Overview of *Advanced Functions 12*

The McGraw-Hill Ryerson *Advanced Functions 12* program has six components.

Student Text

The student text introduces topics in real-world contexts. In each numbered section, **Investigate** activities encourage students to develop their own understanding of new concepts. **Examples** present solutions in a clear, step-by-step manner, and then the **Key Concepts** summarize the new principles. **Communicate Your Understanding** gives students an opportunity to reflect on the concepts of the numbered section, and helps you assess students' grasp of the new ideas and readiness to proceed with the exercises.

Practise questions are knowledge questions and assist students in building their understanding. **Connect and Apply** questions allow students to use what they have learned to solve problems and make connections among concepts. **Extend and Challenge** questions are more challenging and thought-provoking. Answers to Practise, Connect and Apply, and Extend and Challenge questions are provided at the back of the text. Most numbered sections conclude with a few **Math Contest** questions. **Chapter Tasks** are more involved problems that require students to use several concepts from the preceding chapters. Solutions to the Chapter Tasks are provided in this Teacher's Resource.

A **Chapter Review** of skills and concepts is provided at the end of each chapter. Questions are organized by specific numbered sections from the chapter. **Cumulative Reviews** are provided after Chapters 3, 5, and 8 and help prepare students for the Tasks. A course review is provided that encompasses Chapters 1 to 8.

The text includes a number of items that can be used as assessment tools:

- **Communicate Your Understanding** questions assess student understanding of the concepts
- **Achievement Checks** provide opportunities for formative assessment using the four Achievement Chart Categories, Knowledge and Understanding, Thinking, Communication, and Application
- **Practice Tests** contain multiple choice, short response, and extended response questions to help model classroom testing practices
- **Chapter Problem Wrap-Ups** finish each chapter by providing a set of questions that involve all four Achievement Chart Categories
- **Chapter Tasks** are presented after each chapter and may combine concepts from the preceding chapters

Technology is integrated throughout the program and includes the use of scientific calculators, graphing calculators, computer algebra systems, spreadsheet programs, dynamic geometry software, and the Internet.

Teacher's Resource

This Teacher's Resource provides the following teaching and assessment suggestions:

- **Teaching Suggestions** for all the sections
- **Practice** and chapter-specific blackline masters
- Answers to the **Investigate** questions
- Responses for the **Communicate Your Understanding** questions
- Responses for the **Chapter Problem Wrap-Up** and **Chapter Tasks**
- Students' **Common Errors** and suggested remedies
- Solutions and rubrics for the **Achievement Check** questions
- **Differentiated Instruction** strategies for students

Computerized Assessment Bank CD-ROM

The Computerized Assessment Bank CD-ROM (CAB) contains over 1000 questions based on the material presented in the student text, and allows you to create and modify tests. Questions are connected to the chapters in the student text. The question types include: True/False, Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated to the corresponding Achievement Chart Category, specific curriculum expectation, and curriculum strand from the Ontario Mathematics MHF4U Curriculum.

Solutions Manual

The Solutions Manual provides worked-through solutions for all questions in the numbered sections of the student text, except for Achievement Check questions, which are in the Teacher's Resource. In addition, the Solutions Manual provides complete solutions for questions in the Review, Practice Test, and Cumulative Review features.

Study Guide and University Handbook

The Study Guide and University Handbook provides additional information related to the Key Ideas sections in the student text. The guide includes practice questions for all sections of the text, a series of challenge questions for each chapter, and a practice exam that covers the entire course. Answers for all questions and full solutions for a selection of questions are provided at the back of the guide. The University Handbook section consists of additional topics that students can use to prepare for their mathematics courses in university or college. This section includes worked examples and practice questions.

Web site

In addition to our McGraw-Hill Ryerson Web site, teachers can access the password protected site to obtain ready-made files for *The Geometer's Sketchpad*® activities in the text, other files to support the student text activities, further support material for differentiated learners, and other supplemental activities.

To access this site go to:
http://www.mcgrawhill.ca/books/functions12
username: advfunctions
password: teacher12

Structure of the Teacher's Resource

The teaching notes for each chapter have the following structure:

Chapter Opener

The following items are included in the Chapter Opener:
- **Specific Expectations** that apply to the chapter, listed by strand
- Introduction to a **Chapter Problem** that may include questions designed to help students move toward the **Chapter Problem Wrap-Up** at the end of the chapter

Planning Chart

This table provides an overview of each chapter at a glance, and specifies:
- **Student Text Pages** references and **Suggested Timing** for numbered sections
- Related blackline masters available on the Teacher's Resource CD-ROM
- Assessment blackline masters for each section of the chapter
- Special tools and/or technology tools that may be needed

Prerequisite Skills

The following items are included in the margin:
- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The key items in this section include:
- **Teaching Suggestions** for how to use the Prerequisite Skills
- **Assessment** ideas on how to ascertain that students are ready for this chapter
- **Common Errors** and remedies to help you anticipate and deal with common errors that may occur

Numbered Sections

The following items are listed in the margin:
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The **Teaching Suggestions** include the following key elements:
- **Student Text Pages** references and **Suggested Timing**
- **Teaching Suggestions** give insights or point out connections on how to present the material from the text
- **Investigate Answers** and **Communicate Your Understanding Responses** let you know the expected outcomes of these activities
- Notes for the **Practise**, **Connect and Apply**, and **Extend and Challenge** questions in the text provide: comments on specific questions to anticipate any difficulties; ways to deal with students' questions; and hints on how to help students answer the questions
- **Achievement Check Answers**
- **Common Errors** and remedies give you ideas on how to help students who make typical mistakes
- **Ongoing Assessment** refers you to the **Achievement Check Rubric** to assess student achievement
- **Differentiated Instruction** items provide suggestions for alternative ways to approach some key topics for students
- **Mathematical Process Integration** chart lists questions that provide good opportunities for students to use the processes

End of Chapter Items

The **Chapter Review** and **Cumulative Reviews** (at the end of Chapters 3, 5, and 8) include the following items:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement

The **Chapter Problem Wrap-Up** includes the following elements:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice and remediation, assessment, or enhancement
- **Using the Chapter Problem** includes teaching suggestions specific to the problem
- **Summative Assessment** refers you to the **Chapter Problem Rubric** to assess student achievement
- **Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 response

The **Practice Test** has the following key features:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** for extra practice or remediation, assessment, or enhancement
- **Study Guide** directs students who have difficulty with specific questions to appropriate examples to review
- **Summative Assessment** refers you to the **Chapter Test** to assess student performance

A **Chapter Task** occurs at the end of each chapter and includes:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and technology tools needed for the section
- **Related Resources** useful for extra practice or remediation, assessment, or enhancement
- **Teaching Suggestions** with steps for you to follow, a list of questions you can use to help students begin the Task, and a list of questions you should consider when assessing students' responses
- **Ongoing Assessment** refers you to the **Chapter Task Rubric** to assess student achievement
- **Level 3 Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 answer

The **Teacher's Resource CD-ROM** provides various blackline masters, including:

- Generic Masters
- Technology Masters
- Practice Masters
- Assessment Masters
- Chapter-specific Masters

Program Philosophy

Advanced Functions 12 is an exciting new resource.

Advanced Functions 12 program is designed to:

- provide full support in teaching the Ontario MHF4U mathematics curriculum
- support and extend students' progress from concrete to representational and abstract thinking
- offer a diversity of options that collectively deliver student and teacher success

Approaches to Teaching Mathematics

> *Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.*
>
> *Ontario Ministry of Education and Training, 2004*

The concrete and abstract progression is exemplified in the following styles of mathematics teaching.

Students learn best by using a concrete, discovery-oriented approach to develop concepts. Once these concepts have been developed, a connectionist approach helps students consolidate their learning.

Transmission-Oriented

- teaching involves "delivering" the curriculum
- focuses on procedures and routines
- emphasizes clear explanations and practice
- "chalk-and-talk"

Connectionist-Oriented

- teaching involves helping students develop and apply their own conceptual understandings
- focuses on different models and methods and the connections among them
- emphasizes "problematic" challenges and teacher-student dialogue

Discovery-Oriented

- teaching involves helping students learn by "doing"
- focuses on applying strategies to practical problems and using concrete materials
- emphasizes student-determined pacing
- "hands-on"

At this level, some transmission-oriented learning is also useful. This variety of approaches can be seen in the *Advanced Functions 12* program design.

Feature	Teaching Style(s) Supported
Chapter Problem	connectionist
Investigate, Reflect	discovery, connectionist
Examples	transmission, connectionist
Key Concepts	connectionist, transmission
Communicate Your Understanding	connectionist, discovery
Practise	transmission
Connect and Apply	connectionist, transmission
Extend and Challenge	connectionist, transmission
Review	transmission, connectionist
Task	discovery, connectionist

Instructional Practice

The resources available in today's classroom offer opportunities and challenges. Indeed, the principal challenge—one that many teachers of mathematics are reluctant to confront—is to teach successfully to the opportunities available.

Grouping

> *Instructional practice that incorporates a variety of grouping approaches enhances the richness of learning for students.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

At one end of the scale, individual work provides an opportunity for students to work on their own, at their own pace. At the other extreme, class discussion of problems and ideas creates a synergistic learning environment. In between, carefully selected groups bring cooperative learning into play.

Manipulatives and Materials

> *Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.*
>
> *Ontario Ministry of Education and Training, 2003*

Although many teachers feel unsure about teaching with manipulatives and other concrete materials, many students find them a powerful way to learn. The *Advanced Functions 12* program supports the use of manipulatives, where appropriate, and helps teachers adapt to this kind of teaching. The Teaching Suggestions sections in the Teacher Resource provide suggestions for developing student understanding using semi-concrete materials, such as diagrams and charts.

Technology

In the *Advanced Functions 12* program, graphing calculator instructions are provided in parallel with conventional calculations. Use of the computer algebra system, on the TI-89 calculator, is provided.

Special computer software designed for the classroom and licensed by the Ministry of Education for use in Ontario classrooms, such as *The Geometer's Sketchpad*®, provide powerful tools for teaching and learning. The *Advanced Functions 12* program supports the use of such software as an enhancement to the classroom experience. In addition, support for computer algebra systems is included. Multiple solutions for worked-through examples in the text allow teachers to enjoy wide flexibility in lesson planning. As a result, you can plan activities using manipulatives, using software, or any combination of the two.

The *Advanced Functions 12* also supports the use of TI-Navigator™ in the classroom. Materials to support the teacher using this technology are available on the McGraw-Hill Ryerson Web site. The Internet provides great opportunities for enhancing learning. Some recommended Web sites for teachers' reference are provided via links on the McGraw-Hill Ryerson Web site at *http://www.mcgrawhill.ca/books/functions12*. As with many other sources of information, students must be protected from inappropriate content. For students, the McGraw-Hill Ryerson Web site at *http://www.mcgrawhill.ca/links/functions12* has been designed to offer only safe and reliable Web site links for students to explore as an integrated part of the *Advanced Functions 12* program.

The expectations for this course mandate the frequent use of technology. Strong support for technology use has been included in the lesson design.

The technology used in this textbook are primarily graphing calculators, specifically the TI-83 Plus/84 Plus series, a computer algebra system (CAS), specifically the TI-89/89T series, and *The Geometer's Sketchpad*®. As the TI-Nspire™ comes online, support will be provided on the McGraw-Hill Web site. These are the three most common calculators in use in Ontario, and the ones that teachers and students are likely to have the most experience with. If other tools are available, use them, if they are appropriate substitutes.

The Geometer's Sketchpad® is licensed in Ontario for use by students at home. Consider providing each student with a copy of the software to install on a home computer so that it can be used as a tool for homework. This allows for a greater number and type of questions you can assign. This also reduces the problem of access to school computers. Ensure that students without home computers have an alternative. One suggestion is to pair these students with students who are willing to share their home computers. An alternative is to provide the software on public use computers in the school, such as library computers.

Activities involving technology are provided with specific instructions. In addition, you can refer to the Technology Appendix for help or review.

The Web site at *http://www.mcgrawhill.ca/books/functions12* is a source of ready-made sketches, additional Chapter Tasks, and other resource materials. Visit *http://www.mcgrawhill.ca/links/functions12* for the *Advanced Functions 12* textbook for additional materials.

Problems of access to technology will occur in most schools. Every attempt has been made to provide alternative activities. Although this measure ensures that the content can be taught, it is important to use available technology as much as possible.

A scientific calculator provides strong support for students, and is expected to be a standard tool available to the student at all times.

Literacy

Effective mathematics classrooms show students that math is everywhere in their world. For example, students should see that their work in graphing can be used in Science class. The written work they produce explaining their answers is also a language arts product. When connections such as these are made, students begin to see that math is not an isolated subject, but rather a vital part of everyday life. Contextual examples and problems can be linked to students' everyday experiences outside the classroom, as well.

Connections

Connections give students help to understand a symbol, a phrase, or a new word. They also provide suggestions for connecting mathematics to literacy, by connecting terms in mathematics to vocabulary used in other contexts. This feature provides one more way for students to feel successful in mathematics.

Writing and Mathematics

Being able to communicate ideas clearly is an important part of the *Advanced Functions 12* program. Students are asked to write about the mathematics they are learning, and communicate their understanding about what they are learning.

Take time to discuss the importance of being able to communicate understanding. The students' responses are meant to communicate with the teacher and are assessed as part of the mathematics work.

Problem Solving

> *Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.*
>
> *National Council of Teachers of Mathematics, 2000*

Problem solving is an integral part of mathematics learning. The National Council of Teachers of Mathematics recommends that problem solving be the focus of all aspects of mathematics teaching because it encompasses skills and functions, which are an important part of everyday life.

> *NCTM Problem-Solving Standard*
> Instructional programs should enable all students to—
> - Build new mathematical knowledge through problem solving
> - Solve problems that arise in mathematics and in other contexts
> - Apply and adapt a variety of appropriate strategies to solve problems
> - Monitor and reflect on the process of mathematical problem solving

McGraw-Hill Ryerson has made the problem-based learning approach the focus of its program. In *Advanced Functions 12*, a variety of problem-solving opportunities are provided for students:

- The **Mathematical Process Expectations** (Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating) are embedded throughout the student textbook. The Teacher's Resource identifies which questions provide good opportunities to use the mathematical processes for each numbered section.
- **Math Contest** questions are included at the end of many numbered section exercises to give students more opportunities to solve non-routine problems.
- Each chapter begins with an investigation of a real-life problem. The **Chapter Problem** is then revisited throughout the chapter through **Chapter Problem** questions, and ends with the **Chapter Problem Wrap-Up**.
- At the end of each chapter, students are presented with a **Chapter Task** where the solution path is not readily apparent and where solving the problem requires more than just applying a familiar procedure. These cross-curricular tasks require students to apply what they have learned in the current chapter and previous chapters to solve real-life, broad-based problems.
- An additional Task for each chapter can be found on the Teacher's Resource Web site. Go to *http://www.mcgrawhill.ca/books/functions12* and follow the links.

Mathematical Processes

The seven expectations presented at the start of the mathematics curriculum in Ontario describe the mathematical processes that students need to learn and apply as they investigate mathematical concepts, solve problems, and communicate their understanding. Although the seven processes are categorized, they are interconnected and are integrated into student learning in all areas of the *Advanced Functions 12* program.

Achievement Chart Category	Related Math Processes
Knowledge and Understanding	Selecting tools and computational strategies
Communication	Communicating
Application	Selecting tools and computational strategies Connecting
Thinking	Problem solving Reasoning and proving Reflecting

Problem Solving

Problem solving is the basis of the *Advanced Functions 12* program. Students can achieve the expectations by using this essential process, and it is an integral part of the mathematics curriculum in Ontario. Useful problem-solving strategies include: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

Reasoning and Proving

Critical thinking is an essential part of mathematics. As the students investigate mathematical concepts in *Advanced Functions 12*, they learn to: employ inductive reasoning; make generalizations based on specific findings; use counter-examples to disprove conjectures; use deductive reasoning.

Reflecting

Students are given opportunities to regularly and consciously reflect on their thought processes as they work through the problems in *Advanced Functions 12*. As they reflect, they learn to: recognize when the technique they are using is not helpful; make a conscious decision to switch to a different strategy; rethink the problem; search for related knowledge; determine the reasonableness of an answer.

Selecting Tools and Computational Strategies

Students are given many opportunities to use a variety of manipulatives, electronic tools, and computational strategies in the *Advanced Functions 12* program. The student text provides examples of and ways to use various types of technology, such as calculators, computers, and communications technology, to perform particular mathematical tasks, investigate mathematical ideas, and solve problems. These important problem-solving tools can be used to: investigate number and graphing patterns, geometric relationships, and different representations; simulate situations; collect, organize, and sort data; extend problem solving.

Connecting

Advanced Functions 12 is designed to give students many opportunities to make connections between concepts, skills, mathematical strands, and subject areas. These connections help them see that mathematics is much more than a series of isolated skills and concepts. Connecting mathematics to their everyday lives also helps students see that mathematics is useful and relevant outside the classroom.

Representing

Throughout the *Advanced Functions 12* program, students represent mathematical ideas in various forms: numeric, geometric, graphical, algebraic, pictorial, and concrete representations, as well as representation using dynamic software. Students are encouraged to use more than one representation for a single problem, seeing the connections between them.

Communicating

Students use many different ways of communicating mathematical ideas in the *Advanced Functions 12* program, including: oral, visual, writing, numbers, symbols, pictures, graphs, diagrams, and words. The process of communication helps students reflect on and clarify ideas, relationships, and mathematical arguments.

Using Mathematical Processes

Encourage students to use the mathematical processes in their work by prompting them with questions such as the following:

- *How can you tell whether your answer is correct/reasonable?* This promotes reasoning and reflection.
- *Why did you choose this method?* This promotes reflection, reasoning, selecting tools and computational strategies, and communication.
- *Could you have solved the problem another way?* This promotes reasoning, reflection, selecting tools and computational strategies, representing, and communication.
- *In what context have you solved a problem like this before?* This promotes connecting.

Use **Assessment Masters A1** through **A7** to share with students evidence that you may be looking for when you are assessing them on the mathematical processes expectations.

You can also encourage students to use a Think-Pair-Share approach to problem solving (see the **Assessment** section in this Program Overview). They will benefit greatly from brainstorming ideas and comparing methods of approach. A useful life skill is willingness to try different methods of solving a problem, learning from methods that perhaps do not reach the final goal, and being able to change their approach to reach the solution.

Technology

Advanced Functions 12 taps the full power of today's interactive technologies to engage students in math inquiry, research, and problem solving. Technology is a major focus in several of the chapters, providing students with hands-on experience in creating graphs, and constructing and analysing geometric figures using *The Geometer's Sketchpad*®. If at all possible, a classroom environment should be in place in which students are encouraged to reach for and apply technology whenever they feel the situation calls for it. In such an environment, the ongoing use of technology becomes another tool in the student's problem-solving tool kit, rather than a discrete event.

The *Advanced Functions 12* program includes opportunities for students to do research in the library or on the Internet. Consider having a class discussion on Internet Web sites and appropriate sources. Remind students that anyone can create a Web site on any topic on the Internet. Ask students to raise their hands if they have a personal Web site or keep an Internet journal (a *blog*). Explain that Web sites like these contain personal opinions and information contained on them should be looked at critically.

Types of Programs

Several types of software programs are used or suggested in *Advanced Functions 12*:

Interactive Geometry Programs
- *The Geometer's Sketchpad*®

Interactive Statistical Programs
- *Fathom*™

Spreadsheet Programs
- Microsoft® *Excel*

Technology BLMs are also available, providing students with step-by-step directions on how to use technology, such as software and the Computer Algebra System on the TI-89 calculators, to explore the mathematical concepts of the lesson. These BLMs include:
- T–1 Microsoft® *Excel*
- T–2 *The Geometer's Sketchpad*® 4
- T–3 *Fathom*™
- T–4 CAS TI-89
- T–5 Using the CBR (calculator based-ranger)

The **Technology Appendix**, on pages 505–523, of the student text provides clear step-by-step instruction in the basic functions of the TI-83/84Plus graphing calculator, the computer algebra system on the TI-89 Titanium calculator, and the basic features of *The Geometer's Sketchpad*®.

Assessment

The main purpose of assessment is to improve student learning. Assessment data helps you determine the instructional needs of your students during the learning process. Some assessment data is used to evaluate students for the purpose of reporting.

Assessment must be purposeful and inclusive for all students. It should be varied to reflect learning styles of students and be clearly communicated with students and parents. Assessment can be used diagnostically to determine prior knowledge, formatively to inform instructional planning, and in a summative manner to determine how well the students have achieved the expectations at the end of a learning cycle.

Diagnostic Assessment

Assessment for diagnostic purposes can determine where individual students will need support and will help to determine how the classroom time needs to be spent. *Advanced Functions 12* provides you with diagnostic support at the start of the text and the beginning of every chapter.

- The **Prerequisite Skills** section at the beginning of each chapter provides connections to essential concepts and skills needed for the upcoming chapter.
- For students needing support beyond the Prerequisite Skills, **Practice Masters** are provided in this Teacher's Resource that both develop conceptual understanding and improve procedural efficiency.

Diagnostic support is also provided at the start of every section.

- Each numbered section of the text begins with an introduction to facilitate open discussion in the classroom.
- Each activity starts with a question that stimulates prior knowledge and allows you to monitor students' readiness.

Formative Assessment

Formative assessment tools are provided throughout the text and Teacher's Resource. Formative assessment allows you to determine students' strengths and weaknesses and guide your class towards improvement. *Advanced Functions 12* provides blackline masters for student use that complement the text in areas where formative assessment indicates that students need support.

The **Chapter Opener**, visual, and the introduction to the **Chapter Problem** at the beginning of each chapter in the student book provide opportunities for you to do a rough formative assessment of student awareness of the chapter content.

Within each lesson:

- **Key Concepts** can be used as a focus for classroom discussion to determine the students' readiness to continue.
- **Communicate Your Understanding** questions allow you to determine if the student has developed the conceptual understanding and/or skills that were the goal of the section.
- **Connect and Apply** offers you an opportunity to determine students' understanding of concepts through conversations and written work. It also allows you to monitor students' procedural skills, their application of procedures, their ability to communicate their understanding of concepts, and their ability to solve problems related to the section's Key Concepts.
- **Achievement Check** questions allow students to demonstrate their knowledge and understanding and their ability to apply, think of, and communicate what they have learned.

- **Chapter Problem** questions provide opportunities to verify that students are developing the skills and understanding they need to complete the **Chapter Problem Wrap-Up** questions.
- **Extend and Challenge** questions are more challenging and thought-provoking, and are aimed at Level 3 and 4 performance.
- **Chapter Reviews** and **Cumulative Reviews** provide an opportunity to assess Knowledge/Understanding, Thinking, Communication, and Application.

Summative Assessment

Summative data is used for both planning and evaluation.

- A **Practice Test** in each chapter assess students' achievement of the expectations in the areas of Knowledge/Understanding, Thinking, Communication, and Application.
- The **Chapter Problem** provides a problem-solving opportunity using an open-ended question format that is revisited in the **Chapter Problem Wrap-Up** questions. The **Chapter Problem** can be used to evaluate students' understanding of the expectations under the categories of Knowledge and Understanding, Thinking, Communication, and Application.
- **Chapter Tasks** include open-ended investigations with rubrics provided. They are presented at the end of each chapter. The Tasks require students to use and make connections among several concepts from the preceding chapters.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios:

- Help teachers assess students' growth and mathematical understanding
- Provide insight into students' self-awareness about their own progress
- Help parents understand their child's growth

Advanced Functions 12 has many components that provide ideal portfolio items. Inclusion of all or any of these chapter items provides insight into students' progress in a non-threatening, formative manner.
These items include:

- Students' responses to the **Chapter Opener**
- Students' responses to the **Chapter Problem Wrap-Up** assignments
- Responses to **Communicate Your Understanding** questions, which allow students to explore their initial understanding of concepts
- Answers to **Achievement Check** questions, which are designed to show students' mastery of specific expectations
- **Chapter Task** assignments, which show students' understanding across several chapters

Assessment Masters

Advanced Functions 12 provides a variety of assessment tools with the chapter-specific blackline masters, such as Chapter Tests, Chapter Problem Wrap-Up rubrics, and Task rubrics. In addition, the program includes generic assessment blackline masters based on the mathematical processes. These BLMs will help you to effectively monitor student progress and evaluate instructional needs.

Generic Assessment BLM	Type	Purpose
A–1 **Problem** Solving	**Checklist**	**Assess** students' problem solving skills.
A–2 **Reasoning** and Proving	**Checklist**	**Assess** students' reasoning and proving skills.
A–3 **Reflecting**	**Checklist**	**Assess** students' understanding of an expectation.
A–4 **Selecting** Tools and Computational Strategies	**Checklist**	**Assess** students' ability to select the appropriate tool(s) and strategies for solving a problem.
A–5 **Connecting**	**Checklist**	**Assess** students' ability to make connections of concepts learned to problem situations, to connect prior learning with current concepts.
A–6 **Representing**	**Checklist**	**Assess** students' ability to represent the problem and/or solution using an appropriate representation.
A–7 **Communicating**	**Checklist**	**Assess** students' ability to explain and communicate effectively their learning.

Reaching all Students

Advanced Functions 12 accommodates a broad range of needs and learning styles, including those students requiring accommodations, students with limited proficiency in English, and gifted learners. This Teacher's Resource provides support in addressing multiple intelligences and learning styles through:

- **Differentiated Instruction** items in the margin provide suggestions for alternative ways to approach key topics for some students

Accommodations for Students with Language Difficulties

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Give concise, step-by-step directions • Teach students to look for cue words, highlight these words • Use visual models • Use visual representations to accompany word problems • Encourage students to look for common patterns in word problems	• Provide reference charts with operations and formulae stated simply • Post reference charts with math vocabulary • Reinforce learning with visual aids and manipulatives • Using a visual format, post strategies for problem solving • Use a peer tutor or buddy system	• Read instructions/ word problems to students on tests • Extend time lines

Accommodations for Students with Visual/Perceptual/Spatial/Motor Difficulties

Instructional	Environmental	Assessment
• Reduce copying • Provide worksheets • Provide graph paper • Provide concrete examples • Allow use of a number line • Provide a math journal • Encourage and teach self-talk strategies • Chunk learning and tasks	• visual bombardment • a work carrel or work area that is not visually distracting • rest periods and breaks	• Provide graph paper for tests • Extend time lines • Provide consumable tests • Reduce the number of questions required to indicate competency • Provide a scribe when lengthy written answers are required

Accommodations for Students with Memory Difficulties

Instructional	Environmental	Assessment
• Regularly review concepts • Activate prior knowledge • Teach mnemonic strategies (e.g., BEDMAS) • Teach visualization strategies • Colour-code steps in sequence • Teach functional math concepts related to daily living	• Provide reference charts with commonly used facts, formulae, and steps for problem-solving • Allow use of a calculator • Use games and computer programs for drill and repetition	• Allow use of formula sheets • Allow use of other reference charts as appropriate • Allow use of calculators • Extend time lines • Present one concept-type of question at a time

Accommodations for Enrichment

Instructional	Environmental	Assessment
• Structure learning activities to develop higher-order thinking skills (analysis, synthesis, and evaluation) • Provide open-ended questions • Value learner's own interests and learning style, and allow for as much student input into program options as possible • Encourage students to link learning to wider applications • Encourage learners to reflect on the process of their own learning • Encourage and reward creativity • Avoid repetitive tasks and activities	• Encourage a stimulating environment that invites exploration of mathematical concepts • Display pictures of role models who excel in mathematics • Provide access to computer programs that extend learning	• Reduce the number of questions to allow time for more demanding ones • Allow for opportunities to demonstrate learning in non-traditional formats • Pose more questions that require higher-level thinking skills (analysis, synthesis, and evaluation) • Reward creativity

Accommodations for ESL Students

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Explain colloquial expressions and figurative speech • Review comparative forms of adjectives	• Display reference charts with mathematical terms and language • Encourage personal math dictionaries with math terms and formulae	• Allow access to personal math dictionaries • Read instructions to students and clarify terms • Allow additional time

Differentiated Instruction ideas and different learning strategies are recommended as alternative ways to approach some key topics. By addressing these topics in a new or different way, teachers can provide students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Types of Strategies

A number of different types of cooperative learning strategies can be used in the mathematics classroom, and many are suggested in the Differentiated Instruction margin items in this Teacher's Resource.

Think-Pair-Share

Students individually think about a concept, and then pick a partner to share their ideas. For example, students might work on the Communicate Your Understanding questions, and then choose a partner to discuss the concepts with. Working together, the students could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Jigsaw

Individual group members are responsible for researching and understanding a specific part of the information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during calculus, this type of group might have "experts" in making various types of graphs using technology. Group members could then coach each other in making each kind of graph.

Another way of using the Jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching the shapes of various sports' surfaces might have a home group of four in which each member is responsible for researching one of soccer, baseball, hockey, or basketball. Individual members then move to expert groups. Expert groups include all of the students responsible for researching one of the sports. Each of the expert groups researches their particular sport. Once the information has been gathered and prepared for presentation, individual members of the expert group return to their home group and teach other members about their sport.

Placemat

In groups of four, students individually complete their section of a placemat (BLM provided on the *Advanced Functions 12: Teacher's Resource* CD-ROM). The group then pools their responses and completes the centre portion of the placemat with group responses. This method can be used for pre-assessment (diagnostic), review, or to summarize a topic.

Concept Attainment

Based on a list of examples and non-examples of a concept, students identify and define the concept. Then, they determine the critical attributes of the concepts and apply their defined concept to generate their own examples and non-examples.

Think Aloud

Students work through a problem in front of the class, verbalizing their thinking throughout. This method can help develop process thinking in students.

Decision Tree

Students use a graphic organizer flow chart to identify key decisions and consequences.

Carousel

Students at different stations display and explain topics or concepts to other classmates who rotate through the stations, usually in order.

Timed Retell

Students sit in pairs facing each other. After some preparation time, Student A has 30 s to tell what she or he knows about the topic to Student B. Student B then retells the talk for about 30 s and adds additional information. Both students then write a summary of the talk.

Frayer Model

Students complete four quadrants for a specified topic: definition, facts/characteristics, examples, and non-examples. Variation: Give students a completed model and ask them to identify the topic/concept.

Blast Off

This strategy can be used to start a class in an energized way. Students are asked to record: **3** important things they learned last class; **2** questions they have about last class; **1** reflection on what they learned last class; Blast Off!

Inside/Outside Circle
Students face each other in pairs, forming two concentric circles. Students take turns giving information to their partner, then the outside circle rotates one person to the right while the inside circle remains still. Students then share information with their new partners. The process continues until the students in the outside circle have rotated back to their starting point.

Four Corners
Students move from one corner of the room based on preference or opinion. (Example: To differentiate a particular function, students choose from algebraic model using pencil and paper, algebraic model using first principles, algebraic model using technology, dynamic model (GSP).)

Graffiti
There are two different methods of graffiti.

In the first method, students in groups take turns adding information to a sheet of paper passed around the group (for example, adding lines to a solution).

Example:
1. Start with groups of four. Each student gets a different word problem.
2. The first student completes step one, e.g., draw a diagram on a page. The page is passed to the next student.
3. The second student then completes the first line of the solution on the page, e.g., provides an opening statement. The page is passed to the next student.
4. The third student completes the next line of the solution. The page is passed to the next student and so on until the group has completed the solutions to all four problems.

In the second method, groups or pairs of students move around the room and add information to questions posted on chart paper around the room.

Gallery Walk
Groups move from station to station, read what is already there and add information, and eventually return to their original station. (Example: groups might first create a word problem on chart paper at their station, move to the next station and solve the problem there, and eventually return to their first station to see the solution to their problem.)

Graffiti plus Gallery Walk
Combines the Graffiti and Gallery Walk strategies described above.

Anticipation Guide
Prior to a unit of study, students form conjectures about the material they are about to learn. This strategy activates prior knowledge and builds student interest. Return to the conjectures at the end of the unit to compare.

What-So What Double Entry
Use a T-chart to organize information. On the left side, list information. On the right side, list how information is used.

Word Wall/Information Wall
This strategy progressively places terminology, formulas, or information on a classroom wall during a unit of study. Information can be actively re-grouped as the unit progresses to emphasize important concepts for current work.

Curriculum Correlation between McGraw-Hill Ryerson *Mathematics 12 Advanced Functions* and The Ontario Curriculum *Advanced Functions, Grade 12*, (MHF4U)

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs

Prerequisite: Functions, Grade 11, University Preparation, or Mathematics for College Technology, Grade 12, College Preparation

Mathematical Process Expectations.

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

Problem Solving	• develop, select, apply, compare and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., use inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical idea and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representation to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

The mathematical process expectations are integrated throughout *Mathematics 12 Advanced Functions*.

A. Exponential and Logarithmic Functions

Overall Expectations

By the end of this course, students will:

A1. demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;

A2. identify and describe some key features of the graphs of logarithmic functions, make connections between the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;

A3. solve exponential and simple logarithmic equations in one variable algebraically, including those arising from real-world applications.

Specific Expectations

	Chapter/Section	Pages
1. Evaluating Logarithmic Expressions		
By the end of this course students will:		
A1.1 recognize the logarithm of a number with a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions ***Sample problem:*** Why is it not possible to determine $\log_{10}(-3)$or $\log_2 0$? Explain your reasoning.	**6.1**	310–323
A1.2 determine, with technology, the approximate logarithm of a number with any base, including base 10 (e.g., by reasoning that $\log_3 29$ is between 3 and 4 and using systematic trial to determine that $\log_3 29$ is approximately 3.07)	**6.2**	323–331
A1.3 make connections between related logarithmic and exponential equations (e.g., $\log_5 125 = 3$ can also be expressed as $5^3 = 125$), and solve simple exponential equations by rewriting them in logarithmic form (e.g., solving $3^x = 10$ by rewriting the equation as $\log_3 10 = x$)	**6.2**	323–331
A1.4 make connections between the laws of exponents and the laws of logarithms [e.g., use the statement $10^{a+b} = 10^a 10^b$ to deduce that $\log_{10}x + \log_{10}y = \log_{10}(xy)$], verify the laws of logarithms with or without technology (e.g., use patterning to verify the quotient law for logarithms by evaluating expressions such as $\log_{10}1000 - \log_{10}100$ and then rewriting the answer as a logarithmic expression with the same base), and use the laws of logarithms to simplify and evaluate numerical expressions	**6.2, 6.4, 7.3**	323–331 341–349 378–386
2. Connecting Graphs and Equations of Logarithmic Functions		
By the end of this course, students will:		
A2.1 determine, through investigation with technology (e.g., graphing calculator, spreadsheet) and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, increasing/decreasing behaviour) of the graphs of logarithmic functions of the form $f(x) = \log_b x$, and make connections between the algebraic and graphical representations of these logarithmic functions ***Sample problem:*** Compare the key features of the graphs of $f(x) = \log_2 x$, $g(x) = \log_4 x$, and $h(x) = \log_8 x$ using graphing technology.	**6.1**	310–323

	Chapter/Section	Pages
A2.2 recognize the relationship between an exponential function and the corresponding logarithmic function to be that of a function and its inverse, deduce that the graph of a logarithmic function is the reflection of the graph of the corresponding exponential function in the line $y = x$, and verify the deduction using technology ***Sample problem:*** Give examples to show that the inverse of a function is not necessarily a function. Use the key features of the graphs of logarithmic and exponential functions to give reasons why the inverse of an exponential function is a function.	**6.1**	310–323
A2.3 determine, through investigation using technology, and describe the roles of the parameters d and c in functions of the form $y = \log_{10}(x - d) + c$ and the roles of the parameters a and k in functions of the form $y = a\log_{10}(kx)$ in terms of transformations on the graph of $f(x) = \log_{10} x$ (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions) ***Sample problem:*** Investigate the graphs of $f(x) = \log_{10}(x) + c$, $f(x) = \log_{10}(x - d)$, $f(x) = \log_{10}(kx)$, and for various values of c, d, a, and k, using technology, describe the effects of changing these variables in terms of transformations, and make connections to the transformations of other functions such as polynomial functions, exponential functions, and trigonometric functions.	**6.3**	331–340
A2.4 pose problems based on real-world applications of exponential and logarithmic functions (e.g., exponential growth and decay, the Richter scale, the pH scale, the decibel scale), and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation ***Sample problem:*** The pH or acidity of a solution is given by the equation pH = −logC, where C is the concentration of $[H^+]$ ions in multiples of M = 1 mol/L. Use graphing software to graph this function. What is the change in pH if the solution is diluted from a concentration of 0.1M to a concentration of 0.01M. From 0.001M to 0.0001M? Describe the change in pH when the concentration of any acidic solution is reduced to $\frac{1}{10}$ of its original concentration. Rearrange the given equation to determine concentration as a function of pH.	**6.5** **7.5**	349–355 393–407
3. Solving Exponential and Logarithmic Equations		
By the end of this course, students will:		
A3.1 recognize equivalent algebraic expressions involving logarithms and exponents, and simplify expressions of these types ***Sample problem:*** Sketch the graphs of $f(x) = \log_{10}(100x)$ and $g(x) = 2 + \log_{10} x$, compare the graphs, and explain your findings algebraically.	**7.1**	364–369
A3.2 solve exponential equations in one variable by finding a common base (e.g., solving $4^x = 8^{x+3}$ by expressing each side as a power of 2) and by using logarithms (e.g., solving $4^x = 8^{x+3}$ by taking the logarithm base 2 of both sides), recognizing that logarithms base 10 are commonly used (e.g., solving $3^x = 7$ by taking the logarithm base 10 of both sides) ***Sample Problem:*** Solve $300(1.05)^n = 600$ and $2^{x+2} = 12$ by either finding a common base or by taking logarithms, and explain your choice of method in each case.	**7.2**	370–377
A3.3 solve simple logarithmic equations in one variable algebraically [e.g., $\log_3(5x + 6) = 2$, $\log_{10}(x + 1) = 1$];	**7.4**	387–392
A3.4 solve problems involving exponential and logarithmic equations algebraically, including problems arising from real-world applications. ***Sample problem:*** The pH or acidity of a solution is given by the equation pH = −logC, where C is the concentration of $[H^+]$ ions in multiples of M = 1 mol/L. You are given a solution of hdrochloric acid with a pH of 1.7 and asked to increase the pH of the solution by 1.4. Determine how much you must dilute the solution. Does you answer differ if you start with a pH of 2.2?	7.5	393–407

B. Trigonometric Functions

Overall Expectations

By the end of this course, students will:

B1. demonstrate an understanding of the meaning and application of radian measure;

B2. make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;

B3. solve problems involving trigonometric equations and prove trigonometric identities.

Specific Expectations

	Chapter/Section	Pages
1. Understanding and Applying Radian Measure		
By the end of this course students will:		
B1.1 recognize the radian as an alternative unit to the degree for angle measurement, define the radian measure of an angle as the length of the arc that subtends this angle at the centre of the unit circle, and develop and apply the relationship between radian and degree measure	**4.1**	202–210
B1.2 represent radian measure in terms of π (e.g., $\frac{\pi}{3}$ rad, 2π rad) and as a rational number (e.g., 1.05 rad, 6.28 rad)	**4.1**	202–210
B1.3 determine, with technology, the primary trigonometric ratios (i.e., sine, cosine, tangent) and the reciprocal trigonometric ratios (i.e., cosecant, secant, cotangent) of angles expressed in radian measure	**4.2**	211–219
B1.4 determine, without technology, the exact values of the primary trigonometric ratios and the reciprocal trigonometric ratios for the special angles 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and their multiples less than or equal to 2π	**4.3**	220–227
2. Connecting Graphs and Equations of Trigonometric Functions		
By the end of this course, students will:		
B2.1 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in radians, and determine and describe some key properties (e.g., period of 2π, amplitude of 1) in terms of radians	**5.1**	252–260
B2.2 make connections between the tangent ratio and the tangent function by using technology to graph the relationship between angles in radians and their tangent ratios and defining this relationship as the function $f(x) = \tan x$, and describe key properties of the tangent function	**5.1**	252–260
B2.3 graph, with technology and using the primary trigonometric functions, the reciprocal trigonometric functions (i.e., cosecant, secant, cotangent) for angle measures expressed in radians, determine and describe key properties of the reciprocal functions (e.g., state the domain, range, and period, and identify and explain the occurrence of asymptotes), and recognize notations used to represent the reciprocal functions [e.g., the reciprocal of $f(x) = \sin x$ can be represented using $\csc x$, $\frac{1}{f(x)}$, or $\frac{1}{\sin x}$, but not using $f^{-1}(x)$ or $\sin^{-1} x$, which represent the inverse function]	**5.2**	261–269
B2.4 determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$, with angles expressed in radians	**5.3**	270–279

	Chapter/Section	Pages
B2.5 sketch graphs of $y = a\sin(k(x - d)) + c$ and $y = a\cos(k(x - d)) + c$ by applying transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, with angles expressed in radians, and state the period, amplitude, and phase shift of the transformed functions ***Sample problem:*** Transform the graph of $f(x) = \cos x$ to sketch $g(x) = 3\cos(2x) - 1$, and state the period, amplitude, and phase shift of each function.	5.3	270–279
B2.6 represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians ***Sample problem:*** A sinusoidal function has an amplitude of 2 units, a period of π, and a maximum at (0, 3). Represent the function with an equation in two different ways.	5.3	270–279
B2.7 pose problems based on applications involving a trigonometric function with domain expressed in radians (e.g., seasonal changes in temperature, heights of tides, hours of daylight, displacements for oscillating springs) , and solve these and other such problems by using a given graph or a graph generated with or without technology from its equation. ***Sample problem:*** The population size, P, of owls (predators) in a certain region can be modelled by the function $P(t) = 1000 + 100\sin\left(\frac{\pi t}{12}\right)$, where t represents the time in months. The population size, p, of mice (prey) in the same region is given by $p(t) = 20\,000 + 4000\cos\left(\frac{\pi t}{12}\right)$. Sketch the graphs of these functions, and pose and solve problems involving the relationships between the two populations over time.	5.3, 5.5	270–279 290–299
3. Solving Trigonometric Equations		
By the end of this course, students will:		
B3.1 recognize equivalent trigonometric expressions [e.g., by using the angles in a right triangle to recognize that $\sin x$ and $\cos\left(\frac{\pi}{2} - x\right)$ are equivalent; by using transformations to recognize that $\cos\left(x + \frac{\pi}{2}\right)$ and $-\sin x$ are equivalent], and verify equivalence using graphing technology	4.3	220–227
B3.2 explore the algebraic development of the compound angle formulas (e.g., verify the formulas in numerical examples, using technology; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]), and use the formulas to determine exact values of trigonometric ratios [e.g., determining the exact value of $\sin\left(\frac{\pi}{12}\right)$ by first rewriting it in terms of special angles as $\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$]	4.4	228–235
B3.3 recognize that trigonometric identities are equations that are true for every value in the domain (i.e., a counter-example can be used to show that an equation is not an identity), prove trigonometric identities through the application of reasoning skills, using a variety of relationships (e.g., $\tan x = \frac{\sin x}{\cos x}$; $\sin^2 x + \cos^2 x = 1$; the reciprocal identities; the compound angle formulas), and verify identities using technology ***Sample problem:*** Use the compound angle formulas to prove the double angle formulas.	4.5	236–241
B3.4 solve linear and quadratic trigonometric equations, with and without graphing technology, for the domain of real values from 0 to 2π, and solve related problems. ***Sample problem:*** Solve the following trigonometric equations for $0 \le x \le 2\pi$, and verify by graphing with technology: $2\sin x + 1 = 0$; $2\sin^2 x + \sin x - 1 = 0$; $\sin x = \cos 2x$; $\cos 2x = \frac{1}{2}$.	5.4	282–289

C. Polynomial and Rational Functions

Overall Expectations

By the end of this course, students will:

C1. identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;

C2. identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;

C3. solve problems involving polynomial and simple rational equations graphically and algebraically;

C4. demonstrate an understanding of solving polynomial and simple rational inequalities.

Specific Expectations

	Chapter/Section	Pages
1. Connecting Graphs and Equations of Polynomial Functions		
By the end of this course students will:		
C1.1 recognize a polynomial expression (i.e., a series of terms where each term is the product of a constant and a power of x with a non-negative integral exponent, such as $x^3 - 5x^2 + 2x - 1$); recognize the equation of a polynomial function and give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions	**1.1, 1.2**	4–29
C1.2 compare, through investigation using graphing technology, the numeric, graphical, and algebraic representations of polynomial (i.e., linear, quadratic, cubic, quartic) functions (e.g., compare finite differences in tables of values; investigate the effect of the degree of a polynomial function on the shape of its graph and the maximum number of x-intercepts; investigate the effect of varying the sign of the leading coefficient on the end behaviour of the function for very large positive or negative x-values) ***Sample problem:*** Investigate the maximum number of x-intercepts for linear, quadratic, cubic, and quartic functions using graphing technology.	**1.2, 1.3**	15–41
C1.3 describe key features of the graphs of polynomial functions (e.g., the domain and range, the shape of the graphs, the end behaviour of the functions for very large positive or negative x-values) **Sample** problem: Describe and compare the key features of the graphs of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = x^3 + x^2$, and $f(x) = x^3 + x$.	**1.1, 1.2**	4–29
C1.4 distinguish polynomial functions from sinusoidal and exponential functions [e.g., $f(x) = \sin x$, $f(x) = 2^x$], and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions	**1.1, 1.2, 6.1**	4–29 310–322
C1.5 make connections, through investigation using graphing technology (e.g., dynamic geometry software), between a polynomial function given in factored form [e.g., $f(x) = 2(x - 3)(x + 2)(x - 1)$] and the x-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features (e.g., by determining intercepts and end behaviour; by locating positive and negative regions using test values between and on either side of the x-intercepts) ***Sample problem:*** Investigate, using graphing technology, the x-intercepts and the shape of the graph of polynomial functions with one or more repeated factors, for example, $f(x) = (x - 2)(x - 3)$, $f(x) = (x - 2)(x - 2)(x - 3)$, $f(x) = (x - 2)(x - 2)(x - 2)(x - 3)$, and $f(x) = (x - 2)(x - 2)(x - 2)(x - 2)(x - 3)$, by considering whether the factor is repeated an even or an odd number of times. Use your conclusions to sketch $f(x) = (x + 1)(x + 1)(x - 3)(x - 3)$, and verify using technology.	**1.3**	30–41

	Chapter/Section	Pages
C1.6 determine, through investigation using technology, and describe the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$ in terms of transformations on the graphs of $f(x) = x^3$ and $f(x) = x^4$ (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions) ***Sample problem:*** Investigate, using technology, the graph of $f(x) = 2(x - d)^3 + c$ for various values of d and c, and describe the effects of changing d and c in terms of transformations.	**1.4**	42–52
C1.7 determine an equation of a polynomial function that satisfies a given set of conditions (e.g., degree of the polynomial, intercepts, points on the function), using methods appropriate to the situation (e.g., using the x-intercepts of the function; using a trial-and-error process with a graphing calculator or graphing software; using finite differences), and recognize that there may be more than one polynomial function that can satisfy a given set of conditions (e.g., an infinite number of polynomial functions satisfy the condition that they have three given x-intercepts) ***Sample problem:*** Determine an equation for a fifth-degree polynomial function that intersects the x-axis at only 5, 1, and -5, and sketch the graph of the function.	**1.3, 1.4** **2.4**	30–52 113–122
C1.8 determine the equation of the family of polynomial functions with a given set of zeros and of the member of the family that passes through another given point [e.g., a family of polynomial functions of degree 3 with zeros 5, -3, and -2 is defined by the equation $f(x) = k(x - 5)(x + 3)(x + 2)$, where k is a real number, $k \neq 0$; the member of the family that passes through $(-1, 24)$ is $f(x) = 2(x - 5)(x + 3)(x + 2)$] ***Sample Problem:*** Investigate, using graphing technology, and determine a polynomial function that can be used to model the function $f(x) = \sin x$ over the interval $0 \leq x \leq 2\pi$.	**2.4**	113–122
C1.9 determine, through investigation, and compare the properties of even and odd polynomial functions [e.g., symmetry about the y-axis or the origin; the power of each term; the number of x-intercepts; $f(x) = f(-x)$ or $f(-x) = -f(x)$], and determine whether a given polynomial function is even, odd, or neither ***Sample problem:*** Investigate numerically, graphically, and algebraically, with and without technology, the conditions under which an even function has an even number of x-intercepts.	**1.1, 1.3**	4–14 30–41
2. Connecting Graphs and Equations of Rational Functions		
By the end of this course, students will:		
C2.1 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions [e.g., make connections between $f(x) = \frac{1}{x^2 - 4}$ and its graph by using graphing technology and by reasoning that there are vertical asymptotes at $x = 2$ and $x = -2$ and a horizontal asymptote at $y = 0$ and that the function maintains the same sign as $f(x) = x^2 - 4$] ***Sample problem:*** Investigate, with technology, the key features of the graphs of families of rational functions of the form $f(x) = \frac{1}{x + n}$ and $f(x) = \frac{1}{x^2 + n}$, where n is an integer, and make connections between the equations and key features of the graphs.	**3.1, 3.2**	148–167

	Chapter/Section	Pages
C2.2 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that have linear expressions in the numerator and denominator [e.g., $f(x) = \frac{2x}{x-3}$, $h(x) = \frac{x-2}{3x+4}$], and make connections between the algebraic and graphical representations of these rational functions ***Sample problem:*** Investigate, using graphing technology, key features of the graphs of the family of rational functions of the form $f(x) = \frac{8x}{nx+1}$ for $n = 1, 2, 4,$ and 8, and make connections between the equations and the asymptotes.	**3.1, 3.3**	148–155 168–176
C2.3 sketch the graph of a simple rational function using its key features, given the algebraic representation of the function	**3.2, 3.3, 3.4**	157–185
3. Solving Polynomial and Rational Equations		
By the end of this course, students will:		
C3.1 make connections, through investigation using technology (e.g., computer algebra systems), between the polynomial function $f(x)$, the divisor $x - a$, the remainder from the division $\frac{f(x)}{x-a}$, and $f(a)$ to verify the remainder theorem and the factor theorem Sample problem: Divide $f(x) = x^4 + 4x^3 - x^2 - 16x - 14$ by $x - a$ for various integral values of a using a computer algebra system. Compare the remainder from each division with $f(a)$.	**2.1, 2.2**	84–103
C3.2 factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring; difference of squares; trinomial factoring; factoring by grouping; remainder theorem; factor theorem) ***Sample problem:*** Factor: $x^3 + 2x^2 - x - 2$; $6x^3 + 4x^2 + 6x - 5$.	**2.1, 2.2**	84–103
C3.3 determine, through investigation using technology (e.g., graphing calculator, computer algebra systems) the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function, and describe this connection [e.g., the real roots of the equation $x^4 - 13x^2 + 36 = 0$ are the x-intercepts of the graph of $f(x) = x^4 - 13x^2 = 36$] ***Sample problem:*** Describe the relationship between the x-intercepts of the graphs of linear and quadratic functions and the real roots of the corresponding equations. Investigate, using technology, whether this relationship exists for polynomial functions of higher degree.	**2.3**	104–112
C3.4 solve polynomial equations in one variable, of degree no higher than four (e.g., $2x^3 - 3x^2 + 8x - 12 = 0$), by selecting and applying strategies (i.e., common factoring; difference of squares; trinomial factoring; factoring by grouping; remainder theorem; factor theorem), and verify solutions using technology (e.g., using computer algebra systems to determine the roots; using graphing technology to determine the x-intercepts of the corresponding polynomial function)	**2.3**	104–112
C3.5 determine, through investigation using technology (e.g., graphing calculator, computer algebra systems) the connection between the real roots of a rational equation and the x-intercepts of the graph of the corresponding rational function, and describe this connection [e.g., the real root of the equation $\frac{x-2}{x-3} = 0$ is 2, which is the x-intercept of the function $f(x) = \frac{x-2}{x-3} = 0$; the equation has no real roots, and the function $f(x) = \frac{1}{x-3}$ does not intersect the x-axis]	**3.4**	177–185
C3.6 solve simple rational equations in one variable algebraically, and verify solutions using technology (e.g., using computer algebra systems to determine the roots; using graphing technology to determine the x-intercepts of the corresponding rational function)	**3.4**	177–185

	Chapter/Section	Pages
C3.7 solve problems involving applications of polynomial and simple rational functions and equations [e.g., problems involving the factor theorem or remainder theorem, such as determining the values of k for which the function $f(x) = x^3 + 6x^2 + kx - 4$ gives the same remainder when divided by $x - 1$ and $x + 2$] ***Sample problem:*** Use long division to express the given function $f(x) = \frac{x^2 + 3x - 5}{x - 1}$ as the sum of a polynomial function and a rational function of the form $\frac{A}{x - 1}$ (where A is a constant), make a conjecture about the relationship between the given function and the polynomial function for very large and very small x-values, and verify your conjecture using graphing technology.	**2.3, 2.4, 3.5**	104–122 186–191
4. Solving Inequalities		
By the end of this course, students will:		
C4.1 describe, for polynomial and simple rational functions, and explain the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality (e.g., demonstrating numerically and graphically that the solution to $\frac{1}{x + 1} < 5$ is $x < -1$ or $-\frac{4}{5}$)	**2.5**	123–131
C4.2 determine solutions to polynomial inequalities in one variable [e.g., Solve $f(x) \geq 0$, where $f(x) = x^3 - x^2 + 3x - 9$] and to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which x satisfies the inequalities	**2.5**	123–131
C4.3 solve linear inequalities and factorable polynomial inequalities in one variable (e.g., $x^3 + x^2 > 0$) in a variety of ways (e.g., by determining intervals using x-intercepts and evaluating the corresponding function for a single x-value within each interval; by factoring the polynomial and identifying the conditions for which the product satisfies the inequality), and represent the solutions on a number line or algebraically (e.g., for the inequality $x^4 - 5x^2 + 4 < 0$, the solution represented algebraically is $-2 < x < -1$ or $1 < x < 2$)	**2.5, 2.6**	123–139

D. Characteristics of Functions

Overall Expectations

By the end of this course, students will:

D1. demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;

D2. determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;

D3. compare the characteristics of functions, and solve problems by modelling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

Specific Expectations

	Chapter/Section	Pages
1. Understanding Rates of Change		
By the end of this course students will:		
D1.1 gather, interpret, and describe information about real-world applications of rates of change, and recognize different ways of representing rates of change (e.g., in words; numerically; graphically; algebraically);	**1.5**	53–64
D1.2 recognize that the rate of change for a function is a comparison of changes in the dependent variable to changes in the independent variable, and distinguish situations in which the rate of change is zero, constant, or changing by examining applications, including those arising from real-world situations (e.g., rate of change of the area of a circle as the radius increases; inflation rates; the rising trend in graduation rates among Aboriginal youth, speed of a cruising aircraft; speed of a cyclist climbing a hill; infection rates) ***Sample problem:*** The population of bacteria in a sample is 250 000 at 1:00 p.m., 500 000 at 3:00 p.m., and 1 000 000 at 5:00 p.m.. Compare the methods used to calculate the change in the population and the rate of change in the population between 1:00 p.m. to 5:00 p.m.. Is the rate of change constant? Explain your reasoning.	**1.5, 1.6**	53–73
D1.3 sketch a graph that represents a relationship involving rate of change, as described in words, and verify with technology (e.g., motion sensor) when possible ***Sample problem:*** John rides his bicycle at a constant cruising speed along a flat road. He then decelerates (i.e., decreases speed) as he climbs a hill. At the top, he accelerates (i.e., increases speed) on a flat road back to his constant cruising speed, and he then accelerates down a hill. Finally, he comes to another hill and glides to a stop as he starts to climb. Sketch a graph of John's speed versus time and a graph of his distance travelled versus time.	**1.5, 1.6,** **3.5**	53–73 186–191
D1.4 calculate and interpret average rates of change of functions (e.g., linear, quadratic, exponential, sinusoidal) arising from real-world applications (e.g., in the natural, physical, and social sciences), given various representations of the functions (e.g., tables of values, graphs, equations) ***Sample problem:*** Fluorine-20 is a radioactive substance that decays over time. At time 0, the mass of a sample of the substance is 20 g. The mass decreases to 10 g after 11 s, to 5 g after 22 s, and to 2.5 g after 33 s. Compare the average rate of change over the 33-s interval with the average rate of change over consecutive 11-s intervals	**1.5,** **3.5** **5.1**	53–64 186–191 252–260
D1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change (e.g., an average rate of change can be used to approximate an instantaneous rate of change) ***Sample problem:*** In general, does the speedometer of a car measure instantaneous rate of change (i.e., instantaneous speed) or average rate of change (i.e., average speed)? Describe situations in which the instantaneous speed and the average speed would be the same.	**1.6,** **3.2, 3.3, 3.5** **5.5** **6.1**	65–73 157–191 290–299 310–322

	Chapter/Section	Pages
D1.6 determine, through investigation using various representations of relationships (e.g., tables of values, graphs, equations) approximate instantaneous rates of change arising from real-world applications (e.g., in the natural, physical, and social sciences) by using average rates of change and reducing the interval over which the average rate of change is determined ***Sample problem:*** The distance, d metres, travelled by a falling object in t seconds is represented by $d = 5t^2$. When $t = 3$, the instantaneous speed of the object is 30 m/s. Compare the average speeds over different time intervals starting at $t = 3$ with the instantaneous speed when $t = 3$. Use your observations to select an interval that can be used to provide a good approximation of the instantaneous speed at $t = 3$.	**1.6** **3.2, 3.3, 3.5** **5.5** **6.1**	65–73 157–176 186–191 290–299 310–322
D1.7 make connections, through investigation, between the slope of a secant on the graph of a function (e.g., quadratic, exponential, sinusoidal) and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point ***Sample problem:*** Use tangents to investigate the behaviour of a function when the instantaneous rate of change is zero, positive, or negative.	**1.5, 1.6** **3.2, 3.3, 3.5**	53–73 157–176 186–191
D1.8 determine, through investigation using a variety of tools and strategies (e.g., using a table of values to calculate slopes of secants or graphing secants and measuring their slopes with technology), the approximate slope of the tangent to a given point on the graph of a function (e.g., quadratic, exponential, sinusoidal) by using the slopes of secants through the given point (e.g., investigating the slopes of secants that approach the tangent at that point more and more closely), and make connections to average and instantaneous rates of change	**1.5, 1.6** **3.2, 3.3, 3.5** **5.5** **6.1**	53–73 157–176 186–191 290–299 310–322
D1.9 solve problems involving average and instantaneous rates of change, including problems arising from real-world applications, by using numerical and graphical methods (e.g., by using graphing technology to graph a tangent and measure its slope) ***Sample problem:*** The height, h metres, of a ball above the ground can be modelled by the function $h(t) = 5t^2 + 20t$, where t is the time in seconds. Use average speeds to determine the approximate instantaneous speed at $t = 3$.	**1.5, 1.6** **3.2, 3.3, 3.5** **5.5** **6.1**	53–73 157–176 186–191 290–299 310–322
2. Combining Functions		
By the end of this course, students will:		
D2.1 determine, through investigation using graphing technology, key features (e.g., domain, range, maximum/minimum points, number of zeros) of the graphs of functions created by adding, subtracting, multiplying, or dividing functions [e.g., $f(x) = 2^{-x}\sin 4x$, $g(x) = x^2 + 2x$, $h(x) = \frac{\sin x}{\cos x}$], and describe factors that affect these properties ***Sample problem:*** Investigate the effect of the behaviours of $f(x) = \sin x$, $f(x) = \sin 2x$, and $f(x) = \sin 4x$ on the shape of $f(x) = \sin x + \sin 2x + \sin 4x$.	**8.1, 8.2, 8.3, 8.4**	416–460

	Chapter/Section	Pages
D2.2 recognize real-world applications of combinations of functions (e.g., the motion of a damped pendulum can be represented by a function that is the product of a trigonometric function and an exponential function; the frequencies of tones associated with the numbers on a telephone involve the addition of two trigonometric functions), and solve related problems graphically ***Sample problem:*** The rate at which a contaminant leaves a storm sewer and enters a lake depends on two factors: the concentration of the contaminant in the water from the sewer and the rate at which the water leaves the sewer. Both of these factors vary with time. The concentration of the contaminant, in kilograms per cubic metre of water, is given by $c(t) = t^2$, where t is in seconds. The rate at which water leaves the sewer, in cubic metres per second, is given by $w(t) = \frac{1}{t^4 + 10}$. Determine the time at which the contaminant leaves the sewer and enters the lake at the maximum rate.	3.5 8.1, 8.2, 8.5	186–191 416–438 461–471
D2.3 determine, through investigation, and explain some properties (i.e., odd, even, or neither; increasing/decreasing behaviours) of functions formed by adding, subtracting, multiplying, and dividing general functions [e.g., $f(x) + g(x)$, $f(x)g(x)$] ***Sample problem:*** Investigate algebraically, and verify numerically and graphically, whether the product of two functions is even or odd if the two functions are both even or both odd, or if one function is even and the other is odd.	8.1, 8.2	416–438
D2.4 determine the composition of two functions [i.e., $f(g(x))$] numerically (i.e., by using a table of values) and graphically, with technology, for functions represented in a variety of ways (e.g., function machines, graphs, equations), and interpret the composition of two functions in real-world applications ***Sample problem:*** For a car travelling at a constant speed, the distance driven, d kilometres, is represented by $d(t) = 80t$, where t is the time in hours. The cost of gasoline, in dollars, for the drive is represented by $C(d) = 0.09d$. Determine numerically and interpret $C(d(5))$, and describe the relationship represented by $C(d(t))$.	8.1, 8.2, 8.3	416–449
D2.5 determine algebraically the composition of two functions [i.e., $f(g(x))$], verify that $f(g(x))$ is not always equal to $g(f(x))$ [e.g., by determining $f(g(x))$ and $g(f(x))$, given $f(x) = x + 1$ and $g(x) = 2x$], and state the domain [i.e., by defining $f(g(x))$ for those x-values for which $g(x)$ is defined and included in the domain of $f(x)$] and the range of the composition of two functions ***Sample problem:*** Determine $f(g(x))$ and $g(f(x))$, given $f(x) = \cos x$ and $g(x) = 2x + 1$, state the domain and range of $f(g(x))$ and $g(f(x))$, compare $f(g(x))$ with $g(f(x))$ algebraically, and verify numerically and graphically with technology.	8.3, 8.5	439–449 461–471
D2.6 solve problems involving the composition of two functions, including problems arising from real-world applications ***Sample problem:*** The speed of a car, v kilometres per hour, at a time of t hours is represented by $v(t) = 40\ 1\ 3t + t^2$. The rate of gasoline consumption of the car, c litres per kilometre, at a speed of v kilometres per hour is represented by $c(v) = \left(\frac{v}{500} - 0.1\right)^2 + 0.15$. Determine algebraically $c(v(t))$, the rate of gasoline consumption as a function of time. Determine, using technology, the time when the car is running most economically during a four-hour trip.	8.3, 8.5	439–449 461–471

<table>
<tr><th></th><th>Chapter/Section</th><th>Pages</th></tr>
<tr><td>D2.7 demonstrate, by giving examples for functions represented in a variety of ways (e.g., function machines, graphs, equations), the property that the composition of a function and its inverse function maps a number onto itself [i.e., $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$ demonstrate that the inverse function is the reverse process of the original function and that it undoes what the function does]</td><td>8.3, 8.4</td><td>439–460</td></tr>
<tr><td>D2.8 make connections, through investigation using technology, between transformations (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions) of simple functions $f(x)$ [e.g., $f(x) = x^3 + 20$, $f(x) = \sin x$, $f(x) = \log x$] and the composition of these functions with a linear function of the form $g(x) = A(x + B)$
Sample problem: Compare the graph of $f(x) = x^2$ with the graphs of $f(g(x))$ and $g(f(x))$, where $g(x) = 2(x = d)$, for various values of d. Describe the effects of d in terms of transformations of $f(x)$.</td><td>8.1, 8.2, 8.5</td><td>416–439
461–471</td></tr>
<tr><td colspan="3">3. Using Function Models to Solve Problems</td></tr>
<tr><td colspan="3">By the end of this course, students will:</td></tr>
<tr><td>D3.1 compare, through investigation using a variety of tools and strategies (e.g., graphing with technology; comparing algebraic representations; comparing finite differences in tables of values) the characteristics (e.g., key features of the graphs, forms of the equations) of various functions (i.e., polynomial, rational, trigonometric, exponential, logarithmic)</td><td>3.1, 5.1, 6.1, 7.1
8.1, 8.2, 8.3, 8.4</td><td>148–155
252–260
310–322
364–369
416–449</td></tr>
<tr><td>D3.2 solve graphically and numerically equations and inequalities whose solutions are not accessible by standard algebraic techniques
Sample problem: Solve: $2x^2 < 2^x$; $\cos x = x$, with x in radians.</td><td>3.4,
8.4</td><td>177–186
450–460</td></tr>
<tr><td>D3.3 solve problems, using a variety of tools and strategies, including problems arising from real-world applications, by reasoning with functions and by applying concepts and procedures involving functions (e.g., by constructing a function model from data, using the model to determine mathematical results, and interpreting and communicating the results within the context of the problem).
Sample problem: The pressure of a car tire with a slow leak is given in the following table of values:
<table>
<tr><th>Time, t (min)</th><th>Pressure, P (kPa)</th></tr>
<tr><td>0</td><td>400</td></tr>
<tr><td>5</td><td>335</td></tr>
<tr><td>10</td><td>295</td></tr>
<tr><td>15</td><td>255</td></tr>
<tr><td>20</td><td>225</td></tr>
<tr><td>25</td><td>195</td></tr>
<tr><td>30</td><td>170</td></tr>
</table>
Use technology to investigate linear, quadratic and exponential models for the relationship of the tire pressure and time, and describe how well each model fits the data. Use each model to predict the pressure after 60 min. Which model given the most realistic answer?</td><td>1.6
2.3, 2.4
3.5
5.5
6.5
7.5
8.1, 8.3, 8.5, 8.6</td><td>65–73
104–122
186–191
290–299
349–355
393–407
416–428
439–449
461–471</td></tr>
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