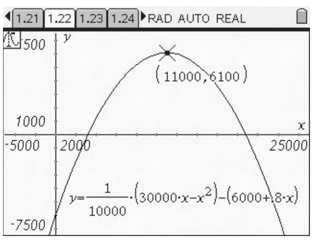
e) Optimum ticket price is \$14.66; maximum profit per ticket is \$3.23

15. a)



b) 2 points of intersection; when $C(x) = R(x)$ means 0 profit for the company **c)** (3190, 18 810); the company makes profit

d)



e) i) 11 000 units **ii)** 0.55 \$/unit **iii)** \$ 6100

f) $R(x)$ is a quadratic function; the graph is a parabola, facing down; after the maximum point the function decreases, and one reason for this is probably because demand for pens decreases

16. Let n be the number of necklaces made in a given week. **a)** $C(n)$ = fixed cost + variable cost $C(n) = 55 + 3.50n$

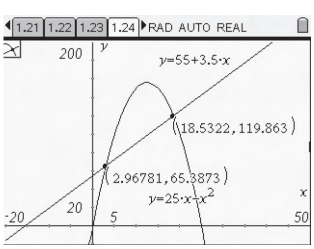
b) $R(n)$ = price per necklace × number of necklaces made per week

$R(n) = (25 - n)n$

$R(n) = 25n - n^2$

c) Stephanie will make profit if her revenue is greater than her cost, $R(n) > C(n)$ or $R(n) - C(n) = 0$

d) Graph the functions $C(n) = 55 + 3.50n$ and $R(n) = 25n - n^2$ on the same graph. Compare the graph visually in order to find the interval(s) for which $R(n) > C(n)$.



The graph of $R(n)$ is above the graph of $C(n)$ between $x \cong 3$ and $x \cong 19$. Therefore, $R(n) > C(n)$ on the interval (3, 19).

Stephanie should make between 3 and 19 necklaces each week in order to make profit.

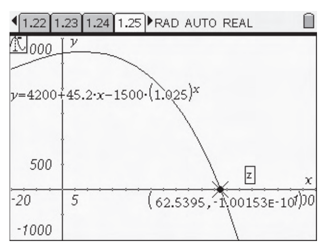
e) $P(n) = R(n) - C(n)$

$$\begin{aligned} P(n) &= (25n - n^2) - (55 + 3.50n) \\ &= 25n - n^2 - 55 - 3.50n \\ &= -n^2 + 21.5n - 55 \\ &= -(n^2 - 21.5n + 55) \quad \text{Complete the square} \end{aligned}$$

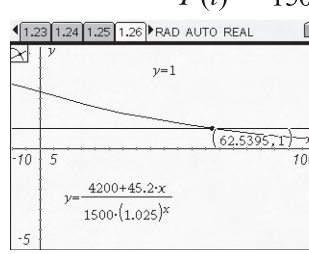
$$\begin{aligned} &= -\left[\left(n^2 - 21.5n + \left(\frac{21.5}{2}\right)^2\right) - \left(\frac{21.5}{2}\right)^2 + 55\right] \\ &= -\left[(n - 10.75)^2 - \frac{462.25}{4} + 55\right] \\ &= -[(n - 10.75)^2 - 115.562\,5 + 55] \\ &= -[(n - 10.75)^2 - 60.562\,5] \\ &= -(n - 10.75)^2 + 60.562\,5 \end{aligned}$$

The maximum of the function $P(n) = R(n) - C(n)$ occurs at $n = 10.75$ and the value is approximately 60.56. Therefore, the optimum number of necklaces is approximately 11 and Stephanie will earn approximately \$60.56.

17. a) $N(t) - P(t) = 4200 + 45.2t - 1500(1.025)^t$



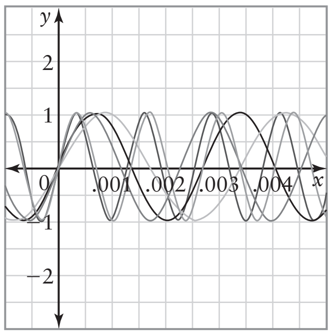
b) availability of local water supply for the town population **c)** In approximately 62.5 years there will be shortage of local water supply. **d)** $\dfrac{N(t)}{P(t)} = \dfrac{4200 + 45.2t}{1500(1.025)^t}$



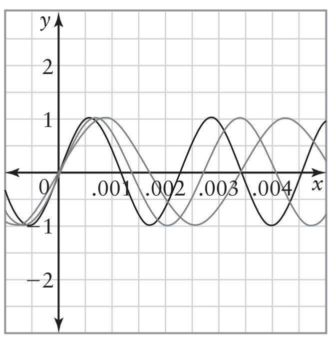
e) availability of local water supply for the town population; If $\frac{N(t)}{P(t)} > 1$, there is enough local water supply; if $\frac{N(t)}{P(t)} < 1$, there is shortage of local water supply.

8.5 Making Connections: Modeling With Combined Functions

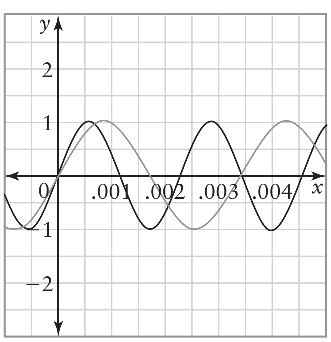
1. a)



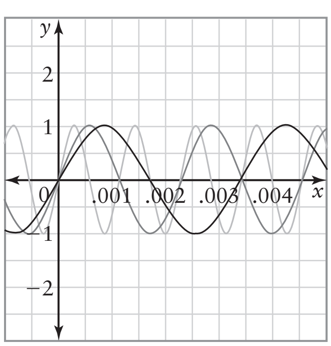
b) the D major triad of D, F# and A:



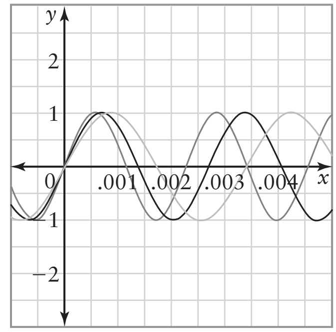
2. a) i)



ii)



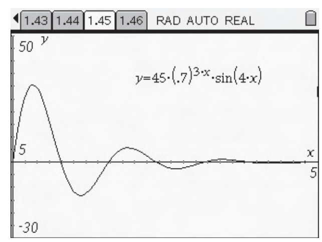
b) Compare to



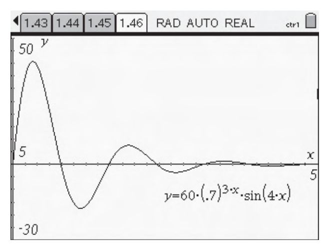
These notes will sound good together since the resultant curve of the combined function is similar to that formed from the individual notes

3. $y = 2x + 200 + 0.4\sin\left(\frac{\pi}{3}x\right)$

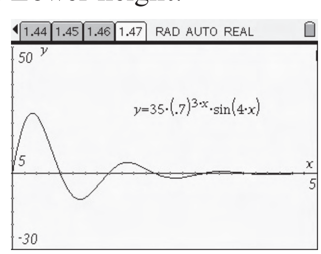
4. Original graph:



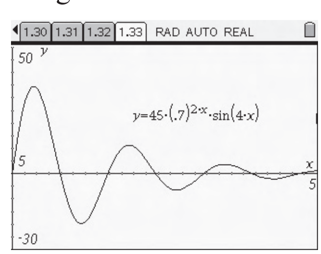
a) Greater height:



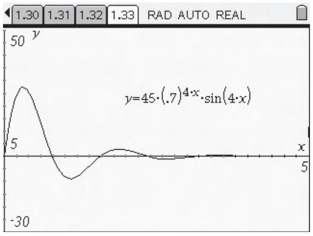
b) Lower height:



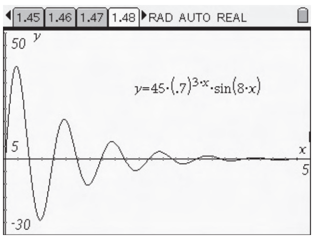
5. a) Longer cord:



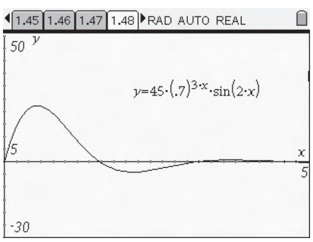
b) Shorter cord:



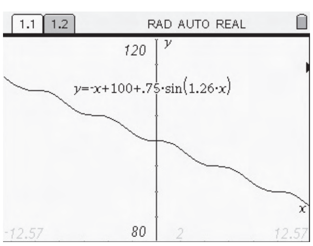
6. a) Springier cord:



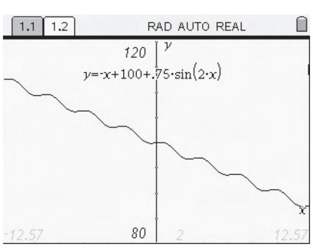
b) Stiffer cord:



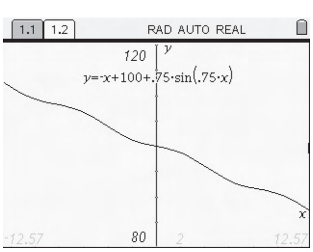
7. Original graph:



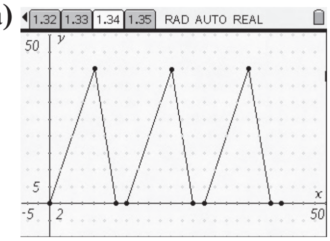
a) Shorter moguls:



b) Farther apart moguls



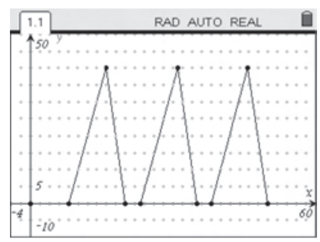
8. a)



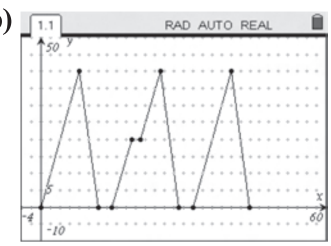
b) In the first region of each cycle, the skier travels up the chairlift. In the second region, assuming that the skier does not wait at the top, the skier skis down the hill. In the final region the skier waits in line for a chairlift.

c) Answers may vary. For example: Use a combination of straight lines with different slopes to represent the three different regions of a cycle.

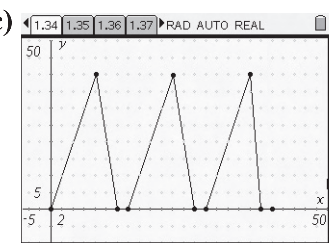
9. a)



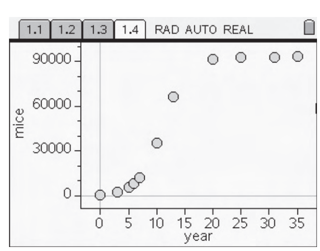
b)



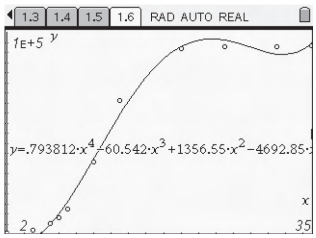
c)



10. a)



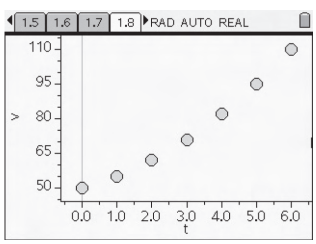
b) Quartic function. When using technology, the regression curve for a quartic function fit best and had the r^2 value closest to 1. $y = 0.793\,812x^4 - 60.542x^3 + 1356.55x^2 - 4692.85x + 1524.55$



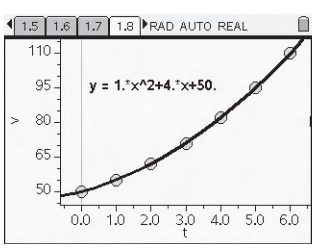
c) $N(t) = 0.793\,812t^4 - 60.542t^3 + 1356.55t^2 - 4692.85t + 1524.55$

d) Removing the data point caused a slight reduction in the coefficient of determination r^2. This implies that a quartic regression works better with that particular data point.

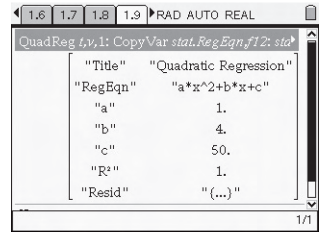
11. a) Use a graphing calculator to plot all the data points.



b) Use the regression function of the graphing calculator to find a curve that best fits the data points. In this case the quadratic function $y = x^2 + 4x + 50$ best fits the data.



To check how close the curve fits the data points, display the value for the coefficient of determination, r^2.



In this case $r^2 = 1$, which means the quadratic function $y = x^2 + 4x + 50$ is the best fit.

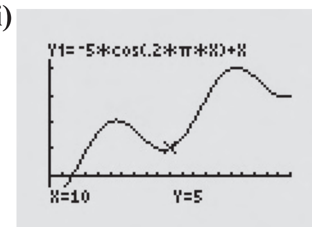
c) $v(t) = t^2 + 4t + 50$

d) $c(v) = \left(\frac{v(t)}{450} - 0.09\right)^2 + 0.18$

$c(v(t)) = \left(\frac{t^2 + 4t + 50}{450} - 0.09\right)^2 + 0.18$

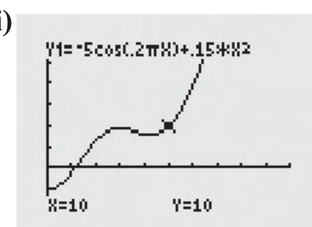
12. a) i) Answers may vary. Sample answer: The motion of the runner would look like a cosine wave, but sketched along the line $y = t$, rather than along the time axis.

ii)



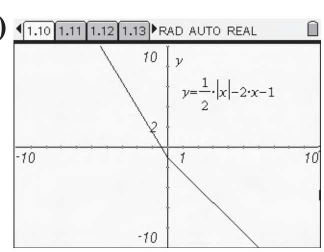
12. b) i) Answers may vary. Sample answer: The motion of the runner would look like a cosine wave, but sketched along the curve $y = t^2$, rather than along the time axis.

ii)

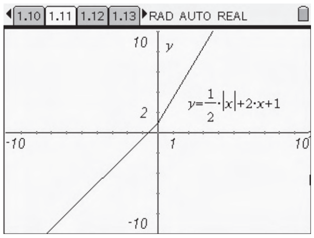


Chapter 8 Challenge Questions

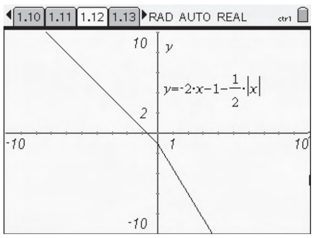
C1.a)



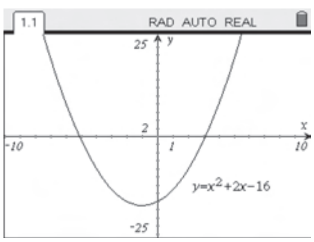
b)



c)

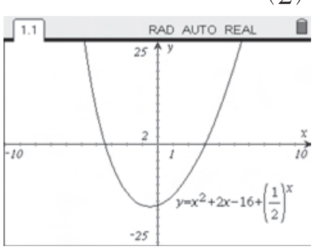


C2. a) $y = x^2 + 2x - 16$



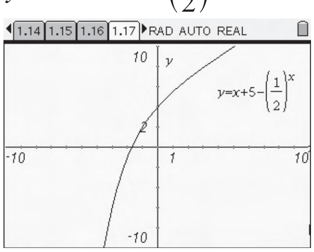
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} | y \geq 5.75\}$

b) $y = x^2 + 2x - 16 + \left(\frac{1}{2}\right)^x$



$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} | y \geq 6.41\}$

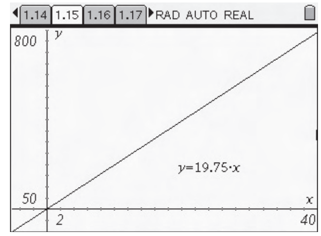
c) $y = x + 5 - \left(\frac{1}{2}\right)^x$



$\{x \in R\}$,$\{y \in R\}$

C3. See answer for question C2

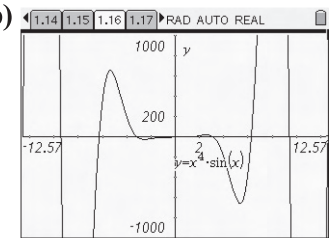
C4. a) $W(t) = 7.50t$ **b)** $T(t) = 12.25t$
c) $E(t) = W(t) + T(t) = 19.75t$



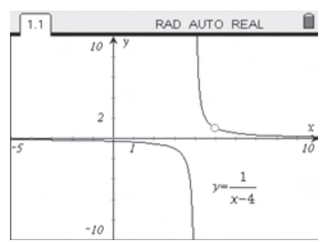
d) \$632

5. a) point symmetry, i.e., the graph is symmetrical about a point

b)

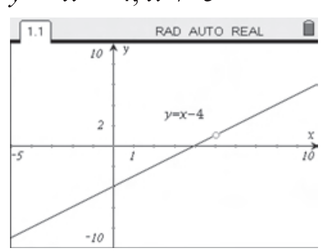


6. a) $y = \dfrac{1}{x-4}, x \neq 5$



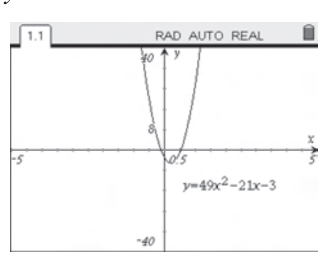
$\{x \in \mathbb{R} \mid x \neq 4, x \neq 5\}, \{y \in \mathbb{R} \mid y \neq 1\}$

b) $y = x - 4, x \neq 5$



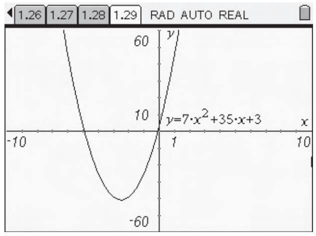
$\{x \in \mathbb{R} \mid x \neq 5\}, \{y \in \mathbb{R} \mid y \neq 0, y \neq 1\}$

7. a) $y = 49x^2 - 21x - 3$



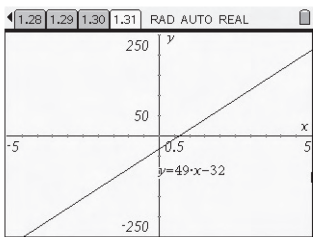
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} \mid y \geq 0.214\}$

b) $y = 7x^2 + 35x + 3$



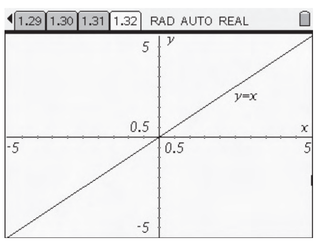
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R} \mid y \geq -2.5\}$

c) $y = 49x - 32$



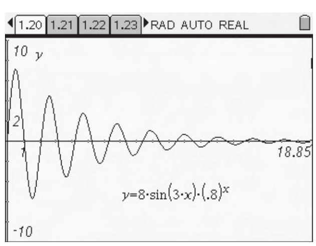
$\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

d) $y = x$



$\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$

8. a)



periodic with decreasing amplitude; domain$\{t \in \mathbb{R} \mid x \geq 0\}$, range $\{x(t) \in \mathbb{R} \mid -5.65 \leq x(t) \leq 7.14\}$

b) i) $8 \sin(3t)$

ii) 0.80^t

c) $x \cong 7.14$

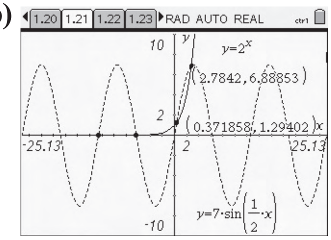
d) $t = \left(\frac{2\pi}{3}\right)\left(\frac{k}{4}\right)$, where k is an odd integer When the pendulum changes direction

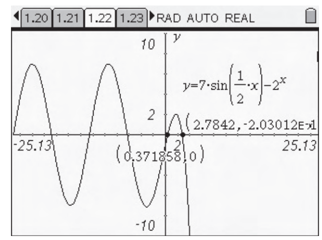
e) $t \cong 5.7$ s

9. a) i) $... \cup (-8\pi, -6\pi) \cup (-4\pi, -2\pi) \cup (0.37, 2.78)$

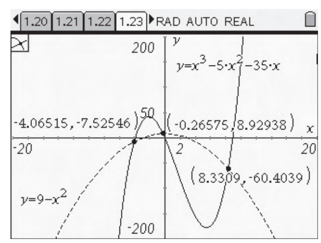
ii) $... \cup (-6\pi, -4\pi) \cup (-2\pi, 0.37) \cup (2.78, +\infty)$

b)





10. a)



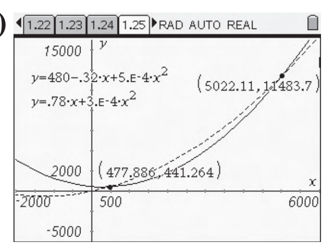
b) $(-4.07, -0.27) \cup (8.33, +\infty)$

c) Using algebraic method, find the points of intersection and then find the intervals for which $f(x) > g(x)$.

11. a) i) $(0, 4) \cup (5.5, \infty)$ **ii)** $(4, 5.5)$

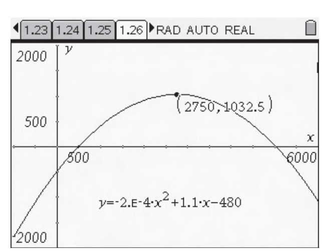
b) running at a loss **c)** decrease costs

12. a)



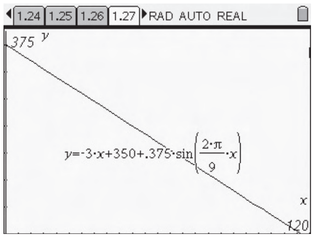
b) Two points of intersection: (477.89, 441.26) and (5022.11, 11 483.7) This is when revenue equals cost.

c) $p(x) - 0.000\,2x^2 + 1.1x - 480$

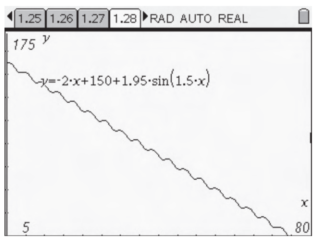


d) \$1032.50

13. $f(t) = -3t + 350 + 1.5\sin(\pi x)$



14. a)



b) $\{t \in \mathbb{R} | 0 \le t \le 74.1\}$,
$\{h \in \mathbb{R} | 0 \le h \le 150\}$

University Preparation

UP 1.1 Factoring Complex Equations

1. **a)** $x(x - 4)$ **b)** $-(2n + 1)$
c) $(3x + y - 1)(3x + y + 1)$
d) $-3(5x - 3)(3x + 1)$
e) $3(x - 1)(x + 9)$ **f)** $-10(7x - 5)(x + 1)$
g) $-(x - 1)(x - 2)$

2. **a)** $-3(x + 9)(x - 1)$
b) $9(2x - 5)$ **c)** $-\frac{1}{9}(20x - 17)(22x - 13)$
d) $-0.11(38x + 9)(2x + 1)$

3. **a)** $2(6x - 7)(2x + 5)$
b) $9(x - 3)(x - 11)(5x^2 - 38x + 101)$
c) $-32(a - b - 6c)(3a - 3b - 2c)$
d) $-16(2x^2 - 12xy - 7y^2)$
$(6x^2 + 4xy + 9y^2)$

4. **a)** $(x + 2 + y)(x + 2 - y)$
b) $(x - 3 - 2y)(x - 3 + 2y)$
c) $(a + 5 - 3b)(a + 5 + 3b)$
d) $4m^2 - 4n^2 - 8np - 4p^2$
$= 4m^2 - (4n^2 + 8np + 4p^2)$
$= 4m^2 - 4(n^2 + 2np + p^2)$
$= 4[(m)^2 - (n + p)^2]$
$= 4(m - n - p)(m + n + p)$
e) $2(p - q + 2r)(p + q - 2r)$
f) $(x^2 - y + z)(x^2 + y - z)$
g) $(x - y - 3a)(x - y + 3a)$

5. **a)** $(3a + 3b - 2x + 2y)(3a + 3b + 2x - 2y)$
b) $(4s - 4t - p - 2q)(4s - 4t + p + 2q)$
c) $(5r - 6s - 3q + 2h)(5r - 6s + 3q - 2h)$
d) $s^2 - 14st + 49t^2 - a^2 - 18ab - 81b^2$
$= s^2 - 14st + 49t^2 - (a^2 + 18ab + 81b^2)$
$= (s - 7t)^2 + (a + 9b)^2$
$= (s - 7t - a - 9b)(s - 7t + a + 9b)$

6. **a)** $(x + 5)(x + 8)$ **b)** $4(x + 1)(x + 6)$
c) $6(x^2 - 1)^2 + 23(x^2 - 1) + 7$
Let $n = x^2 - 1$
$6n^2 + 23n + 7$
$= (3n + 1)(2n + 7)$
$= (3(x^2 - 1) + 1)(2(x^2 - 1) + 7)$
$= (3x^2 - 3 + 1)(2x^2 - 2 + 7)$
$= (3x^2 - 2)(2x^2 + 5)$
d) $3(13x + 3)(8x - 15)$

7. **a)** $(x^{2n} - y^{3n})(x^{2n} + y^{3n})$
b) $(3x^{3n} - 2y^{2n})^2$ **c)** $(4x^{2n+1} + 3y^{4n})^2$

UP 1.2 Techniques for Solving Complex Equations

1. **a)** $m^2 - 17m + 16 = 0$
b) $x = \pm 4$ or $x = \pm 1$

2. $x = \pm 5$ or $x = \pm 1$

3. **a)** $n^2 - 6n + 8 = 0$ **b)** $x = 2$ or $x = 1$

4. **a) i)** $n = x^2$ **ii)** $x = \pm 2$ or $x = \pm 1$
b) i) $n = x^2$ **ii)** $x = \pm 2$
c) i) $n = x^2$ **ii)** $x = \pm\sqrt{10}$ or $x = \pm\sqrt{6}$
d) i) $n = x^2$ **ii)** $x = \pm\sqrt{35}$ or $x = \pm 1$
e) i) $n = x^2$ **ii)** $x = \pm 2$ or $x = \pm 4$
f) i) $n = x^2$ **ii)** $x = \pm 2$ or $x = \pm 5$
g) i) $n = x^2$ **ii)** $x = \pm\sqrt{2}$ or $x = \pm\sqrt{3}$
h) i) $n = x^2$ **ii)** $x = \pm\sqrt{5}$

5. **a) i)** $n = 5^x$ **ii)** $x = 0$ or $x = 1$
b) i) $n = 3^x$ **ii)** $x = 1$ or $x = 3$
c) i) $n = 2^x$ **ii)** $x = 2$ or $x = 3$
d) i) $n = 2^x$ **ii)** $x = 1$ or $x = 4$
e) i) $n = 2^x$ **ii)** $x = 1$ or $x = 3$
f) i) $n = 3^x$ **ii)** $x = 1$ or $x = 2$
g) $5^{2x} - 30(5^x) + 125 = 0$
Let $n = 5^x$.
$n^2 - 30n + 125 = 0$
$(n - 25)(n - 5) = 0$
$n - 25 = 0 \qquad n - 5 = 0$
$n = 25 \qquad n = 5$
$5^x = 25 \qquad 5^x = 5$
$x = 2 \qquad x = 1$

6. **a)** $x = 1$ or $x = 0$ **b)** $x = 1$ or $x = -1$
c) $x = \pm 1$ or $x = \pm 3$

7. **a)** $x = 3$ or $x = 0$ **b)** $x = 1$ or $x = -\frac{2}{3}$
c) $x = -3$, $x = 1$, $x = -4$ or $x = 2$
d) $x = 4$, $x = -1$, $x = 1$ or $x = 2$
e)
$$\begin{aligned} (x^2 - 2x)^2 - 4 &= 2(x^2 - 2x) - 1 \\ (x^2 - 2x)^2 - 2(x^2 - 2x) - 3 &= 0 \end{aligned}$$
Let $n = x^2 - 2x$.
$$\begin{aligned} n^2 - 2n - 3 &= 0 \\ (n - 3)(n + 1) &= 0 \\ (x^2 - 2x - 3)(x^2 - 2x + 1) &= 0 \\ (x - 3)(x + 1)(x - 1)^2 &= 0 \end{aligned}$$
$x - 3 = 0$ $\quad x + 1 = 0$ $\quad x - 1 = 0$
$x = 3$ $\quad x = -1$ $\quad x = 1$

8. **a)** $x = \pm 2$ or $x = \pm 3$ **b)** $x = -\frac{1}{2}$ or $x = -3$
c) $x = \frac{1}{3}$ or $x = -\frac{1}{4}$ **d)** $x = \pm\frac{1}{2}$ or $x = \pm\frac{\sqrt{5}}{5}$
e) $x = \pm\frac{\sqrt{2}}{4}$ or $x = \pm 1$

9. **a)** $x = \frac{15}{7}$ or $x = \frac{11}{5}$ **b)** $x = -\frac{13}{3}$ or $x = -6$
c) $x = -2$, $x = 1$, $x = -4$ or $x = 3$

10. **a)** $x = 6$, $x = 1$, $x = -2$ or $x = -3$
b) $x = 2$, $x = 1$ or $x = 4$
c) $x = 2 \pm \sqrt{5}$ or $x = \frac{3 \pm \sqrt{13}}{2}$

11. **a)** $n = x + \frac{1}{x}$
b) $x = -2 \pm \sqrt{3}$ or $x = \frac{3 \pm \sqrt{5}}{2}$

UP 2.1 Solving Equations Involving Absolute Value

1. **a)** 6 **b)** 8 **c)** 7 **d)** 11 **e)** 4

2. **a)** $x = \pm 2$ **b)** $x = \pm 9$ **c)** $x = \pm 9$
d) $x = \pm 33$ **e)** $x = \pm 12.5$ **f)** $x = \pm 39$
g) $x = \pm 35$ **h)** $x = \pm 8$ **i)** no solution

3. **a)** $x = -1$ or $x = -7$ **b)** $x = 9$ or $x = -3$
c) $x = 9$ or $x = -5$ **d)** $x = 4$ or $x = -6$
e) no solution **f)** $x = 9$ or $x = -7$

4. **a)** $x = 5$ or $x = -4$ **b)** $x = 1$ or $x = -\frac{1}{2}$
c) $x = 0$ or $x = -6$ **d)** $x = -3$ or $x = \frac{13}{3}$
e) no solution **f)** $x = \frac{1}{4}$ or $x = -\frac{5}{4}$
g) no solution

5. **a)** $x = \frac{2}{3}$ **b)** $x = -\frac{5}{3}$ **c)** $x = \frac{1}{2}$ **d)** $x = 1$
e) $x = \frac{1}{4}$ **f)** $x = -\frac{3}{2}$ or $x = \frac{9}{2}$

6. **a)** $x = \frac{2}{3}$ **b)** no solution **c)** $x = 4$ **d)** $x = 3$
e) $16x - 3|2x - 1| = |10x - 5|$
Both $2x - 1$ and $10x - 5$ are equal to zero at $x = \frac{1}{2}$, which sets up two cases:
Case 1
$$x < \frac{1}{2}$$
$$\begin{aligned} 16x - 3[-(2x - 1)] &= -(10x - 5) \\ 16x - 3(-2x + 1) &= -10x + 5 \\ 16x + 6x - 3 &= -10x + 5 \\ 16x + 6x + 10x &= 5 + 3 \\ 32x &= 8 \\ x &= \frac{1}{4} \end{aligned}$$
Case 2
$$x > \frac{1}{2}$$
$$\begin{aligned} 16x - 3(2x - 1) &= (10x - 5) \\ 16x - 6x + 3 &= 10x - 5 \\ 16x - 6x - 10x &= -5 - 3 \\ 0x &= -8 \quad \textit{no solution} \end{aligned}$$
Therefore, the only solution is $x = \frac{1}{4}$.
f) $x = -\frac{6}{11}$

7. **a)** $x = 0$ or $x = -2$ **b)** $x = 1$ or $x = -5$
c) $x = 5$ or $x = -\frac{5}{3}$ **d)** $x = 2$ or $x = \frac{2}{5}$
e) $x = 9$ or $x = -11$

8. **a)** $x = 1$ or $x = \frac{1}{3}$ **b)** $x = 2$ or $x = 4$
c) $x = -\frac{3}{4}$ or $x = \frac{7}{2}$ **d)** $x = \frac{4}{3}$ or $x = 2$
e) $|3x - 7| = |2 - x|$
For intervals:
$3x - 7 = 0$ *and* $2 - x = 0$
$3x = 7$ $\qquad 2 = x$
$x = \frac{7}{3}$
Case 1
$$x < 2$$
$$\begin{aligned} -(3x - 7) &= (2 - x) \\ -3x + 7 &= 2 - x \\ -3x + x &= 2 - 7 \\ -2x &= -5 \\ x &= \frac{5}{2} \end{aligned}$$
Since this solution is not in the interval of $x < 2$, it is not a valid solution.

Case 2

$$2 < x < \frac{7}{3}$$
$$-(3x - 7) = -(2 - x)$$
$$-3x + 7 = -2 + x$$
$$-3x - x = -2 - 7$$
$$-4x = -9$$
$$x = \frac{9}{4}$$

This solution is in the interval, so it is a solution.

Case 3

$$x > \frac{7}{3}$$
$$(3x - 7) = -(2 - x)$$
$$3x - 7 = -2 + x$$
$$3x - x = -2 + 7$$
$$2x = 5$$
$$x = \frac{5}{2}$$

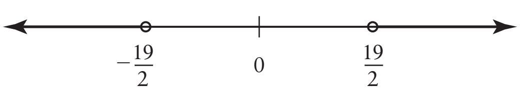
This solution is in the interval, so it is a solution.

9. **a)** $x = \pm 4$ **b)** no solution

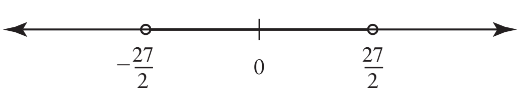
UP 2.2 Solving Inequalities Involving Absolute Value

1. **a)** $x < -3$ or $x > 3$ **b)** $-2 < x < 2$
c) $-5 \leq x \leq 5$ **d)** all values of x
e) $-\frac{7}{4} \leq x \leq \frac{7}{4}$ **f)** $x < -4$ or $x > 4$
g) $-\frac{14}{5} < x < \frac{14}{5}$
h) $-7 < x < 7$ **i)** $-4 \leq x \leq 4$

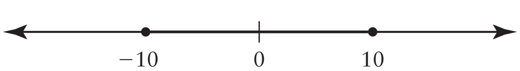
2. **a)** $x < -\frac{19}{2}$ or $x > \frac{19}{2}$



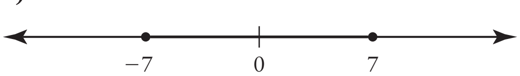
b) no solution
c) $-\frac{27}{2} < x < \frac{27}{2}$



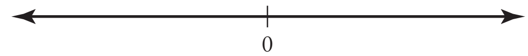
d) $-10 \leq x \leq 10$



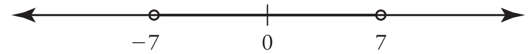
e) $-7 \leq x \leq 7$



f) $x \in \mathbb{R}$

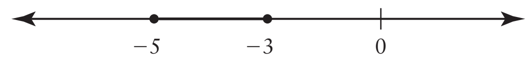


g) $-7 < x < 7$

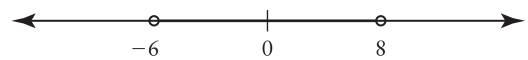


3. **a)** $x < 0$ or $x > 6$

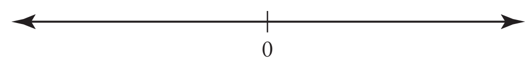
b) $-5 \leq x \leq -3$



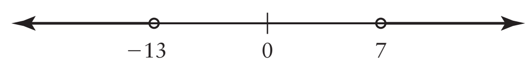
c) $-6 < x < 8$



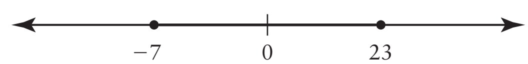
d) $x \in \mathbb{R}$



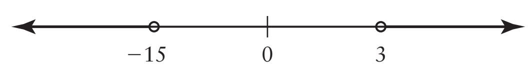
e) $x < -13$ or $x > 7$



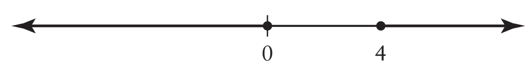
f) $-7 \leq x \leq 23$



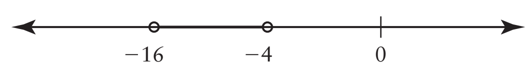
g) $x < -15$ or $x > 3$



h) $x \leq 0$ or $x \geq 4$



i) $-16 < x < -4$



4. **a)** $x < -2$ or $x > 1$ **b)** $x \leq -1$ or $x \geq \frac{11}{5}$
c) $\frac{1}{7} < x < \frac{3}{7}$ **d)** $1 \leq x \leq 4$
e) $-\frac{10}{3} < x < \frac{8}{3}$ **f)** $x \leq -\frac{3}{2}$ or $x \geq 5$
g) $x < -7$ or $x > 4$ **h)** $x \leq \frac{1}{6}$ or $x \geq \frac{7}{6}$
i) $x < -\frac{67}{3}$ or $x > \frac{65}{3}$
j) $x \leq -\frac{22}{5}$ or $x \geq -\frac{18}{5}$

5. **a)** $x > \frac{1}{2}$ **b)** $x > 1$ **c)** $x = 2$

d) $2 - 3x = 0$

$2 = 3x$

$x = \frac{2}{3}$

Two cases:

Case 1

$x < \frac{2}{3}$

$2 - 3x > 3x - 6$

$-3x - 3x > -6 - 2$

$-6x > -8$

$x < \frac{-8}{-6}$

$x < \frac{4}{3}$

This is in the interval for the case, so all values of x are solutions in this interval.

Case 2

$x > \frac{2}{3}$

$-(2 - 3x) > 3x - 6$

$3x - 2 > 3x - 6$

$3x - 3x > -6 + 2$

$0x > -4$

This is always true, so all values of x are solutions in this interval.

As a result of the two cases, the solution is $x \in \mathbb{R}$.

e) $x \in \mathbb{R}$

f) $x < -\frac{1}{4}$ **g)** $x \geq \frac{3}{7}$

6. **a)** $-\frac{17}{5} \leq x \leq 19$ **b)** $-1 < x < 1$

c) $|x + 2| > 4 + |x|$

$x + 2 = 0$ $\quad x = 0$

$x = -2$

The solutions to the expressions equal to zero set up the intervals for the cases of the solution.

Case 1

$x < -2$

$-(x + 2) > 4 - (x)$

$-x - 2 > 4 - x$

$-x + x > 4 + 2$

$0x > 6$ *no solution*

Case 2

$-2 < x < 0$

$(x + 2) > 4 - (x)$

$x + 2 > 4 - x$

$x + x > 4 - 2$

$2x > 2$

$x > 1$

This is not in the interval of the case, so there is no solution in this interval.

Case 3

$x > 0$

$(x + 2) > 4 + (x)$

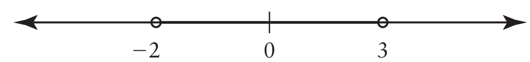
$x + 2 > 4 + x$

$x - x > 4 - 2$

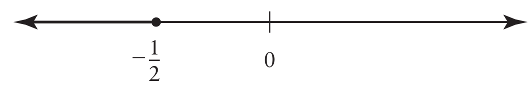
$0x > 2$ *no solution*

Since there is no solution in any interval, there is no solution to the original inequality.

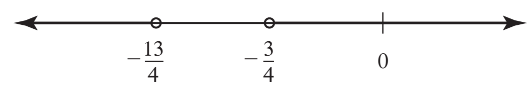
7. **a)** $-2 < x < 3$



b) $x \leq -\frac{1}{2}$



c) $x < -\frac{13}{4}$ or $x > -\frac{3}{4}$



8. **a)** $x > -1$ **b)** $-\frac{5}{6} \leq x \leq \frac{1}{10}, x \neq -\frac{1}{4}$

c) $-12 \leq x \leq -\frac{8}{3}$

UP 3.1 Introduction to Matrices

1. **a)** 2×3 **b)** 3; −2

c) $\begin{bmatrix} 4 & -3 & -8 \\ -5 & -1 & 2 \end{bmatrix}$ **d)** $\begin{bmatrix} -24 & 18 & 48 \\ 30 & 6 & -12 \end{bmatrix}$

e) $\begin{bmatrix} -12 & 9 & 24 \\ 15 & 3 & -6 \end{bmatrix}$

2. **a)** ***BC***; ***B*** is a 2×3 matrix and ***C*** is a 2×2 matrix. Since the number of columns of ***B*** does not match the number of rows of ***C***, this product does not exist.

b) ***CB***; ***C*** is a 2×2 matrix and ***B*** is a 2×3 matrix. Since the number of columns of ***C*** equals the number of rows of ***B***, this product exists. The new matrix will have dimensions 2×3.

c) B^2; B is not a matrix with an equal number of rows and columns, and so cannot be multiplied by itself.

d) C^2 ; C is a matrix with an equal number of rows and columns, and so can be multiplied by itself. The new matrix will have dimensions 2 × 2.

3. b) $\begin{bmatrix} -28 & 20 & -23 \\ -4 & 8 & 28 \end{bmatrix}$ **d)** $\begin{bmatrix} 9 & 1 \\ 0 & 16 \end{bmatrix}$

4. a) $\begin{bmatrix} -7 & 5 & -1 \\ -3 & -2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$ **b)** $\begin{bmatrix} -21 & 15 & -3 \\ -9 & -6 & 15 \\ 9 & 12 & 15 \end{bmatrix}$

c) does not exist because R and Q have different dimensions

d) $\begin{bmatrix} -8 & -2 \\ 0 & \frac{2}{3} \\ 2 & -\frac{14}{3} \end{bmatrix}$

5. a) $\begin{bmatrix} -66 & 24 & 30 \\ -2 & -4 & 2 \\ 32 & 25 & -23 \end{bmatrix}$ **b)** $\begin{bmatrix} -58 & 22 & 30 \\ 3 & -11 & 1 \\ 29 & 26 & -34 \end{bmatrix}$

c) does not exist because number of columns of Q is not equal to number of rows of P

d) $\begin{bmatrix} 15 & -7 \\ 12 & 43 \\ 18 & -47 \end{bmatrix}$ **e)** $\begin{bmatrix} -104 & -13 & 59 \\ 14 & 184 & -50 \\ -202 & -170 & 148 \end{bmatrix}$

f) $\begin{bmatrix} 7 & -29 & 17 \\ -6 & 107 & -62 \\ 10 & -75 & 56 \end{bmatrix}$

g) does not exist because RS and P^2 have different dimensions

h) $\begin{bmatrix} -33 & 53 & -25 \\ 47 & -170 & 97 \\ -53 & 116 & -117 \end{bmatrix}$ **i)** $\begin{bmatrix} 20 & -48 & 16 \\ 24 & -94 & 24 \\ -8 & 76 & -106 \end{bmatrix}$

6. a) false

Example:

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 11 & 10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 5 & 8 \end{bmatrix}$$

b) true

Example:

$$(AB)C = \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 17 & 17 \\ 41 & 41 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\right) = \begin{bmatrix} 17 & 17 \\ 41 & 41 \end{bmatrix}$$

c) true

Example:

$$k(AB) = 2\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right) = \begin{bmatrix} 10 & 8 \\ 22 & 20 \end{bmatrix}$$

$$(kA)B = \left(2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ 22 & 20 \end{bmatrix}$$

$$A(kB) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left(2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right) = \begin{bmatrix} 10 & 8 \\ 22 & 20 \end{bmatrix}$$

7. $a = 5, b = 2, c = 2, d = -1$

8. a) $\begin{bmatrix} -186 & 264 \\ -10 & 2 \end{bmatrix}$

b) $[3 \;\; -4]\begin{bmatrix} -1 & 1 & 2 \\ 8 & 3 & -5 \end{bmatrix}$

$+\, 4[2 \;\; -1 \;\; -2]$

$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 0 & 1 \\ 5 & 6 & -2 \end{bmatrix}$

$[3 \times -1 - 4 \times 8 \quad 3 \times 1 - 4 \times 3 \quad 3 \times 2 - 4 \times -5] + 4[2 \times 3 - 1 \times -1 - 2 \times 5 \quad 2 \times 2 - 1 \times 0 - 2 \times 6 \quad 2 \times 4 - 1 \times 1 - 2 \times -2]$

$= [-3 - 32 \quad 3 - 12 \quad 6 + 20] + 4[6 + 1 - 10 \quad 4 - 12 \quad 8 - 1 + 4]$

$= [-35 \quad -9 \quad 26] + 4[-3 \quad -8 \quad 11]$

$= [-35 \quad -9 \quad 26] + [-12 \quad -32 \quad 44]$

$= [-35 - 12 \quad -9 - 32 \quad 26 + 44]$

$= [-47 \quad -41 \quad 70]$

9. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10. a) $x = \frac{19}{13}, y = \frac{43}{13}$ **b)** $x = -11, y = -26, z = 3$

11. $M^2 = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ -30 & 15 \end{bmatrix}$

$5M = 5\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ -30 & 15 \end{bmatrix}$

12. No.

Counterexample:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}$

$(A + B)(A - B) = \begin{bmatrix} -8 & -3 \\ -15 & -7 \end{bmatrix}$

$A^2 - B^2 = \begin{bmatrix} -9 & -8 \\ -9 & -6 \end{bmatrix}$

UP 3.2 Determinants

1. a) unique solution: $\left(\frac{9}{4}, -\frac{17}{4}\right)$ **b)** no unique solution **c)** unique solution: $\left(-\frac{22}{9}, -4\right)$

2. Determinant is a real number. Matrix is a rectangular array of number(s) arranged in rows and columns.

3. **a)** 48 **b)** 22 **c)** −263 **d)** 58

e) $5\begin{vmatrix} 9 & 10 \\ -7 & 4 \end{vmatrix} - \frac{2}{3}\begin{vmatrix} 6 & 27 \\ 18 & -15 \end{vmatrix}$

$= 5[9 \times 4 - (-7) \times 10]$

$\quad - \frac{2}{3}[6 \times (-15) - 27 \times 18]$

$= 5(106) - \frac{2}{3}(-576)$

$= 530 + 384$

$= 914$

4. **a)** unique solution: $\left(\frac{25}{23}, \frac{28}{23}\right)$ **b)** no unique solution **c)** unique solution: $\left(-\frac{11}{7}, -\frac{12}{7}\right)$ **d)** no unique solution **e)** unique solution: $\left(\frac{10}{33}, -\frac{31}{11}\right)$

5. $\begin{vmatrix} 1 & 1-x \\ x-4 & 2 \end{vmatrix} = 0$

$1(2) - (x-4)(1-x) = 0$

$2 - (x - x^2 - 4 + 4x) = 0$

$2 - (-x^2 + 5x - 4) = 0$

$2 + x^2 - 5x + 4 = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3 \qquad x = 2$

6. $a = 1, b = 7$

7. **a)** $x = -\frac{37}{58}, y = -\frac{20}{29}, z = \frac{141}{58}$ **b)** no unique solution **c)** $x = 1, y = -2, z = -4$

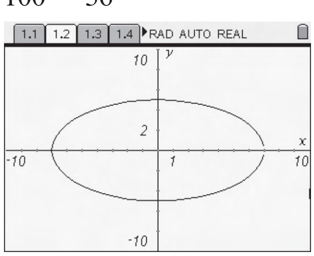
8. $f(x) = 2x^2 - 5x + 8$

UP 4.2 The Ellipse

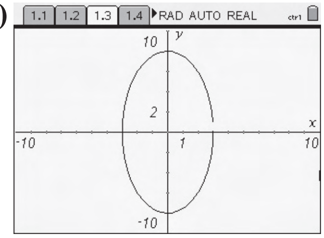
1. **a)** $\frac{x^2}{49} + \frac{y^2}{25} = 1$ **b)** $\frac{x^2}{9} + \frac{y^2}{64} = 1$

c) $\frac{x^2}{100} + \frac{y^2}{36} = 1$

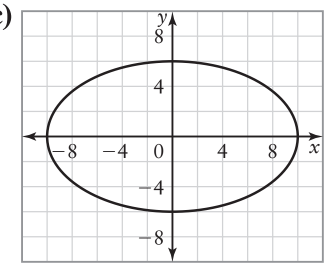
2. **a)**



b)



c)



	$\frac{x^2}{49} + \frac{y^2}{25} = 1$	$\frac{x^2}{9} + \frac{y^2}{64} = 1$	$\frac{x^2}{100} + \frac{y^2}{36} = 1$
Centre	(0, 0)	(0, 0)	(0, 0)
Vertices	(±7, 0)	(0, ±8)	(±10, 0)
Intercepts	$y = \pm 5$	$x = \pm 3$	$y = \pm 6$
Major axis and length	x-axis, 14	y-axis, 16	x-axis, 20
Minor axis and length	y-axis, 10	x-axis, 6	y-axis, 12
Foci	$(\pm 2\sqrt{6}, 0)$	$(0, \pm\sqrt{55})$	(±8, 0)
Distance between foci	$4\sqrt{6}$	$2\sqrt{55}$	16

3. **a) i)** $(0, \pm 4\sqrt{3})$ **ii)** Major axis: y-axis; Minor axis: x-axis
b) i) $(\pm\sqrt{19}, 0)$ **ii)** Major axis: x-axis; Minor axis: y-axis
c) i) $(0, \pm 3\sqrt{5})$ **ii)** Major axis: y-axis; Minor axis: x-axis
d) i) $(0, \pm\sqrt{11})$ **ii)** Major axis: y-axis; Minor axis: x-axis
e) i) $(\pm\sqrt{21}, 0)$ **ii)** Major axis: x-axis; Minor axis: y-axis
f) i) $(0, \pm\sqrt{18.75})$ **ii)** Major axis: y-axis; Minor axis: x-axis

4.

	$\frac{x^2}{121}+\frac{y^2}{64}=1$	$\frac{x^2}{16}+\frac{y^2}{81}=1$	$\frac{x^2}{144}+\frac{y^2}{169}=1$	$\frac{x^2}{18}+\frac{y^2}{44}=1$
Vertices	$(\pm 11, 0)$	$(0, \pm 9)$	$(0, \pm 13)$	$(0, \pm 2\sqrt{11})$
Intercepts	$y = \pm 8$	$x = \pm 4$	$x = \pm 12$	$x = \pm 3\sqrt{2}$
Major axis and length	x-axis, 22	y-axis, 18	y-axis, 26	y-axis, $4\sqrt{11}$
Minor axis and length	y-axis, 16	x-axis, 8	x-axis, 24	x-axis, $6\sqrt{2}$
Foci	$(\pm\sqrt{57}, 0)$	$(0, \pm\sqrt{65})$	$(0, \pm 5)$	$(0, \pm\sqrt{26})$
Distance between foci	$2\sqrt{57}$	$2\sqrt{65}$	10	$2\sqrt{26}$

5. **a)** **i)** $\frac{x^2}{25}+\frac{y^2}{16}=1$ **ii)** $\frac{x^2}{25}+\frac{y^2}{9}=1$

iii) $\frac{x^2}{25}+\frac{y^2}{49}=1$

b) **i)** Yes, if the y-axis is the major axis.

ii) No, the y-axis has to be the minor axis because the one vertex is on the x-axis.

iii) No, the x-axis has to be minor axis and the y-axis has to be major axis in order to satisfy the given intercept and vertex.

6. **a)** Let $\boldsymbol{P}(x, y)$ be any point on the ellipse, such that $\boldsymbol{PF}_1 + \boldsymbol{PF}_2 = 20$

$$\sqrt{(x+8)^2+y^2}+\sqrt{(x-8)^2+y^2}=20$$

$$\sqrt{(x+8)^2+y^2}=20-\sqrt{(x-8)^2+y^2}$$

Square both sides.

$$(x+8)^2+y^2=$$
$$\left(20-\sqrt{(x-8)^2+y^2}\right)^2$$
$$x^2+16x+64+y^2=$$
$$400-40\sqrt{(x-8)^2+y^2}+(x-8)^2+y^2$$
$$x^2+16x+64+y^2=$$
$$400-40\sqrt{(x-8)^2+y^2}+$$
$$x^2-16x+64+y^2$$
$$16x+64=464-40\sqrt{(x-8)^2+y^2}-16x$$
$$32x-400=-40\sqrt{(x-8)^2+y^2}$$
$$4x-50=-5\sqrt{(x-8)^2+y^2}$$

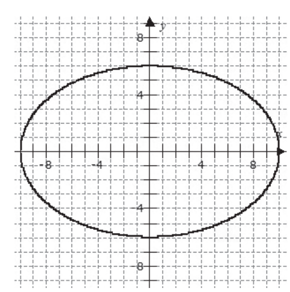
Square both sides.

$$(4x-50)^2=\left(-5\sqrt{(x-8)^2+y^2}\right)^2$$
$$16x^2-400x+2500=$$
$$25(x^2-16x+64+y^2)$$
$$16x^2-400x+2500=$$
$$25x^2-400x+1600+25y^2$$
$$9x^2+25y^2=900$$
$$\frac{x^2}{100}+\frac{y^2}{36}=1$$

b) In $\frac{x^2}{100}+\frac{y^2}{36}=1$, $a^2 = 100$ and $b^2 = 36$, so $a = 10$ and $b = 6$. As well, $c^2 = a^2 - b^2$, so $c^2 = 100 - 36$ or $c^2 = 64$. This gives $c = \pm 8$.

	$\frac{x^2}{100}+\frac{y^2}{36}=1$
centre	(0, 0)
vertices	(−10, 0) and (10, 0)
intercepts	$y = -6$ and $y = 6$
major axis length	$2a = 2(10)$ or 20
minor axis length	$2b = 2(6)$ or 12
foci	$c = \pm 8$
distance between foci	$2(8) = 16$

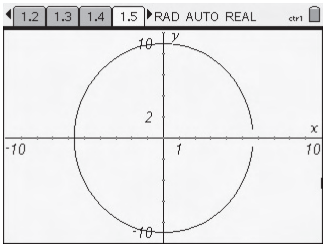
c)



7. **a)** $\frac{x^2}{32}+\frac{y^2}{81}=1$

b)

	$\frac{x^2}{32}+\frac{y^2}{81}=1$
Vertices	$(0, \pm 9)$
Intercepts	$x = \pm 4\sqrt{2}$
Major axis and length	y-axis, 18
Minor axis and length	x-axis, $8\sqrt{2}$
Foci	$(0, \pm 7)$
Distance between foci	14

c) 

8. **a)** Assume that the major axis is on the x-axis, therefore, $a = 5$ and $b = 4$. This gives $a^2 = 25$ and $b^2 = 16$.
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *becomes*
$\frac{x^2}{25} + \frac{y^2}{16} = 1$ *or* $16x^2 + 25y^2 = 400$

b) To find the width of the pool at a point on the major axis that is 2 m from the centre, we substitute $x = 2$ into the equation of the ellipse and solve for y. We will then multiply this value by 2 to find the width.

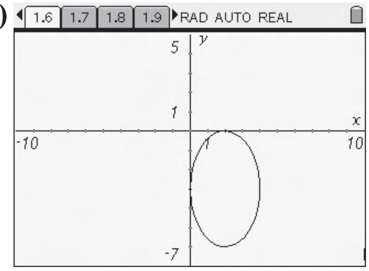
$$16x^2 + 25y^2 = 400$$
$$16(2)^2 + 25y^2 = 400$$
$$16(4) + 25y^2 = 400$$
$$64 + 25y^2 = 400$$
$$25y^2 = 400 - 64$$
$$25y^2 = 336$$
$$y^2 = \frac{336}{25}$$
$$y = \sqrt{\frac{336}{25}}$$
$$y \doteq 3.7$$

9. **a)** $e \doteq 0.69$ **b)** $e \doteq 0.90$ **c)** $e \doteq 0.38$ **d)** $e \doteq 0.77$

10. $\frac{x^2}{36} + \frac{y^2}{20} = 1$

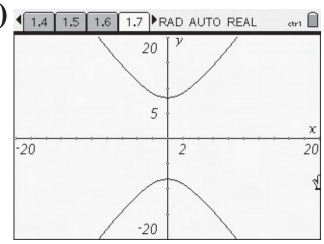
11. **a)**

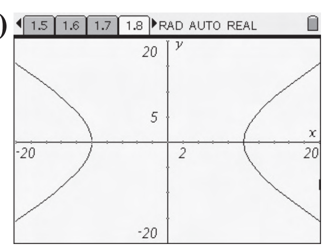
	$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$
Centre	$(2, -3)$
Vertices	$(2, -3 \pm 3)$
Intercepts	$x = 2 \pm 2$
Major axis and length	y-axis, 6
Minor axis and length	x-axis, 4
Foci	$(2, -3 \pm \sqrt{5})$
Distance between foci	$2\sqrt{5}$

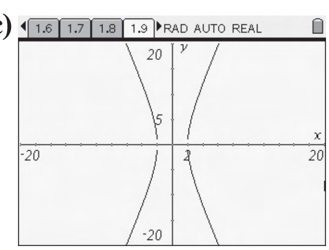
b) 

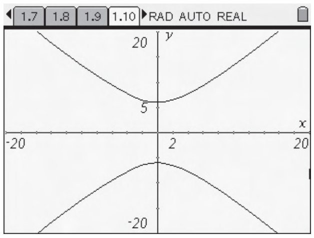
4.3 The Hyperbola

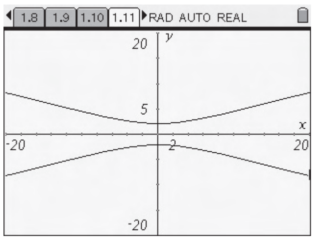
1. **a)** Transverse axis and length: y-axis, 16;
Conjugate axis and length: x-axis, 8
b) Transverse axis and length: x-axis, 20;
Conjugate axis and length: y-axis, 18
c) Transverse axis and length: x-axis, 4;
Conjugate axis and length: y-axis, 14
d) Transverse axis and length: y-axis, 12;
Conjugate axis and length: x-axis, 10
e) Transverse axis and length: y-axis, 4;
Conjugate axis and length: x-axis, 10
f) Transverse axis and length: x-axis, 5;
Conjugate axis and length: y-axis, 10

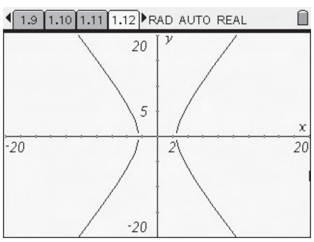
2. **a)** 

b) 

c) 

d) 

e) 

f) 

3. **a)** $\frac{x^2}{49} - \frac{y^2}{16} = -1$ **b)** $\frac{x^2}{36} - \frac{y^2}{9} = -1$
c) $\frac{x^2}{25} - \frac{y^2}{64} = -1$ **d)** $\frac{x^2}{81} - \frac{y^2}{16} = -1$
e) $\frac{x^2}{2.25} - \frac{y^2}{9} = -1$

4. **a)** $\frac{x^2}{100} - \frac{y^2}{9} = 1$ **b)** $\frac{x^2}{64} - \frac{y^2}{36} = 1$
c) $\frac{x^2}{81} - \frac{y^2}{121} = 1$ **d)** $\frac{x^2}{49} - \frac{y^2}{25} = 1$
e) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

5.

	a) $\frac{x^2}{16} - \frac{y^2}{64} = -1$	**b)** $\frac{x^2}{100} - \frac{y^2}{81} = 1$	**c)** $\frac{x^2}{4} - \frac{y^2}{49} = 1$
Centre	(0, 0)	(0, 0)	(0, 0)
Vertices	$(0, \pm 8)$	$(\pm 10, 0)$	$(\pm 2, 0)$
Transverse axis and length	y-axis, 16	x-axis, 20	x-axis, 4
Conjugate axis and length	x-axis, 8	y-axis, 18	y-axis, 14
Foci	$(0, \pm 4\sqrt{5})$	$(\pm\sqrt{181}, 0)$	$(\pm\sqrt{53}, 0)$
Distance between foci	$8\sqrt{5}$	$2\sqrt{181}$	$2\sqrt{53}$
Asymptotes	$y = \pm 2x$	$y = \pm\frac{9}{10}x$	$y = \pm\frac{7}{2}x$

	d) $\frac{x^2}{25} - \frac{y^2}{36} = -1$	**e)** $\frac{x^2}{25} - \frac{y^2}{4} = -1$	**f)** $\frac{x^2}{6.25} - \frac{y^2}{25} = 1$
Centre	(0, 0)	(0, 0)	(0, 0)
Vertices	$(0, \pm 6)$	$(0, \pm 2)$	$(\pm 2.5, 0)$
Transverse axis and length	y-axis, 12	y-axis, 4	x-axis, 5
Conjugate axis and length	x-axis, 10	x-axis, 10	y-axis, 10
Foci	$(0, \pm\sqrt{61})$	$(0, \pm\sqrt{29})$	$(\pm\sqrt{31.25}, 0)$
Distance between foci	$2\sqrt{61}$	$2\sqrt{29}$	$2\sqrt{31.25}$
Asymptotes	$y = \pm\frac{6}{5}x$	$y = \pm\frac{2}{5}x$	$y = \pm 2x$

6.

	$\frac{x^2}{81} - \frac{y^2}{64} = 1$	$\frac{x^2}{4} - \frac{y^2}{36} = -1$	$\frac{x^2}{144} - \frac{y^2}{121} = 1$	$\frac{x^2}{9} - \frac{y^2}{25} = -1$
Vertices	$(\pm 9, 0)$	$(0, \pm 6)$	$(\pm 12, 0)$	$(0, \pm 5)$
Transverse axis and length	x-axis, 18	y-axis, 12	x-axis, 24	y-axis, 10
Conjugate axis and length	y-axis, 16	x-axis, 4	y-axis, 22	x-axis, 6
Foci	$(\pm\sqrt{145}, 0)$	$(0, \pm 2\sqrt{10})$	$(\pm\sqrt{265}, 0)$	$(0, \pm\sqrt{34})$
Distance between foci	$2\sqrt{145}$	$4\sqrt{10}$	$2\sqrt{265}$	$2\sqrt{34}$
Asymptotes	$y = \pm\frac{8}{9}x$	$y = \pm 3x$	$y = \pm\frac{11}{12}x$	$y = \pm\frac{5}{3}x$

7. **a)** Let $P(x, y)$ be any point on the hyperbola, such that $|PF_1 - PF_2| = 10$

$\sqrt{(x + 6)^2 + y^2} - \sqrt{(x - 6)^2 + y^2} = 10$

$-\sqrt{(x - 6)^2 + y^2} = 10 - \sqrt{(x + 6)^2 + y^2}$

Square both sides.

$(x - 6)^2 + y^2 = \left(10 - \sqrt{(x + 6)^2 + y^2}\right)^2$

$x^2 - 12x + 36 + y^2$
$= 100 - 20\sqrt{(x + 6)^2 + y^2}$
$+ (x + 6)^2 + y^2$

$x^2 - 12x + 36 + y^2$
$= 100 - 20\sqrt{(x + 6)^2 + y^2}$
$+ x^2 + 12x + 36 + y^2$

$-12x = 100 - 20\sqrt{(x + 6)^2 + y^2} + 12x$

$20\sqrt{(x + 6)^2 + y^2} = 24x + 100$

$5\sqrt{(x + 6)^2 + y^2} = 6x + 25$

Square both sides.

$25[(x + 6)^2 + y^2] = (6x + 25)^2$
$25(x^2 + 12x + 36 + y^2)$
$= 36x^2 + 300x + 625$
$25x^2 + 300x + 900 + 25y^2$
$= 36x^2 + 300x + 625$
$9x^2 - 25y^2 = 275$
$\frac{9x^2}{275} - \frac{25y^2}{275} = 1$
$\frac{9x^2}{275} - \frac{y^2}{11} = 1$
Here $a^2 = \frac{275}{9}$ so $a = \pm\frac{5\sqrt{11}}{3}$ and $b^2 = 11$ so $b = \pm\sqrt{11}$. As well,

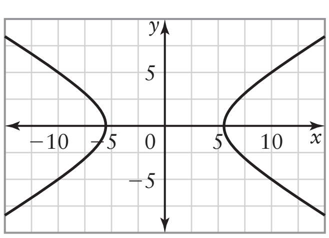
$$c^2 = \frac{275}{9} + 11 = \frac{275}{9} + \frac{99}{9} = \frac{374}{9}$$

$$c = \pm\frac{\sqrt{374}}{3}$$

b) $\frac{9x^2}{275} - \frac{y^2}{11} = 1$

	$\frac{9x^2}{275} - \frac{y^2}{11} = 1$
Centre	(0, 0)
Vertices	$\left(-\frac{5\sqrt{11}}{3}, 0\right)$ and $\left(\frac{5\sqrt{11}}{3}, 0\right)$
Transverse axis and length	x-axis, $\frac{10\sqrt{11}}{3}$
Conjugate axis and length	y-axis, $2\sqrt{11}$
Foci	$\left(-\frac{\sqrt{374}}{3}, 0\right), \left(\frac{\sqrt{374}}{3}, 0\right)$
Distance between foci	$\frac{2\sqrt{374}}{3}$
Asymptotes	$y = \pm\frac{b}{a}x$ so $y = \pm\left(\sqrt{11} \div \frac{5\sqrt{11}}{3}\right)x$ or $y = \pm\frac{3}{5}x$

c)

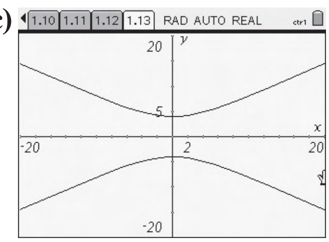


8. a) $\frac{x^2}{33} - \frac{y^2}{16} = -1$

b)

	$\frac{x^2}{33} - \frac{y^2}{16} = -1$
Centre	(0, 0)
Vertices	(0, ±4)
Transverse axis and length	y-axis, 8
Conjugate axis and length	x-axis, $2\sqrt{33}$
Foci	(0, ±7)
Distance between foci	14
Asymptotes	$y = \pm\frac{4}{\sqrt{33}}x$

c)



9. Since the vertex is $(\sqrt{6}, 0)$, then $a = \sqrt{6}$. As well, if the curve passes through the point (9, 5), substitute all of this into the equation.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\frac{(9)^2}{(\sqrt{6})^2} - \frac{(5)^2}{b^2} = 1$$
$$\frac{81}{6} - \frac{25}{b^2} = 1 \quad \text{Multiply by } 6b^2.$$
$$\frac{81(6b^2)}{6} - \frac{25(6b^2)}{b^2} = 1(6b^2)$$
$$81b^2 - 150 = 6b^2$$
$$75b^2 = 150$$
$$b^2 = 2$$

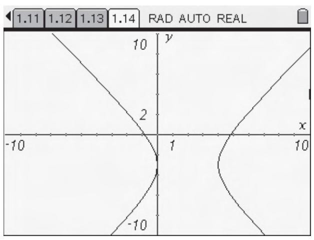
Therefore, the equation is $\frac{x^2}{6} - \frac{y^2}{2} = 1$.

10. a) $e \doteq 1.34$ **b)** $e \doteq 1.05$ **c)** $e \doteq 1.36$ **d)** $e \doteq 1.17$

11. $\frac{x^2}{16} - \frac{y^2}{20} = 1$

12.

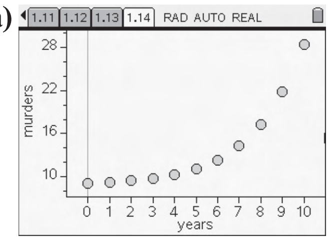
	$\frac{(x-2)^2}{4} - \frac{(y+3)^2}{9} = 1$
Centre	$(2, -3)$
Vertices	$(2 \pm 2, -3)$
Transverse axis and length	x-axis, 4
Conjugate axis and length	y-axis, 6
Foci	$(2 \pm \sqrt{13}, -3)$
Distance between foci	$2\sqrt{13}$
Asymptotes	$y = \pm\frac{3}{2}(x-2) - 3$



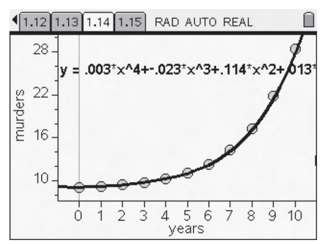
13. $\frac{(x-1)^2}{\frac{90}{7}} - \frac{(y-3)^2}{45} = 1$

Practice Exam

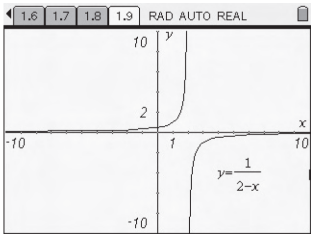
1. **i) a)** 6 **b)** extends from quadrant 2 to 1
c) -1 (order 1), 1 (order 1), -0.867 (order 1), 1.867 (order 1)
ii) a) 8 **b)** extends from quadrant 2 to 1
c) 4 (order 3), 2 (order 2), -11 (order 2), -2 (order 1)
2. Answers may vary. Sample answer: $f(x) = -5(x+1)^2(x-1)$
3. **a)** $f(x) = \frac{1}{20}(x+5)^2(x+3)(x-2)$ shape suggests a polynomial function; extends from quadrant 2 to quadrant 1, therefore polynomial of even degree with positive coefficient; zeros: -5 (order 2), -3 (order 1); 2 (order 1); y-intercept: -7.5
b) $f(x) = \frac{3}{(2x-1)^2}$; shape suggests a reciprocal of a quadratic function; vertical asymptote: $x = \frac{1}{2}$; horizontal asymptote: $y = 0$; y-intercept: 3
c) $f(x) = \frac{x-5}{(2x-1)(x-3)}$; shape suggests a reciprocal of a quadratic function; vertical asymptotes: $x = \frac{1}{2}$, $x = 3$; horizontal asymptote: $y = 0$; y-intercept: $-\frac{5}{3}$; minimum of the parabolic part (middle branch) is at point (2, 1)
4. $-\sqrt{2} \le x \le \sqrt{2}$ or $x \le -3$ or $x \ge 3$
5. **a)**



b) quartic **c) i)** fourth (degree is 4) **ii)** 0.07 (fourth difference is a constant)
iii) 0.003 (4^{th} difference = leading coefficient $\times 4 \times 3 \times 2 \times 1$) **d)** $f(x) = 0.003x^4 - 0.023x^3 + 0.114x^2 + 0.013x + 9.11$



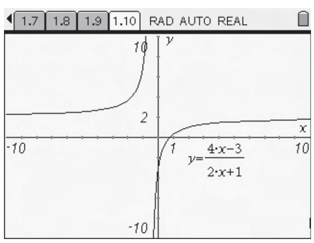
e) find the zeros **f)** never
6. **a)** \$468.20 **b)** 1087.27 **c)** 1076.87; total value of investments, T, at the point when the interest rate, x, reaches the average annual rate (6.25%) plus one
7. 37.8 cm $\times$ 52.8 cm
8. **a)** $x = 4$, $x = \frac{-4 \pm 2\sqrt{7}}{3}$
b) $x = 2$, $x = -1.582$, $x = 0.277$, $x = 1.306$
9. **i) a)** no x-intercept, y-intercept: $\frac{1}{2}$
b) vertical asymptote: $x = 2$, horizontal asymptote: $y = 0$, no oblique asymptote
c) domain: $\{x \in \mathbb{R}, x \neq 2\}$, range: $\{y \in \mathbb{R}, y \neq 0\}$

d) 

ii) a) x-intercept: $\frac{3}{4}$, y-intercept: -3

b) vertical asymptote: $x = -\frac{1}{2}$, horizontal asymptote: $y = 2$, no oblique asymptote

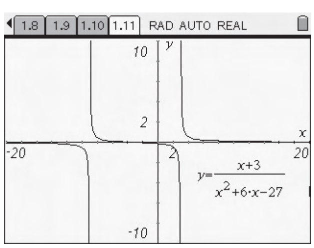
c) domain: $\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}$, range: $\{y \in \mathbb{R}, y \neq 2\}$

d) 

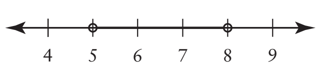
iii) a) x-intercept: -3, y-intercept: $-\frac{1}{9}$

b) vertical asymptotes: $x = -9$, $x = 3$, horizontal asymptote: $y = 0$, no oblique asymptote

c) domain: $\{x \in \mathbb{R}, x \neq -9, x \neq 3\}$, range: $\{y \in \mathbb{R}, y \neq 0\}$

d) 

10. a) $x \leq \frac{7}{2}$ or $x > 4$ 

b) $5 < x < 8$ 

11. $r \cong 15.9$ cm **12.** 28.9 cm²

13.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
a) $\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
b) $\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2	$-\frac{1}{\sqrt{3}}$
c) $\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
d) π	0	-1	0	Undefined	-1	Undefined

14. a) 0.959 4 (apply addition formula for cosine)

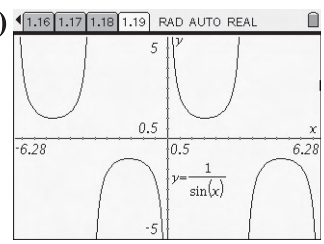
b) 0.989 8 (apply cofunction identity)

15. $f(x) = 3\sin\left[\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right]$

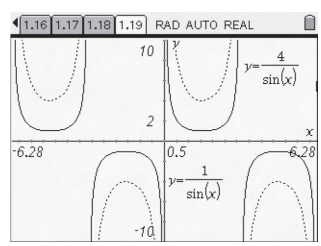
16. Answers may vary.

17. a) $-\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{2}$ **c)** 0 **d)** $-\frac{1}{2}$

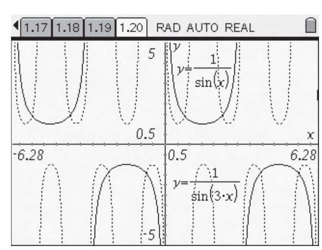
18. $f(x) = -\frac{1}{2}\cos\left[4\left(x + \frac{\pi}{6}\right)\right] + 3$

19. a) 

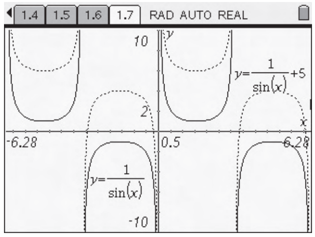
b) i) amplitude changes to 4



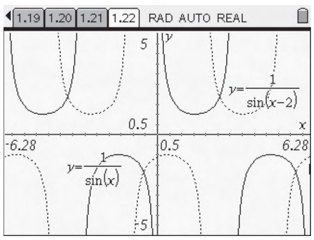
ii) period changes to $\frac{2\pi}{3}$



iii) vertical translation of 5 units up

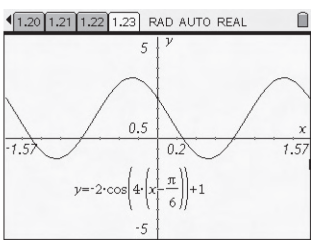


iv) phase shift of 2 units to the right



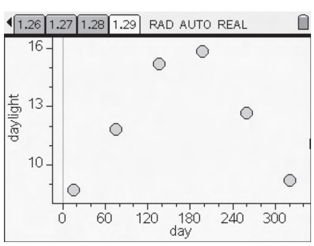
20. a) 2 **b)** $\frac{\pi}{2}$ **c)** $\frac{\pi}{6}$ to the right **d)** 1 unit up

e)



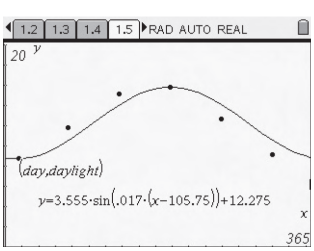
21. a) $\frac{3\pi}{2}$ **b)** $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ **c)** 0.564, 5.075

22. a)



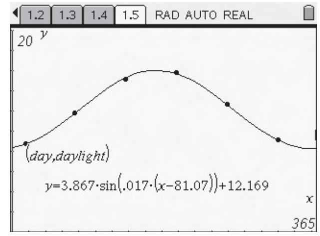
b) Answers may vary. For example:
$y = 3.555 \sin[0.017(x - 105.75)] + 12.275$

c)



The model does not fit well. It is shifted too much to the right; therefore the phase shift needs to be adjusted.

d) $y = 3.867 \sin[0.017(x - 81.070)] + 12.169$



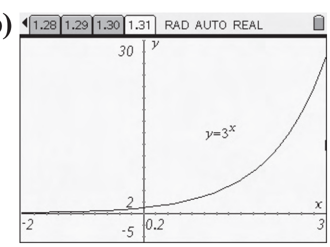
The regression equation is a much better fit than the model from part b)

e) 30%

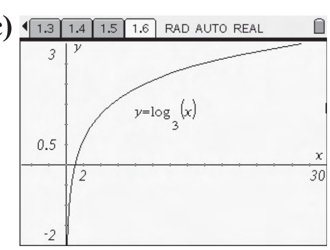
23. a)

x	$y = 3^x$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
$\frac{1}{2}$	$\sqrt{3}$
1	3
2	9
3	27

b)



c)



24. a) 3 **b)** $\frac{4}{5}$ **c)** 2 **d)** 2 **e)** 1
25. a) 2.77 **b)** 3
26. a) \$35 000 **b)** approximately 3.1 years
27. a) 88.4 dB **b)** 3.98×10^{10} W/m^2

28. a) $\frac{19}{4}$ b) 1 **c)** -1

29. a) 1.40 **b)** 3.57 **c)** 2.72 **d)** 3.83 **e)** -3.58
f) 0.45

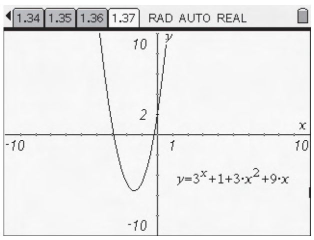
30. a) 191 h **b)** 2.7 h

31. a) $\log(x - 4)$, $x > 4$ **b)** $\log(64x^6)$, $x > 0$
c) $\log\left(\frac{2x^2 + 3xy + 4y^2}{x^4}\right)$, $x > 0$,
$2x^2 + 3xy + 4y^2 > 0$

32. a) 4 **b)** 21 **c)** 3

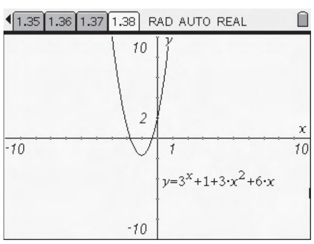
33. a) $P(t) = 48\,000(1.09)^t$ **b)** 12.7 years
c) approximately 3.9 years

34. a) $y = 3^x + 1 + 3x^2 + 9x$



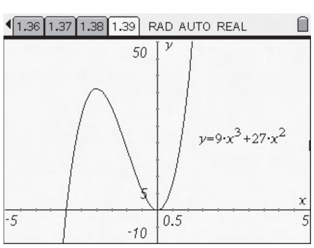
domain: $\{x \in \mathbb{R}\}$,
range: $\{y \in \mathbb{R}, y \geq -5.561\}$

b) $y = 3^x + 1 + 3x^2 + 6x$



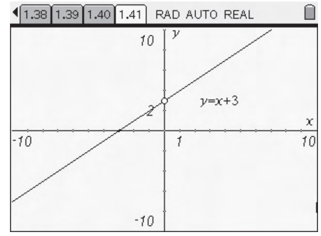
domain: $\{x \in \mathbb{R}\}$,
range: $\{y \in \mathbb{R}, y \geq -1.677\}$

c) $y = 9x^3 + 27x^2$



domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

d) $y = x + 3$

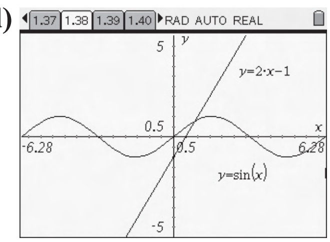


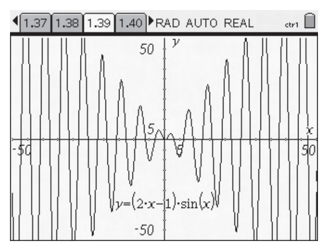
domain: $\{x \in \mathbb{R}, x \neq 0\}$,
range: $\{y \in \mathbb{R}, y \neq 3\}$

35. a) $2.295t^2 + 66.6t + 153$
b) 1668 connections / week

36. a) straight line with slope 2, x-intercept $\frac{1}{2}$, and y-intercept -1; neither **b)** continuous wave with constant frequency and amplitude; odd **c)** continuous wave with constant frequency and variable amplitude

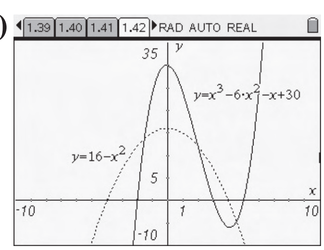
d)





e) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

37. a)

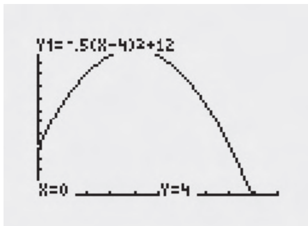


b) $\frac{3 - \sqrt{37}}{2} < x < 2$ or $x > \frac{3 + \sqrt{37}}{2}$

38. a) $f(g(x)) = \frac{1}{x}\sqrt{4 - 6x^2}$, $-\frac{2}{\sqrt{6}} < x < \frac{2}{\sqrt{6}}$
domain: $\left\{x \in \mathbb{R}, -\frac{2}{\sqrt{6}} < x < \frac{2}{\sqrt{6}}\right\}$,
range: $\{y \in \mathbb{R}, y > 0\}$

b) $g(f(x)) = \frac{1}{x - 3}$, $x > 3$
domain: $\{x \in \mathbb{R}, x > 3\}$,
range: $\{y \in \mathbb{R}, y > 0\}$

39. a) 19 600

b) 

Initially a poor team, winning only 4 of the 14 games in the first season, then becomes a good team after 4 years, then becomes poor again.

c) $0 \leq t < 9$ starts in 2009, number of wins cannot be less than 0 **d)** No. $N(W(4)) = 48\ 400$ **e)** It will make team better right away, so would move vertex to the left. **f)** It will make team better in 4 years, so would move vertex up.