Advanced Functions 12 Practice Exam

1. For each of these polynomial functions,

i) $f(x) = x^6 - 2x^5 - x^4 + 3x^3 - x^2 - x + 1$

ii) $g(x) = 5(x - 4)^3 (x - 2)(x + 11)^2 (x^2 - 4)$

a) determine the degree

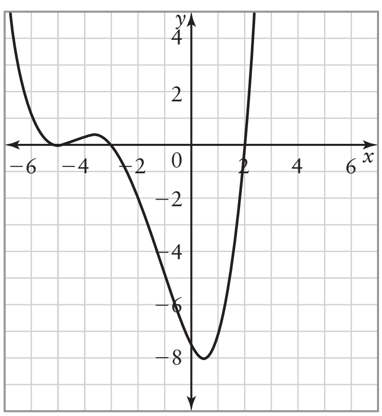
b) describe the end behaviour

c) list the real zeros of the polynomial and the order of each zero

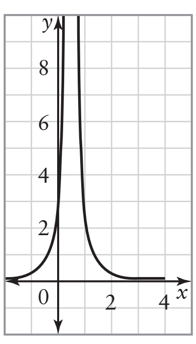
2. Find a polynomial of degree 3 such that $f(-1) = 0$, $f(1) = 0$, and $f(0) = 5$.

3. Determine an equation for each of the following functions. Give reasons for your choices.

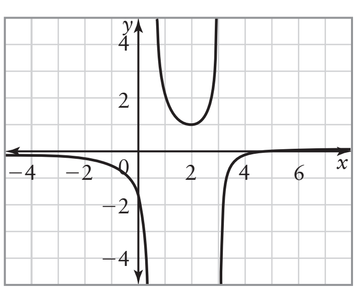
a)



b)



c)



4. Solve the inequality $x^4 - 11x^2 + 18 \geq 0$.

5. Use Technology A national security review studied the number of murders that took place over a ten-year period.

Time in Years, *t*	Number of Murders, *n* (per 100 000 population)
0	9.110 4
1	9.217 2
2	9.456 4
3	9.799 8
4	10.294
5	11.058
6	12.289
7	14.225
8	17.301
9	21.844
10	28.376

a) Graph the data on a graphing calculator.

b) Use the graph to predict the degree of the polynomial function that models this data.

c) i) Determine which finite differences will be constant for this polynomial function.

ii) What is the value of the constant finite differences?

iii) What is the value of the leading coefficient? Explain how you know.

d) Use the regression feature on the graphing calculator to determine an equation for a function that models this situation.

e) How can you use the equation to determine the number of times the function crosses the x-axis?

f) If the study began in 1990, when was the rate 8 people per 100 000?

6. The value of Laura's college fund is the sum of the three investments that she has made over the last five years. To estimate the total value of those investments, Laura used the equation $T(x) = 100x^5 + 56x^3 + 250x$, where T is the total current value of the stocks, and x represents the interest rate plus 1 ($r + 1$).

   a) What is the current value of her investments at an average annual rate of 6.25%?

   b) Determine the average rate of change of the total as the interest rate increases from 6.25% to 7%.

   c) What is the instantaneous rate of change at the average annual rate? What does this value tell Laura?

7. The scenery for a high school drama includes a house with a painted window. Special gloss paint covers the area representing the window to make it look like real glass. If the expensive gloss paint covers only 2000 cm^2 and the window must be 15 cm taller than it is wide, how large should the scenery painters make the window?

8. Use the Integral or Rational Root Theorem to determine a zero of each of these polynomials. Then, solve for the other real roots.

   a) $3x^3 - 4x^2 - 36x + 16 = 0$

   b) $21x^4 - 42x^3 - 45x^2 + 102x - 24 = 0$

9. For each of the following functions,

   i) $y = \dfrac{1}{2 - x}$

   ii) $y = \dfrac{4x - 3}{2x + 1}$

   iii) $f(x) = \dfrac{x + 3}{x^2 + 6x - 27}$

   a) determine the x- and y-intercepts

   b) determine the equations of any vertical, horizontal, or oblique asymptotes

   c) state the domain and range of the function

   d) sketch a graph of the function

10. Solve each of the following inequalities and illustrate each solution on a number line.

   a) $\dfrac{1}{x - 4} \geq -2$

   b) $\dfrac{1}{x^2 - 13x + 40} \leq 0$

11. An arc of 10-cm length has a central angle of $\frac{\pi}{5}$ radians. Determine the radius of the circle.

12. Find the area of a sector of a circle that has a central angle of $\frac{3\pi}{8}$ and a radius of 7 cm. Round your answer to the nearest tenth.

13. Determine exact values of the six trigonometric ratios for each of the following angles:

   a) $\dfrac{11\pi}{6}$

   b) $\dfrac{2\pi}{3}$

   c) $\dfrac{5\pi}{4}$

   d) π

14. Given that $\sin \frac{5\pi}{11} \doteq 0.9898$, determine the following, to four decimal places, without using a calculator. Justify your answers.

   a) $\cos \dfrac{21\pi}{11}$

   b) $\cos \dfrac{\pi}{22}$

15. Write an equation for a sine function with an amplitude of 3, a period of 4π, and a phase shift of $+\frac{\pi}{3}$.

16. Prove the following identities:

a) $\cos^2 x + 2\sin^2 x - 1 = \sin^2 x$

b) $\sin(x + y) = \dfrac{\tan x + \tan y}{\sec x \sec y}$

c) $\sec^2 x - 1 = \tan^2 x$

17. If θ is an angle in the second quadrant where $\sec\theta = -\dfrac{2}{\sqrt{3}}$, the angle ϕ is in the first quadrant, and $\tan\phi = \dfrac{1}{\sqrt{3}}$, determine an exact value for each of the following:

a) $\sin 2\theta$

b) $\cos 2\phi$

c) $\sin(\theta + \phi)$

d) $\cos(\theta - \phi)$

18. Write an equation for a cosine function, reflected in the x axis, with an amplitude of $\frac{1}{2}$, a period of $\frac{\pi}{2}$, a phase shift of $\frac{-\pi}{6}$, and a horizontal translation of $+3$.

19. a) Sketch a graph of the function $y = \csc x$.

b) Predict the shape of each function, and then check by graphing.

i) $y = 4\csc x$

ii) $y = \csc 3x$

iii) $y = \csc x + 5$

iv) $y = \csc(x - 2)$

20. Consider the function $y = -2\cos\left[4\left(x - \frac{\pi}{6}\right)\right] + 1$.

a) What is the amplitude?

b) What is the period?

c) Describe the phase shift.

d) Describe the vertical translation.

e) Sketch a graph of the function over two cycles.

21. Solve the following equations on the interval $x \in [0, 2\pi]$.

a) $\sin^2 x - \sin x = 2$

b) $2\cos^2 x - 3\cos x + 1 = 0$

c) $2\sec x + \tan x = 3$

22. The following table shows the number of hours of daylight per day on various days of the year in Thunder Bay, Ontario.

Day of the Year	Hours of Daylight
16	8.72
75	11.82
136	15.18
197	15.83
259	12.68
320	9.18

a) Make a scatter plot of the data.

b) Write a sine function to model the data.

c) Graph your model on the same set of axes as in part a). Comment on the fit.

d) Check your model using a sinusoidal regression. How does the regression equation compare with the model?

e) Find the percent of days in the year with less than 10 h of daylight. Round your answer to the nearest percent.

23. a) Copy the following table and complete the values for the function $y = 3^x$.

x	y
−2	
−1	
0	
$\frac{1}{2}$	
1	
2	
3	

b) Graph the function.

c) Sketch a graph of the inverse of $y = 3^x$.

24. Evaluate each logarithm.

a) $\log_2 2^3$

b) $\log_2 \sqrt[5]{16}$

c) $\log 40 + \log 2.5$

d) $2 \log_3 12 - 2 \log_3 4$

e) $\log_5 \sqrt{225} - \log_5 \sqrt{9}$

25. Solve for x. Round each answer to two decimal places, if necessary.

a) $3^x = 21$

b) $x = \log_3 27$

26. The value of a vehicle depreciates by 20% each year. The value, V, in dollars, as a function of time, t, in years, can be modelled by the function $V(t) = 35\,000(0.80)^t$.

a) What is the initial value of the vehicle?

b) How long, to the nearest tenth of a year, will it take for the vehicle to depreciate to half its initial value?

27. The decibel rating of a sound is $\beta = \log\frac{I}{I_0}$, where I is the intensity, in watts per square metre, and I_0 is the minimum intensity of sound audible to the average person, or 1.0×10^{-12} W/m^2.

a) Find the decibel rating of a pneumatic drill with an intensity rating of 6.98×10^{-4} W/m^2.

b) The sound of a student whispering in class has a decimal rating of 26 dB. What is the intensity of the whisper?

28. Use Technology Solve. Check your answers using graphing technology.

a) $64^{2x-3} = 16^{x+5}$

b) $9^{4x-1} = 27^{2x}$

c) $\frac{27^x}{9^{2x-1}} = 3^{x+4}$

29. Solve for x. Round each answer to two decimal places.

a) $4^x = 7$

b) $7.3^x = 1200$

c) $15^{x-2} = 7$

d) $2^{-x+7} = 9$

e) $375^{3x+5} = 25^{x-7}$

f) $5^{3x} = 0.75^{x-8}$

30. Various radioactive substances are used in medical tests.

a) In thyroid deficiency tests, a 50-mg sample of radioactive material decays to 38.5 mg in 72 h. What is the half-life of the material to the nearest hour?

b) In bone density tests, a 200-mg sample of radioactive material decays to 120 mg in 2 h. Determine the half-life of the substance to the nearest tenth of an hour.

31. Simplify. State any restrictions on the variable(s).

a) $\log(x^2 - x - 12) - \log(x + 3)$

b) $3 \log 4x + \log x^5 - 2 \log x$

c) $\log\left(2 + \frac{3y}{x} + \frac{4y^2}{x^2}\right) - 2 \log x$

32. Use Technology Solve. Identify and reject any extraneous roots. Check your solutions using graphing technology.

a) $\log(x + 1) + \log(x - 2) = 1$

b) $\log(x^2 - 1) - \log(x + 1) = 1 + \log 2$

c) $2 \log(x + 1) - \log(5x + 1) + \log(x - 1) = \log 2$

33. The population of a city is 48 000 and is increasing by 9% annually.

a) Write an equation to show the population as a function of time, t, in years.

b) How long, to the nearest tenth of a year, will it take for the population to triple?

c) After how many years will the population reach 67 000?

34. Let $f(x) = 3^x + 1$, $g(x) = 3x^2 + 9x$, and $h(x) = 3x$. Determine an algebraic and a graphical model for each function. Identify the domain and range of each.

a) $y = f(x) + g(x)$

b) $y = f(x) + g(x) - h(x)$

c) $y = g(x)h(x)$

d) $y = \dfrac{g(x)}{h(x)}$

35. An Internet service provider has modelled its projected number of business subscribers as $B(t) = 85t + 3700$, and the projected average number of Internet connections per business subscriber as $C(t) = 0.009t^2 + 1.8$, where t is the time, in weeks, over the next year. The projected total number of business Internet connections is then $N(t) = B(t) \times C(t)$.

a) Determine the growth rate of the number of business connections after t weeks.

b) Determine the growth rate of the number of business connections after 15 weeks.

36. Consider $f(x) = 2x - 1$ and $g(x) = \sin x$, where x is in radians.

a) Describe and sketch a graph of $f(x)$. Is this function even, odd, or neither?

b) Describe and sketch a graph of $g(x)$. Is this function even, odd, or neither?

c) Predict the shape of $y = f(x)g(x)$. Sketch the graph of your prediction.

d) **Use Technology** Check your work by graphing the three functions in parts a) to c) using graphing technology.

e) Give the domain and range of $y = f(x)g(x)$.

37. Let $f(x) = x^3 - 6x^2 - x + 30$ and $g(x) = 16 - x^2$.

a) Graph these functions on the same set of axes.

b) Identify, by inspecting the graphs, the interval or intervals for which $f(x) > g(x)$.

c) Check your answer to part b) using another method.

38. Let $f(x) = \sqrt{2x - 6}$ and $g(x) = \dfrac{2}{x^2}$. Write a simplified algebraic model for each composite function. State the domain and range of each.

a) $y = f(g(x))$

b) $y = g(f(x))$

39. The season ticket sales for a football team depend on the number of wins that the team has in the previous year according to the formula $N(w) = 10\,000\left(\dfrac{10 + w}{10}\right)^2$, where the number of season tickets sold is N and the number of wins in the previous season is w. The number of wins that the team has in the previous year is currently given by $w(t) = -\frac{1}{2}(t - 4)^2 + 12$, where t is the year, with $t = 0$ in 2009.

a) Calculate the number of season tickets sold in 2009.

b) Graph $y = w(t)$. If the season has 14 games, describe what the model predicts concerning the winning success of the team.

c) What domain restrictions must be placed on $w(t)$? Explain.

d) The team's stadium holds 50 000 fans. Does the model predict a need for expansion? Justify your answer.

e) The team plans to sign several older star free agents for next year. How might this affect $w(t)$?

f) The team has just drafted four rookies who are projected to be stars in 4 years. How might this affect $w(t)$?