Prerequisite Skills

1. a) –19   **b)** 11
  c) $15x - 9$ **d)** $-5m - 14$

2. a) –8   **b)** –3
  c) $-4x^2 - 8x - 3$ **d)** $-2m^2 + 12m - 16$

3. a) $m = -4, b = -1$ **b)** $m = 3, b = \dfrac{2}{3}$

  c) $m = \dfrac{1}{2}, b = 3$ **d)** $m = \dfrac{5}{2}, b = -3$

4. a) $y = \dfrac{2}{5}x - 2$ **b)** $y = \dfrac{1}{3}x - 4$

  c) $y = -x + 1$

5. a) quadratic   **b)** neither

6. a) $\{\, x \in \square \,\}, \{\, y \in \square, y \le -1 \,\}$

  b) $x \in \square, x \le -\dfrac{4}{3} \,\}, \{\, y \in \square, y \ge 0 \,\}$

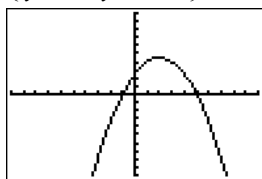**7.** $y = -\dfrac{2}{3}x^2 + \dfrac{2}{3}x + 4$

8. a) x-intercepts 2, 0; vertex (1, –3); opens
    upward; $\{\, x \in \square \,\}; \{\, y \in \square, y \ge -3 \,\}$

  b) x-intercepts – 4, 2; vertex (–1, –9);
    opens upward; $\{\, x \in \square \,\}$,
    $\{\, y \in \square, y \ge -9 \,\}$
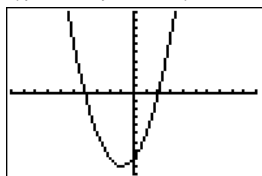
  c) x-intercepts –1, 5; vertex (2, 4.5);
    opens downward; $\{\, x \in \square \,\}$;
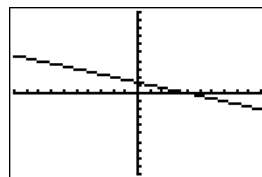 $\{\, y \in \square, y \le 4.5 \,\}$

9. a) vertical stretch by a factor of 2 and a
    translation 5 units to the right
  b) translations 1 unit to the left and 4
    units down

  c) reflection in the x-axis, horizontal

    compression by a factor of $\dfrac{1}{3}$ and a

    translation 1 unit up
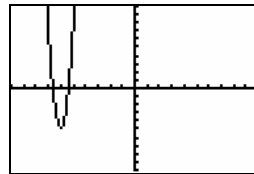
10. a) i) $y = -\dfrac{1}{3}(x - 4)$

  ii)

  iii) $\{\, x \in \square \,\}; \{\, y \in \square \,\}$

  b) $y = 3[2(x + 6)]^2 - 5$

  ii)

  iii) $\{\, x \in \square \,\}; \{\, y \in \square, y \ge -5 \,\}$

11. reflection in the x-axis, horizontal stretch
    by a factor of 2 and translation 8 units to
    the right and 3 units up

1.1 Power Functions

1. a) degree 5; leading coefficient –2
  b) degree 2; leading coefficient 5

  c) degree 4; leading coefficient $\dfrac{1}{3}$

2. quadrant 3 to 1: $y = 3x, y = 4x^5$; reason:
    odd degree with positive leading

    coefficient; quadrant 2 to 4: $y = -\dfrac{1}{6}x^3, y$

    $= -2x^7$; reason: odd degree with negative
    leading coefficient; quadrant 2 to 1: $y =$

    $\dfrac{2}{3}x^6, y = 2x^{10}$ reason: even degree with

    positive leading coefficient; quadrant 3 to
    4: $y = -x^2, y = -3x^4$; reason: even degree
    with negative leading coefficient

3. i) a) exponential function
    b) power function
    c) power function
    d) not a function
    e) none of the functions mentioned
    f) a periodic function

  ii) a) symmetry: none
    b) line symmetry about the y-axis

c) line symmetry about the y-axis
d) symmetry in the x-axis
e) point symmetry about $(-3, 0)$
f) point symmetry about the origin

4. a) Window: $x \in [-4, 4]$, $y \in [-10, 10]$
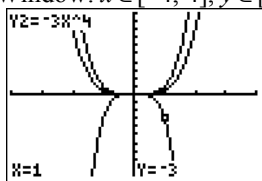
b) $y = x^4$: quadrant 2 to 1, line symmetry about the y-axis ; $y = -3x^4$: quadrant 3 to 4, line symmetry about the y-axis;
$y = \dfrac{1}{2}x^4$: quadrant 2 to 1, line symmetry about the y-axis

5. a)
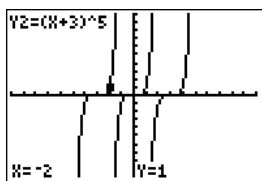
b) similarities: same end behaviour: quadrant 3 to 1; same shape; same domain and range
differences: $y = x^5$: point symmetry about the origin; $y = (x + 3)^5$: point symmetry about $(-3, 0)$; $y = (x - 3)^5$: point symmetry about $(3, 0)$; different x- and y-intercepts

c) They will have the same end behaviour and shape, but different point symmetry.

6. a)
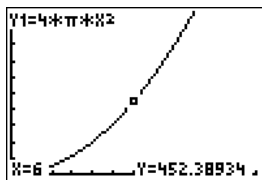
b) $\{ r \in \Box, \ 0 \le r \le 12 \}$;
$\{ S \in \Box, \ 0 \le S \le 576\pi \}$

c) Similarities: end behaviour to the right of the y-axis is in quadrant 1 ; Differences: $S(r)$ is positive only, narrower than $y = x^2$ with no symmetry; $y = x^2$ has positive and negative values with symmetry in the y-axis.

7. a) $y = -0.5(x + 1)^3 - 4$ is compressed vertically by a factor of 0.5, reflected in the x-axis and translated 1 unit to the left and 4 units down ; the shape is the same ; the end behaviour and symmetry differ

b) same transformations and same relationship

c) Window: $x \in [-4, 4]$, $y \in [-10, 10]$
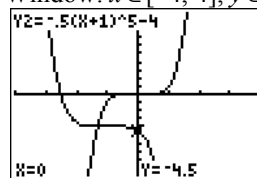
1.2 Characteristics of Polynomial Functions
1. a) least degree 3; one x-intercept, goes from quadrant 3 to quadrant 1
b) least degree 4; two x-intercepts of order 1 and one x-intercept order 2 and 1 local maximum, 2 local minima with a total of 3 which is one less than the degree
c) least degree 4: two x-intercepts but 2 local maxima, 1 local minimum with a total of 3 which is one less than the degree
d) least degree 5: five x-intercepts and 2 local maxima, 2 local minima with a total of 4 which is one less than the degree

2. i) a) positive – quadrant 3 to 1
b) positive – graph opens upward, quadrant 2 to 1
c) negative – graph opens downward, quadrant 3 to 4
d) negative – quadrant 2 to 4
ii) no line symmetry, a) has point symmetry

3. a) i) quadrant 3 to 1   **ii)** third   **iii)** 12
b) i) quadrant 3 to 4   **ii)** second   **iii)** −6
c) i) quadrant 3 to 4   **ii)** fourth   **iii)** −24
d) i) quadrant 3 to 1   **ii)** first   **iii)** 2
e) i) quadrant 3 to 1   **ii)** fifth   **iii)** 120
4. a) 3, 4   **b)** 5, −2   **c)** 2, 6   **d)** 4, −4
5. a) i) quadrant 3 to 4   **ii)** fourth   **iii)** − 48

b)
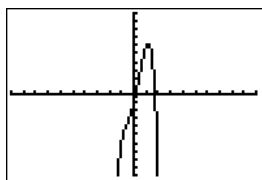
6. a) i) 3 **ii)** 2   **b) i)** 2 **ii)** –6
7. a) third, 6
   b) quadrant 3 to 1
   c) $\{ x \in \square,\ 0 < x < 8 \}$ Window: $x \in [0, 10]$,
      $y \in [0, 300]$, Yscl = 20
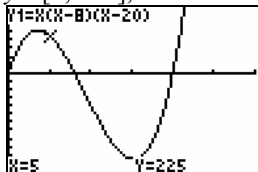
8. Answers may vary.

1.3 Equations and Graphs of Polynomial Functions

1. a) i) 3; negative   **ii)** quadrant 2 to 4
   iii) –2, –1, 3
   b) i) 4; positive   **ii)** quadrant 2 to 1
   iii) 0, 2, 4(order 2)
   c) i) 6; negative   **ii)** quadrant 3 to 4
   iii) –3(order 2), 0, 0.5(order 3)
   d) i) 5; positive   **ii)** quadrant 3 to 1
   iii) –1, 2(order 2), 2.5(order 2)
2. a) i) –1, 2, 4   **ii)** 3, positive
   iii) positive $-1 < x < 2$, $x > 4$; negative
      $x < -1$, $2 < x < 4$
   b) i) –2, 3   **ii)** 4, negative
   iii) negative $x < -2$, $2 < x < 3$, $x > 3$
   c) i) –3, 1.5   **ii)** 2, negative
   iii) positive $-3 < x < 1.5$;
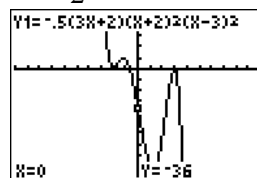   negative: $x < -3$, $x > 1.5$
3. i) a) 0, 2, 3 (all order 1)
   b) –1 (order 3), $\frac{2}{5}$ (order 2)
   c) –2 (order 2)
   d) –3, 3 (both order 2)
   ii) a) neither   **b)** neither
   c) neither   **d)** even
4. i) a) odd   **b)** even   **c)** neither
   d) even   **e)** even   **f)** odd
5. a) $y = (x + 3)(x - 2)^2$
   b) $y = -x(x + 3)(x - 2)(x - 1)$
   c) $y = 0.5(x + 5)(x - 2)$

6. a) $y = -\frac{1}{15}(x + 1)(x - 3)^2(x - 5)$; neither
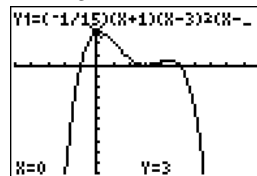
b) $y = -2(x + 3)(2x + 1)(x - 4)$; neither
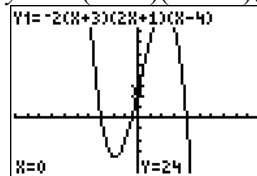
c) $y = -\frac{1}{2}(3x + 2)(x + 2)^2(x - 3)^2$; neither

7. Answers may vary. Sample answers:
   a) $y = -(2x - 1)(x - 3)^2(x - 5)$
   b) $y = (2x - 1)^2(x - 3)(x - 5)^2$
   c) $y = -(2x - 1)(x - 3)(x - 5)$
8. i) a) 3 **b)** –8, –4, 1, 8 **c)** –1, 1, 1.5
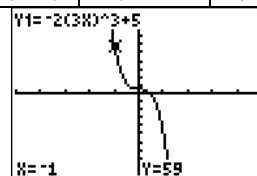**9. a)** $y = x(2x + 7)(2x - 7)$
   c) $y = \frac{1}{3} x(2x + 7)(2x - 7)$

1.4 Transformations

1. a) *a:* vertical stretch by a factor of 2 with
      a reflection in the *x*-axis; *k*: horizontal
      compression by a factor of $\frac{1}{3}$; *c:*
      translation 5 units up

$y = x^3$	$y = (3x)^3$	$y = -2(3x)^3$	$y = -2(3x)^3 + 5$
(−2, −8)	(−2, −216)	(−2, 432)	(−2, 437)
(−1, 1)	(−1, −27)	(−1, 54)	(−1, 59)
(0, 0)	(0, 0)	(0, 0)	(0, 5)
(1, 1)	(1, 27)	(1, −54)	(1, −49)
(2, 8)	(2, 216)	(2, −432)	(2, −427)

2. a) $a = 0.5$: vertical compression by a factor of 0.5;
$k = 1$: no horizontal stretch or compression; $d = -3$: horizontal translation 3 units to the left; $c = -5$: vertical translation 5 units down; $n = 2$

b) $a = 1$: no vertical stretch or compression; $k = 2$: horizontal compression by a factor of $\frac{1}{2}$; $d = 2$: horizontal translation 2 units to the right; $c = 0.5$: vertical translation 0.5 units up; $n = 3$

c) $a = -1$: reflection in the x-axis; $k = 3$: horizontal compression by a factor of $\frac{1}{3}$; $d = 2$: horizontal translation 2 units to the right; $c = -2$: vertical translation 2 units down; $n = 4$

d) $a = 4$: vertical stretch by a factor of 4; $k = -1$: reflection in the y-axis; $d = -6$: horizontal translation 6 units to the left; $c = 1$: vertical translation 1 unit up; $n = 3$

3. i) and b); ii) and d); iii) and c); iv) and a)

4. a) vertical stretch by a factor of 2 with translations 3 units to the right and 4 units down; $y = 2(x - 3)^3 - 4$

b) reflection in the x-axis, horizontal compression by a factor of 0.5 with translations 3 units to the left and 1 unit up; $y = -(2x + 6)^4 + 1$
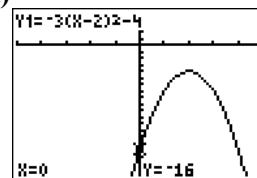
5. a) $a = 0.5$, $k = 3$, $d = 1$, $c = 2$

b) vertical compression by a factor of 0.5, a horizontal compression by a factor of $\frac{1}{3}$ and translations 1 unit to the right and 2 units up

c) $\{x \in \square\}$; $\{y \in \square, y \geq 2\}$; vertex; (1, 2); axis of symmetry: $x = 1$

6. a) i) a vertical stretch by a factor of 3, a reflection in the x-axis and translations 2 units to the right and 4 units down
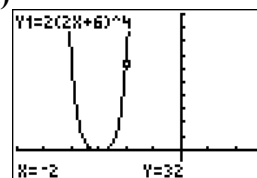**ii)** $y = -3(x - 2)^2 - 4$

iii)

Y1=-3(X-2)2-4
X=0   Y=-16

iv) base: $\{x \in \square\}$; $\{y \in \square, y \geq 0\}$
new: $\{x \in \square\}$; $\{y \in \square, y \leq -4\}$

b) i) a vertical stretch by a factor of 2, a horizontal compression by a factor of 0.5 and a translation 3 units to the left
ii) $y = 2(2x + 6)^4$

iii)
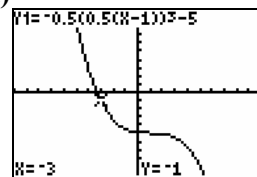
Y1=2(2X+6)^4
X=-2   Y=32

iv) both: $\{x \in \square\}$; $\{y \in \square, y \geq 0\}$

c) i) a vertical compression by a factor of 0.5, a reflection in the x-axis, a horizontal stretch by a factor of 2 and translations 1 unit to the right and 5 units down
ii) $y = -0.5[0.5(x - 1)]^3 - 5$

iii)

Y1=-0.5(0.5(X-1))3-5
X=-3   Y=-1

iv) both: $\{x \in \square\}$; $\{y \in \square\}$

7. a) $y = 0.5(x - 3)^4 + 4$
b) $y = -x^3 + 3$ **c)** $y = [-(x - 3)]^3$

8. a) i) $f(x) = 0.5x^5 - 3$ **ii)** $\{x \in \square\}$; $\{y \in \square\}$

b) i) $y = -2(x - 4)^3 + 1$
ii) $\{x \in \square\}$; $\{y \in \square\}$

c) i) $y = -[\frac{1}{3}(x + 5)]^4$
ii) $\{x \in \square\}$; $\{y \in \square, y \leq 0\}$; (-5, 0); $x = -5$

d) i) $y = 3[-0.5(x - 1)]^4 - 6$
ii) $\{x \in \square\}$; $\{y \in \square, y \geq -6\}$; (1, -6); $x = 1$

9. a) transformed graph is translated 3 units to the left and 1 unit down
**c)** original: 0, -1 ; new: -4.4, -2.2

d) original: $\{x \in \square\}$; $\{y \in \square, y \geq -0.1\}$;
new: $\{x \in \square\}$; $\{y \in \square, y \geq -1.1\}$

1.5 Slopes and Secants and Average Rate of Change

1. a) constant and negative
  b) constant and positive   **c)** zero
2. a) –1   **b)** 2   **c)** 0
3. a) i) 1529.36   **ii)** 1526.28   **iii)** 1522.28
  b) The average rate of change (the slope of the secant line) is positive over each time period, but gradually decreasing.
4. a) i) –24.5 m/s   **ii)** – 49   **iii)** –63.7
  b) velocity
  c) The answers are negative because the ball is moving towards and getting closer to the ground and the ball falls faster as it gets closer to the ground.
5. a) i) 419   **ii)** 88   **iii)** 250   **iv)** 252
  b) Answers will vary.

1.6 Slopes of Tangents and Instantaneous Rate of Change

1. a) $(2, -8)$ **b)** $(6, 0)$ **c)** $m = 2$ **d)** $m = -4$
  e) Slopes of tangent lines drawn from any point on this curve will be positive to the right of the minimum, negative to the left of the minimum and zero at the minimum.
2. a) $\dfrac{\Delta h}{\Delta t}$: $-14.7, -14.25, -14.2$ **b)** -14.2
3. a) 2304 bacteria per hour
  b) 6912 bacteria per hour
4. a) 10 m/s   **b)** -2 m/s
  c) Window: $x \in [0, 5]$, $y \in [-20, 50]$, Yscl = 5
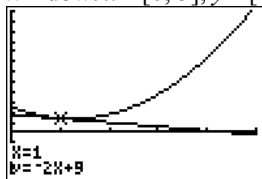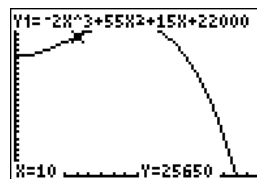

  d) The average rate of change is the slope of the secant line drawn from $t = 1$ to $t = 4$ which is positive and represents the average velocity between those time values. The instantaneous rate of change is the slope of the tangent line drawn at $t = 1$ which is negative and represents the instantaneous velocity after 1 s.

5. a) $\dfrac{f(10+h) - f(10)}{h} = -2h^2 - 5h + 515$
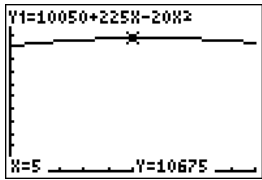 **b) i)** 482 **ii)** 440   **c)** 515
  d)

  e) increased population, 25 650, so store justified
 **f)** decreased population, 17 950, store probably not justified

Chapter 1 Review

1. a) degree: 3; leading coefficient: 2
  b) degree: 1; leading coefficient: 5
  c) degree: 4; leading coefficient: –5
  d) degree: 5; leading coefficient: –3
  e) degree: 3; leading coefficient: –6
2. a) i) even degree   **ii)** negative
    iii) $\{x \in \square\}$; $\{y \in \square, y \leq 15\}$
    iv) no line or point symmetry
    v) quadrant 3 to 4
  b) i) odd degree   **ii)** positive
    iii) $\{x \in \square\}$; $\{y \in \square\}$
    iv) point symmetry about $(0, 0)$
    v) quadrant 3 to 1
3. $y = 3x^7$: quadrant 3 to 1, odd degree with positive leading coefficient; $y = -\dfrac{1}{2}x^3$: quadrant 2 to 4, odd degree with negative leading coefficient; $y = 2x^4$: quadrant 2 to 1, even degree with positive leading coefficient; $y = -0.25x^6$: quadrant 3 to 4, even degree with negative leading coefficient
4. i) and b); ii) and a)
5. a) i) fourth **ii)** fifth
  b) i) – 48 **ii)** –120
6. a) $3, -\dfrac{2}{3}$ **b)** 1, 6   **c)** 6, –1
  d) 4, 4   **e)** 2, –6
7. a) fourth   **b)** positive   **c)** 1
8. a) i) –1, 0, 4; $x(x + 1)(x - 4)$
    ii) 3, negative
    iii) positive $x < -1$, $0 < x < 4$; negative $-1 < x < 0$, $x > 4$

b) i) $-3, -2, 3; (x + 3)(x + 2)(x - 3)^2$
ii) 4, positive
iii) positive $x < -3, -2 < x < 3, x > 3$;
negative $-3 < x < -2$

10. $y = -\dfrac{1}{3}(x + 4)(5x + 2)(x - 3)$

11. a) point symmetry about the origin
b) neither
c) line symmetry about the y-axis

12. a) i) a vertical stretch by a factor of 2, a
reflection in the x-axis and translations
1 unit to the right and 4 units up;
$y = -2(x - 1)^4 + 4$
ii) $\{x \in \Box\}$, $\{y \in \Box, y \le 4\}$
b) i) a vertical compression by a factor of
$\dfrac{1}{3}$, a horizontal compression by a
factor of $\dfrac{1}{2}$ and translations 3 units to
the left and
5 units down; $y = \dfrac{1}{3}(2x + 6)^3 - 5$
ii) $\{x \in \Box\}$, $\{y \in \Box\}$

13. a) $y = 5[4(x + 2)]^5 - 1$
b) $y = 0.5[-(x - 4)]^6 + 3$

14. a) i) 125 **ii)** -35 **iii)** -135
b) The population grew in the first 5
years towards its maximum; then it
began to decrease in the next 3 years
and decreased considerably in the
following 2 years.
c) Window: $x \in [0, 10], y \in [0, 13\ 000]$,
Yscl $= 1000$

15. a) \$999.30   **b)** \$999.00
c) The slope of the secant line from
$x = 20$ to $x = 50$ approaches the slope
of the value of the tangent line at
$x = 50$, i.e. the average rate of change
from $x = 20$ to $x = 50$ is quite close in
value to the instantaneous rate of
change at $x = 50$.

Chapter 1 Test
1. B

2. D
3. i) and **c):** odd degree, negative leading
coefficient; **ii)** and **b):** even degree,
positive leading coefficient; **iii)** and **a):**
odd degree, positive leading coefficient
4. i) fifth; -120 **ii)** fourth; 24 **iii)** third; 6
5. a) $y = a(x + 4)(x + 2)(x - 1)^2$, $a \ne 0$
b) $y = -\dfrac{3}{4}(x + 4)(x + 2)(x - 1)^2$
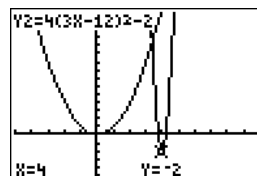**c)**
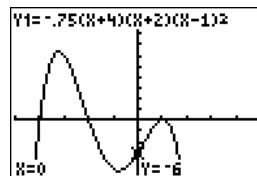
positive $-4 < x < -2$;
negative $x < -4, -2 < x < -1, x > 1$
6. a) $a = 4$: a vertical stretch by a factor of 4;
$k = 3$: a horizontal compression by a
factor of $\dfrac{1}{3}$; $d = 4$: a horizontal
translation
4 units to the right; $c = -2$: a vertical
translation 2 units down
b) $\{x \in \Box\}$; $\{y \in \Box, y \ge -2\}$; vertex: $(4, -2)$; axis of symmetry: $x = 4$
c)

7. $y = -(x + 2)^4 + 4$
8. a) cubic function with a positive leading
coefficient ; end behaviour: quadrant 3
to 1; no point or line symmetry;
x-intercept -16.4; y-intercept 25 000;
no local maximum or minimum points
b) 25 000
c) 36
d) $t \ge 0$, $P(t) \ge 0$
e) 25.3 years
9. a) -20.3 m/s   **b)** -32.5 m/s
c) Both values are negative because the
stone is moving downward towards
the ground.