

McGraw-Hill Ryerson

Advanced Functions 12

Study Guide and University Handbook

AUTHORS

Antonietta Lenjosek

B.Sc., B.Ed.

Ottawa Catholic School Board

Paula Thiessen

B.Math, B.Ed.

District School Board of Niagara

Laurissa Werhun

B.Sc., B.Ed., M.A.

Toronto District School Board

REVIEWERS

Kirsten Boucher

Durham District School Board

Dr. Steven J. Desjardins

Department of Mathematics and Statistics
University of Ottawa

Dr. Gilles Lamothe

Department of Mathematics and Statistics
University of Ottawa

Carol Miron

Toronto District School board



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COPYEDITOR: Dianne Brassolotto

ANSWER PREPARER: Daniela Spiroska, Ken Stewart

ANSWER CHECKER: Maria Stewart

MANAGER, EDITORIAL SERVICES: Crystal Shortt

SUPERVISING EDITOR: Jaime Smith

EDITORIAL ASSISTANT: Erin Hartley

MANAGER, PRODUCTION SERVICES: Yolanda Pigden

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Contents

Overview.....	IX
Formulas.....	X

Chapter 1 Polynomial Functions

1.1 Power Functions	1
• Polynomial Expressions	
• Graphs of Power Functions	
• Recognize Polynomial Functions	
1.2 Characteristics of Polynomial Functions	6
• Key Features of Graphs of Polynomial Functions	
• Relationship Between Finite Differences and the Equation of a Polynomial Function	
1.3 Equations and Graphs of Polynomial Functions	11
• Analyse Graphs of Polynomial Functions	
• Analyse Equations to Sketch Graphs of Polynomial Functions	
• Symmetry	
1.4 Transformations.....	14
• Roles of a , k , d , and c in Polynomial Functions	
• Apply Transformations to Sketch a Graph	
• Describe Transformations From an Equation	
• Determine an Equation Given the Graph of a Transformed Function	
1.5 Slopes of Secants and Average Rate of Change.....	18
• Connection Between Average Rate of Change and Slope	
• Calculate and Interpret Average Rates of Change From a Graph	
• Calculate and Interpret Average Rates of Change From a Table of Values	
• Calculate and Interpret Average Rates of Change From an Equation	
1.6 Slopes of Tangents and Instantaneous Rate of Change	23
• Connection Between Slopes of Secants, Slope of a Tangent, and Instantaneous Rate of Change	
• Estimate Instantaneous Rate of Change From a Graph	
• Estimate Instantaneous Rate of Change From a Table of Values	
• Estimate Instantaneous Rate of Change From an Equation	
Challenge Questions	28
Chapter 1 Checklist	29

Chapter 2 Polynomial Equations and Inequalities

2.1 The Remainder Theorem.....	30
• Divide a Polynomial by a Binomial	
• Apply and Verify the Remainder Theorem	
2.2 The Factor Theorem	34
• Use the Factor Theorem to Find Factors of a Polynomial	
• Strategies to Factor a Polynomial	
• Combine Factor Theorem and Factoring by Grouping	
• Integral Zero Theorem	
• Rational Zero Theorem	

2.3 Polynomial Equations	37
• Factoring Polynomial Equations	
• Use the Factor Theorem to Solve Polynomial Equations	
• Determine the Roots of a Polynomial Equation	
2.4 Families of Polynomial Functions.....	40
• Represent a Family of Functions Algebraically	
• Families of Functions	
• Quartic Functions	
2.5 Solve Inequalities Using Technology.....	44
• Solve Polynomial Inequalities Graphically	
• Solve Polynomial Inequalities Numerically	
• Solve Problems Involving Inequalities	
2.6 Solve Factorable Polynomial Inequalities Algebraically	49
• Solve Linear Inequalities	
• Solve Polynomial Inequalities Algebraically	
• Solve Problems using Factorable Polynomial Inequalities	
Challenge Questions	52
Chapter 2 Checklist	53
Chapter 3 Rational Functions	
3.1 Reciprocal of a Linear Function	54
• Domain, Range, and Asymptotes	
• Intercepts	
• Rate of Change	
3.2 Reciprocal of a Quadratic Function.....	58
• Domain, Range, and Asymptotes	
• Rate of Change	
• Key Features of a Function	
3.3 Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$	62
• Key Features of Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$	
3.4 Solve Rational Equations and Inequalities	66
• Solve Rational Equations Algebraically	
• Solve Rational Equations Using Technology	
• Solve a Simple Rational Inequality	
• Solve a Quadratic Over a Quadratic Rational Inequality	
3.5 Making Connections with Rational Functions and Equations.....	70
• Solve Problems Using Rational Functions and Equations	
Challenge Questions	
Chapter 3 Checklist	76

Chapter 4 Trigonometry

4.1 Radian Measure	77
• Convert Degree Measure to Radian Measure	
• Convert Radian Measure to Degree Measure	
• Arc Length for a Given Angle	
• Angular Velocity of a Rotating Object	
4.2 Trigonometric Ratios and Special Angles	81
• Apply Trigonometric Ratios for Special Angles	
• Trigonometric Ratios for a Multiple of a Special Angle	
4.3 Equivalent Trigonometric Expressions	85
• Use Equivalent Trigonometric Expressions to Evaluate Primary Trigonometric Expressions	
• Use an Equivalent Trigonometric Expression to Evaluate a Reciprocal Trigonometric Expression	
• Use Technology to Verify Equivalent Trigonometric Expressions	
• Trigonometric Identities	
4.4 Compound Angle Formulas	89
• Addition and Subtraction Formulas for Cosine	
• Addition and Subtraction Formulas for Sine	
• Compound Angle Formulas	
4.5 Prove Trigonometric Identities	93
• Basic Trigonometric Identities	
• Provide Formulas and Identities	
Challenge Questions	
Chapter 4 Checklist	98

Chapter 5 Trigonometric Functions

5.1 Graphs of Sine, Cosine, and Tangent Functions	99
• Graphs of the Form $y = \sin x + c$	
• Graphs of the Form $y = a \sin x$	
• Graphs of the Form $y = \sin(x - d)$	
• Graphs of the Form $y = \sin kx$	
5.2 Graphs of Reciprocal Trigonometric Functions	102
• Determine Values on the Graph of $y = \csc x$	
• Determine Values on the Graph of $y = \cot x$	
• Reciprocal and Inverse Notation	
5.3 Sinusoidal Functions of the Form $f(x) = a \sin [k(x - d)] + c$ and $f(x) = a \cos [k(x - d)] + c$	105
• Transform a Cosine Function	
• Transform a Sine Function	
5.4 Solve Trigonometric Equations	109
• Solve a Quadratic Trigonometric Equation	
• Solve a Quadratic Trigonometric Equation by Factoring	
• Solve an Equation Involving Reciprocal Trigonometric Ratios	

5.5 Making Connections and Instantaneous Rate of Change	111
• Average and Instantaneous Rates of Change for a Sinusoidal Function	
• Solve Problems Using Instantaneous Rate of Change	
Challenge Questions	114
Chapter 5 Checklist	116
Chapter 6 Exponential and Logarithmic Functions	
6.1 The Exponential Function and Its Inverse	117
• Features of Exponential Functions	
• Write Equations to Fit Data	
• Graph Inverse Functions	
6.2 Logarithms.....	122
• Logarithmic Function	
• Write Exponential Equations in Logarithmic Form	
• Evaluate Logarithms	
• Write Logarithmic Equations in Exponential Form	
• Approximate Logarithms	
6.3 Transformations of Logarithmic Functions	125
• Translations	
• Stretches, Reflections, and Translations	
• Transformations	
6.4 Power Law of Logarithms.....	129
• The Power Law of Logarithms	
• Solve Problems Using Logarithms	
• Evaluate Logarithms	
• Graph Logarithmic Functions	
6.5 Making Connections: Logarithmic Scales in the Physical Sciences.....	131
• Solving Problems Using Logarithmic Scales	
Challenge Questions	133
Chapter 6 Checklist	135
Chapter 7 Tools and Strategies for Solving Exponential and Logarithmic Equations	
7.1 Equivalent Forms of Exponential Equations	136
• Model Exponential Growth	
• Change the Base of Powers	
• Solve Equations by Changing the Base	
7.2 Techniques for Solving Exponential Equations	139
• Half-Life	
• Powers With Different Bases	
• Apply the Quadratic Formula	
• Extraneous Roots	
7.3 Product and Quotient Laws of Logarithms.....	143
• Product Law of Logarithms	
• Quotient Law of Logarithms	
• Simplify Algebraic Expressions	
7.4 Techniques for Solving Logarithmic Equations.....	147
• Solve Equations Using Logarithms	

7.5 Making Connections: Mathematical Modelling With Exponential and Logarithmic Equations.....	150
• Select and Apply Mathematical Models	
• Solve Problems Using Exponential and Logarithmic Equations	
Challenge Questions	154
Chapter 7 Checklist	156
Chapter 8 Combining Functions	
8.1 Sums and Differences of Functions.....	157
• The Superposition Principle	
• The Profit Function	
8.2 Products and Quotients of Functions	162
• Solve Problems Using Products and Quotients of Functions	
• Combined Functions	
8.3 Composite Functions	166
• Determine Equations for Composite Functions	
• Evaluate Composite Functions	
8.4 Inequalities of Combined Functions.....	170
• Techniques for Illustrating Inequalities	
• Solve Inequalities	
• Solve Problems Using Inequalities	
8.5 Making Connections: Modelling With Combined Functions.....	173
• Solve Problems Using Combined Functions	
• Develop Models using Combined Functions	
Challenge Questions	177
Chapter 8 Checklist	179
University Preparation	
UP 1 Extending Algebraic Skills.....	180
UP 1.1 Factoring Complex Equations	
UP 1.2 Techniques for Solving Complex Equations	
UP 2 Absolute Value.....	184
UP 2.1 Solving Equations Involving Absolute Value	
UP 2.2 Solving Inequalities Involving Absolute Value	
UP 3 Matrices	189
UP 3.1 Introduction to Matrices	
UP 3.2 Determinants	
UP 4 Conics.....	196
UP 4.1 Introduction	
UP 4.2 The Ellipse	
UP 4.3 The Hyperbola	
Practice Exam.....	206
Answers	211

Overview

Advanced Functions play an important role in many activities, from business and economics to the social, medical, and physical sciences. *McGraw-Hill Ryerson Advanced Functions 12 Study Guide and University Handbook* is designed for students planning to qualify for college or university. The study guide is designed to either complement the *McGraw-Hill Ryerson Advanced Functions 12* student book, or to stand alone as a thorough review of the course.

Study Guide Organization

- Chapter 1 introduces polynomial functions and the process of using secants and tangents to analyse average and instantaneous rates of change. Characteristics of polynomial functions are explored, including key features of their graphs and the relationship between finite differences and equations of polynomial functions. The chapter also examines the connection between equations and graphs of polynomial functions, along with transformations.
- In Chapter 2 equation-solving skills and graphing skills are combined to solve polynomial equations and inequalities. The relationship between the Remainder Theorem and the Factor Theorem is identified. Techniques for factoring polynomial functions of degree greater than two are examined and also applied to determine the roots of polynomial equations. Families of polynomial functions are analysed, including representing families of functions algebraically and exploring quartic functions. Finally, inequalities are solved graphically using technology, and algebraic methods for solving factorable polynomial inequalities are demonstrated.
- Chapter 3 focuses on analysing properties of those rational functions created by taking the reciprocal of linear functions and quadratic functions. The equations and key features of the graphs of these rational functions are analysed, including finding their vertical and horizontal asymptotes. Different forms of rational functions are explored and, rational equations and inequalities are solved using a variety of methods (e.g., algebraically, using technology). Connections between real-world situations and rational functions are explored in the last section of the chapter through problem solving.
- Chapter 4 extends concepts of trigonometry by defining trigonometric ratios (both primary and reciprocal ratios) of any angle using radians for angle measures. Methods to convert between radian measure and degree measure are defined. The connection between trigonometric ratios and special angles is identified, along with equivalent trigonometric expressions. Compound angle formulas, including addition and subtraction formulas for cosine and sine, are examined and finally, the chapter ends with proofs of trigonometric identities.
- Chapter 5 applies the concepts from chapter 4 to analyse trigonometric functions. The graphs of the sine, cosine, and tangent functions, along with graphs of the reciprocal trigonometric functions are analysed and their key features are identified. Transformations of the graphs of the cosine and sine functions are examined. Trigonometric equations are solved by combining factoring techniques with knowledge of trigonometric ratios of special angles. The chapter ends with applications of instantaneous rates of change to problems involving trigonometry.
- Chapter 6 introduces two new functions, the exponential and logarithmic functions. The chapter begins by studying the exponential function and its inverse, including writing equations to fit data and graphing inverse functions. Then, logarithms and transformations of logarithmic functions are explored. The power law of logarithms is examined, including solving problems, evaluating logarithms, and graphing logarithmic functions. Finally, problems and applications connecting logarithms and the physical sciences, are solved.
- Chapter 7 builds on the concepts related to exponents and logarithms from chapter 6. Equivalent forms of exponential equations are identified, and techniques to solve exponential equations are investigated and applied. The Product and Quotient Laws of logarithms are developed and techniques to solve logarithmic equations are demonstrated. The final section of this chapter examines mathematical modelling with exponential and logarithmic equations, including solving problems using these equations.

- Chapter 8 integrates concepts from the seven preceding chapters to examine combined functions. The key features of the graphs and equations of the sum, difference, product, and quotient of different functions are examined. Composite functions are evaluated, and solutions to inequalities of combined functions are found. The chapter ends with an exploration of modelling with combined functions, including solving problems and developing models.
- In the University Preparation section, a collection of important Advanced Functions topics are explored. The section examines conics, including the ellipse and hyperbola. The connection between the graphs and equations of the ellipse and hyperbola are identified, as well as the development of the equations based on given information. Another topic is solving absolute value equations and inequalities. Matrices are also introduced and the operations of addition, subtraction, scalar multiplication and matrix multiplication are studied. The final topic of this section deals with extending important algebraic skills, including factoring and solving complex equations.

Study Guide Features

- Each section begins with a page of Key Concepts that summarize the concepts needed to complete the exercises.
- Exercises are organized into three sections: A (practice), B (connect and apply), and C (extend and challenge).
- Each chapter includes additional challenge questions that cover the concepts in the chapter, as well as extend your thinking and combine concepts from previous chapters.
- Selected questions in each section are marked by a star that indicates that full worked solutions are provided at the back of the book. Answers to all other questions are also provided.
- Each chapter ends with a checklist of concepts that specify what you should be able to do by the end of the chapter.
- A practice exam at the end of the study guide gives you the opportunity to determine if you are ready for the final examination.

Formulas

SYMBOLS

\mathbb{R}	real numbers	Greek Lower Case Letters	
\mathbb{N}	natural numbers	α	alpha (a)
\mathbb{Z}	integers	β	beta (b)
∞	infinity	γ	gamma (g)
\in	belongs to	δ	delta (d) lower case
$[a,b]$	$a \leq x \leq b$ (closed interval)	Δ	delta (d) upper case
(a,b)	$a < x < b$ (open interval)	λ	lamda (l)
$\overrightarrow{AB}, \vec{u}$	vector	ρ	rho (r)
$ \vec{v} $	magnitude of a vector	θ	theta (th)
$\vec{u} \cdot \vec{v}$	dot product of vectors	τ	tau (t)
$\vec{u} \times \vec{v}$	cross product of vectors	π	pi (p)
$\frac{d}{dx}$	derivative operator	ω	omega (o)